NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for
MA0301 Elementær diskret matematikk

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Examination date: 26-05-2016

Examination time (from–to): 09:00–13:00

Permitted examination support material: D: No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Date Signature

Problem 1

a) How many non-negative integer solutions does the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$

have?

b) How many of these solutions satisfy the additional requirements $x_2 \geq 2$ and $x_3 \geq 3$?

c) How many of the solutions to (a) satisfy $x_4 \leq 4$ as an additional requirement?

Problem 2

a) Determine whether or not the following statement is a tautology, by either proving it or giving a counterexample:

$$\Big((p \to q) \wedge (r \to \neg q)\Big) \to (p \wedge r)$$

b) Negate the statement given in (a). (In your final answer, the $\neg$ symbol may only appear directly in front of p, q and/or r.)

c) Establish the validity of the following argument using the rules of inference

$$\begin{array}{l} p \to \neg q \\ \neg r \vee q \\ \underline{r} \\ \therefore\ \neg p \end{array}$$

Problem 3 Let $r \in \mathbb{R}$ with $r \neq 1$. Use induction to prove that

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}$$

for all $n \in \mathbb{Z}^+$.

Problem 4

a) Give 6 different strings in the language $\{00\}\{101\}^* \cup \{011\}\{0\}^*\{01\}$

b) Construct a finite state machine that recognises this language (with $\{0, 1\}$ as input and output alphabet).

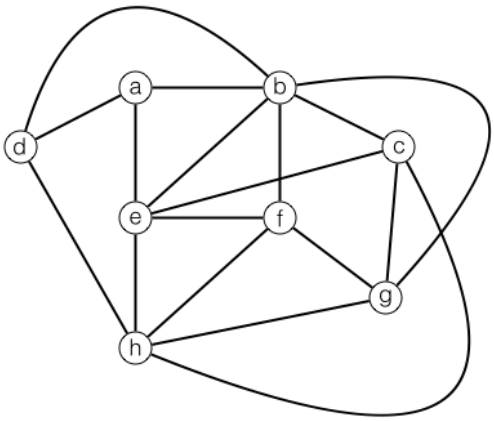

Figure 1: This figure belongs to Oppgave 5

Problem 5

a) Find a rooted spanning tree with root a for this graph using a Breadth First Search Algorithm and vertex order a,b,c,d,e,f,g,h.

b) Study the graph in Figure 1 and determine whether or not is it planar. Prove your answer.

c) Does the graph in Figure 1 have an Euler Trail? Prove your answer.

Problem 6 Let A be the set of all functions from $\mathbb{Z}^+$ to $\{1, 2, 3\}$.

a) Give the three properties of an equivalence relation. You may give the names of these properties, or their definitions.

b) Define a relation $\mathcal{R}_1$ on A by setting $f\mathcal{R}_1 g$ if and only if $f(5) = g(5)$. Prove that $\mathcal{R}_1$ is an equivalence relation.

c) Define a relation $\mathcal{R}_2$ on A by setting $f\mathcal{R}_2 g$ if and only if there exists an $n \in \mathbb{Z}^+$ such that $f(n) = g(n)$. Prove that $\mathcal{R}_2$ is not an equivalence relation.