NTNU
Institutt for datateknikk
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Examination paper for TDT4120 Algorithms and Data Structures

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Examination date	Aug 12, 2014
Examination time (from–to)	0900–1300
Permitted examination support material	D. No printed/handwritten materials permitted. Specific, simple calculator permitted.
Language	English
Number of pages	4
Number of pages enclosed	0
Checked by	Ole Edsberg

See instructions on page 4.

[05] 1. If $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$, then we know that ______________

[05] 2. Solve the recurrence $T(n) = 3T(n/3) + 3$: $T(n) = \Theta($______________$)$

[05] 3. Solve the recurrence $T(n) = T(\sqrt{n}) + \lg n$: $T(n) = \Theta($______________$)$

[05] 4. ______________ can sort n numbers with d digits in the range $1 \ldots k$ with a running time of $\Theta(d(n+k))$

[05] 5. Search in a linked list with n elements has a running time of ______________

[05] 6. A seemingly random hash function is meant to minimize ______________

[05] 7. If ______________ persons shake hands there are 5050 handshakes in total

[05] 8. Dijkstra's algorithm is correct if the graph has ______________

[05] 9. Dynamic programming increases performance when we have ______________

[05] 10. If we increase the capacity of each individual edge in a flow network by k, will the value of the maximum flow always increase by a multiple of k? Explain briefly.

[07] 11. You have an empty binary heap. Insert the following values in order, one by one:

[2, 4, 5, 9, 1, 10, 7, 3, 8, 6]

Then perform Heap-Extract-Max twice on the heap. Assume that you represent the heap as a table. How does it look now?

[07] 12. What does the following algorithm do?

for $\alpha = 1 \ldots \omega$
    for $\beta = 1 \ldots \omega$
        for $\gamma = 1 \ldots \omega$
            $\pi_{\beta\gamma} = \max(\pi_{\beta\gamma}, \pi_{\beta\alpha} \cdot \pi_{\alpha\gamma})$

[08] 13. Assume that you have n geographic regions, where many of the regions overlap, and where each region is defined by a set of $\mathcal{O}(1)$ points. Assume that you have a procedure for combining two regions with k and ℓ points respectively, forming a new region that is the union of the two, defined by at most $k+\ell$ points, adn that this procedure has a running time of $\mathcal{O}(k+\ell)$.

You wish to combine all the regions. How would you do this? What is the running time?

[08] 14. In a graph, you wish to start in a node u and end up in a different node v, and visit each of the other nodes exactly once on the way. Briefly show that this is NP-hard.

[10] 15. Two words are anagrams of each other if they consist of the same characters, but in different orders. Each character must occur the same number of times, and we define a word to be an anagram of itself. Assume that you are given a text, a start-word A and an end-word B. You wish to construct a sequence of words starting with A and ending with B, with the following requirement: If two words X and Y are next to each other in this sequence, an anagram for X should occur in the same sentence as an anagram for Y. Describe briefly an algorithm that decides if such a sequence exists.

[10] 16. A group of knights want to decide who is the best fencer among them. Each knight will participate in k fencing matches with each of the others, and the one who wins the most matches in total is the winner. At some point after a few of the matches have been held, Lady Lurwicke is wondering if she has a chance to win, or at at least tie for first place. Describe an algorithm that lets her decide this.

Hint: How many matches can Lady Lurwicke win in the best case? Is there a possible scenario where no one else wins mor matches in total?

Some important points:

(*i*) Please read the entire exam thoroughly before you start. Plan your time and prepare any questions before the teacher arrives at the exam location.

(*ii*) Feel free to use a pencil! Or perhaps write a separate draft to avoid strikouts.

(*iii*) Make assumptions where necessary. Write your answers on the exam itself, as indicated, either on lines where words are missing from the problem, or in answer boxes below the problem text. Keep your answers short and concise, if possible. Long explanations that do not directly answer the questions are given little or no consideration.

(*iv*) Unless something else is stated, you may describe your algorithms in prose, pseudocode or program code, depending on your preferences, as long as it is clear how the algorithms works. Short, abstract explanations can be just as good as detailed pseudocode, as long as they are clear and precise. In most cases, such a brief answer will require less work and have fewer sources of error.

(*v*) Your algorithms should be as efficient as possible, unless otherwise stated. Running times are given in asymptotic notation, as precisely as possible.

The exam has 16 problems, for a total of 100 points. The point values are listed next to each problem.