TDT4120 Algorithms and Data Structures

Examination, August 7, 2019, 09:00–13:00

Academic kontakt Magnus Lie Hetland

Support material code D

Problems

6 % **1** Your friend Klokland claims to have found a top secret algorithm, where parts of the pseudocode is redacted (see below). You immediately recognize it as an algorithm from the curriculum. Which one?

```
[redacted](A, p, r)
1  x = A[r]
2  i = p − 1
3  for j = p to r − 1
4       if A[j] ≤ x
5            i = i + 1
6            [redacted] A[i] [redacted] A[j]
7  [redacted] A[i + 1] [redacted] A[r]
8  [redacted] i + 1
```

6 % **2** Klokland has also found a redacted description of *parts* of another algorithm, which you recognize as MST-Prim (see below). How is line 7 supposed to look, unredacted? (Here a brief explanation of what the line is supposed to do will suffice, if you do not remember the exact notation from the curriculum.)

```
 6  while Q ≠ ∅
 7       u = [redacted]
 8       for each v ∈ G.Adj[u]
 9            if v ∈ Q and w(u, v) < v.key
10                 v.π = u
11                 v.key = w(u, v)
```

6 % **3** Klokland scrounges up a redacted description of parts of *yet another* algorithm, which you recognize as Floyd-Warshall (see below). How is line 7 supposed to look, unredacted? (Here a brief explanation of what the line is supposed to do will suffice, if you do not remember the exact notation from the curriculum.)

```
5       for i = 1 to n
6            for j = 1 to n
7                 d_ij^(k) = [redacted]
```

6 % **4** Klokland has a last redacted algorithm fragment, this time the end of counting sort (CountingSort, see below). What does it mean that counting sort is *stable*? What is it one must ensure in line 10 in order for the algorithm to be stable? Explain briefly.

```
10  for [redacted]
11      B[C[A[j]]] = A[j]
12      C[A[j]] = C[A[j]] − 1
```

6 % **5** Give a very brief explanation of how binary search (Bisect) works. What is the running time? (You need not use code or pseudocode. A brief textual explanation will suffice.)

5 % **6** Let W, below, be the weight matrix of an undirected graph G.

W

	1	2	3	4	5
1	0	4	5	∞	1
2	4	0	∞	2	4
3	5	∞	0	7	8
4	∞	2	7	0	3
5	1	4	8	3	0

Execute Kruskal's algorithm (MST-Kruskal) on the graph G. List the edges in the order in which they are added to the solution. (If $i < j$, give the edge between i and j as (i, j). Write one edge per line in your answer.)

6 % **7** Let's say you were to solve the recurrence $T(n) = 3T(n/4) + n$ using the method of repeated substitution (*iterasjonsmetoden*), where you repeatedly expand the recursive part, with each expansion on a separate line. How many lines would you need? (Give your answer in Θ notation. Note: You are *not* supposed to solve the recurrence.)

6 % **8** You sort n numbers by inserting them (in random order) into a binary search tree, and then running Inorder-Tree-Walk. What is the expected total running time for insertion and subsequent traversal? Briefly explain how you arrive at this answer.

6 % **9** You are to select video channels for the front page of a video website. You have a set of channels that each has a target audience, given as an estmiated age range (e.g., *from 10 to 25 years*). You are to select as many as possible, but are not permitted to select any whose age ranges overlap. How would you proceed? Explain briefly.

6 % **10** Your friends Lurvik and Smartnes are discussing the relationship between abstract decision problems, concrete decision problems and formal languages. Both agree that decision problems may be represented as formal languages, but while Lurvik thinks that we then call them abstract, Smartnes claims that this is what we refer to as concrete problems. Who is correct?

6 % **11** If the edge (u, v) in a flow network has a flow of 4 and a capacity of 11, what edges will exist between u and v in the residual network, and what capacities will they have?

6 % **12** Your friend Klokland has designed the hash function $h : \mathrm{U} \to \{0, \ldots, m-1\}$ for use in the hash table $\mathrm{T}[0\,..\,m-1]$, where the universe U is all positive integers:

$$h(k) = \min\big(\lfloor m(k\mathrm{A} \bmod 1)\rfloor + 2^k, m\big) - 1$$

Here $\mathrm{A} \approx \big(\sqrt{5}-1\big)/2$. Is h a good hash function? Discuss briefly.

7 % **13** Your friend Gløgsund is pondering the shortest path problem and negative cycles, and has managed to confuse herself profoundly. Is the problem NP hard or is it just meaningless? And what role do simple paths play in all of this? Give a brief summary.

7 % **14** Your friend Gløgsund thinks she has found a verification algorithm with polynomial running time for the complement of HAM-CYCLE. If she is correct, what consequences would this have for the relationship between the classes P, NP and co-NP? Explain.

7 % **15** You are to solve a problem in two phases. In the first phase you are given a sequence of m equations of the type $x_i = x_j$, where i and j are integers in the range $1, \ldots, n$. In the second phase, you are given a sequence of inequation of the type $x_i \neq x_j$, and you are to determine whether these could be true, given the first sequence of equations. Assume that you wish to process both the equations and the inequations as efficiently as possible, in the worst case. How would you proceed? How would you proceed if you were permitted to use arbitrarily long time in the first hase (i.e., treat the equations as a *static dataset*)?

7 % **16** You are given two sequences $\mathrm{X} = \langle x_1, \ldots, x_m\rangle$ and $\mathrm{Y} = \langle y_1, \ldots, y_n\rangle$ and wish to construct two sequences $\mathrm{A} = \langle a_1, \ldots, a_k\rangle$ and $\mathrm{B} = \langle b_1, \ldots, b_k\rangle$ that are as long as possible (i.e., where k is as large as possible), and where the following holds:

1. $a_i \in \{1, \ldots, m\}$ og $b_i \in \{1, \ldots, n\}$ (for $i = 1, \ldots, k$)
2. $a_i < a_{i+1}$ og $b_i < b_{i+1}$ (for $i = 1, \ldots, k-1$)
3. $x_{a_i} < y_{b_i}$ (for $i = 1, \ldots, k$)

Describe an algorithm that solves the problem. What is the running time?

(A major part of the problem is to understand the problem text.)