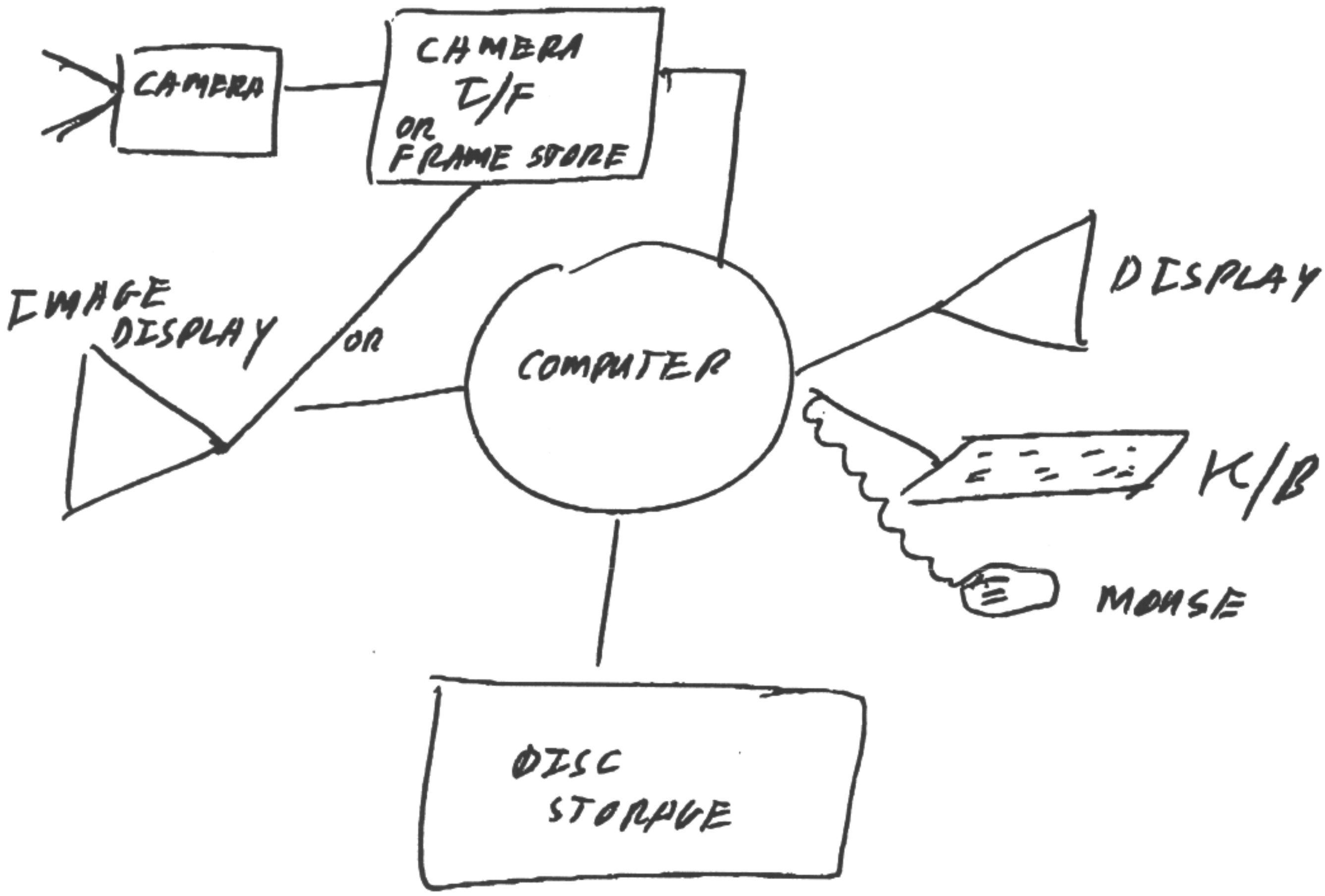
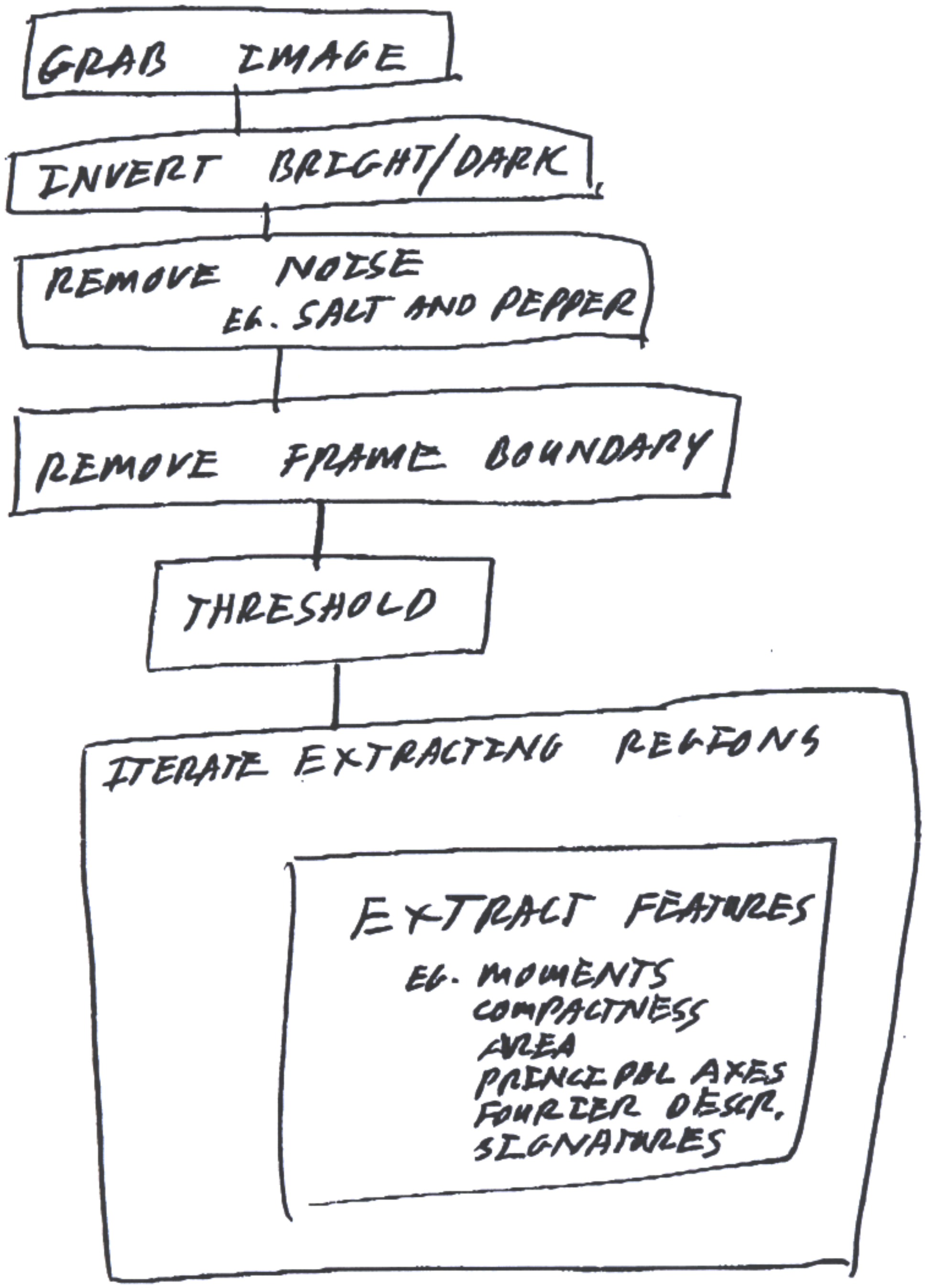
Solutions to image processing questions,
SIF8043, May15th. 2003.

Qn 4.

a. Draw a labelled block diagram showing the organisation of the hardware of an image processing workstation



b. Draw a diagram that shows a sequence of steps needed to extract information for the recognition of shapes of objects that are dark on a light background.



c. A colour image that is 256*256 pixels, and has three planes of byte sized pixels, is to be transmitted along a parallel data path that is 8 bits wide. If the maximum capacity of the path is 100k bytes per second, what is the minimum time to transfer the image?

The image contains 3 planes, each with 65536 bytes. The total size of the image is 196608 bytes

Each pixel can be transmitted as a one byte parallel transfer on the data path. Ths 196608/100000 is the minimum number of seconds for the transfer, ie 1.96608 seconds

d. what numerical data type is required for Fourier transform calculations?

Floating point values are required to preserve accuracy due to the large numerical range involved in Fourier transform calculations.

Qn 5.

a. Give a definition of the one dimensional discrete Fourier transform of a function f(x) taken over N points, x=0 to x=N-1.

The one dimensional discrete Fourier transform of f(x) is F(u) where

$$F(u) = \sum_{x=0}^{N} f(x) \exp(2 \pi j u x / N)$$

b. Prove that the discrete Fourier transform of a function, f(x), taken over n points, x=0 to x=N-1, is periodic with period N.

The discrete Fourier transform of f(x) over N points, x=0 to x=N is defined by

$$F(u) = \sum_{x=0}^{N} f(x) \exp(2 \pi j u x / N)$$

To prove periodicity of the transform we need to show that F(u)=F(u+N).

From the definition, F(u+N) is given by

$$F(u+N) = \sum_{x=0}^{N} f(x) \exp(2 \pi j (u+N) x / N)$$

Rearranging the exp function as the product of two exp functions by separating (u+N)

$$F(u+N) = \sum_{x=0}^{N} f(x) \exp(2 \pi j u x / N) \exp(2 \pi j N x / N)$$

or

$$F(u+N) = \sum_{x=0}^{N} f(x) \exp(2 \pi j u x / N) \exp(2 \pi j x)$$

Consider the term exp(2 π j x). x is an integer and for any integer, x, exp(2 π j x)= 1. The expression for F(u+N) can be simplified to

$$F(u+N) = \sum_{x=0}^{N} f(x) \exp(2 \pi j u x / N)$$

The RHS is exactly the definition of F(u), so

$$F(u+N) = F(u)$$

c. A two dimensional image function, f(x,y) x=0 to x=127 and y=0 to y=127 is given by

f(x,y) = 100 + 50sin(PIx/8) cos(PIy/32)

What will be the coordinates of the peaks in the power spectrum when mod F(u,v) in plotted in u, v space?

TWO SOLUTIONS ARE POSSIBLE HERE BECAUSE OF A MISPRINT ON THE QUESTION PAPER.

INTENDED SOLUTION:

Sinusoidal variations sin(PIx/8) cos(PIy/32) should be expressed as

sin(2*8*PIx/128) cos(2*2*PIy/128)

so the arguments of the functions are 2*PI*u*x/N and 2*PI*v*y/N

The frequencies u and v can then be read off as u=8 and v=2.

The spectral peaks are found at <±u, ±v>. Thus the spectral peaks for this example occur at <±8, ±2>.

SOLUTION DUE TO THE MISPRINT:

Sinusoidal variations sin(PIx/8) cos(PIx/32) should be expressed as

sin(2*8*PIx/128) cos(2*2*PIx/128)

so the arguments of the functions are 2*PI*u*x/N.
Due to the misprint there is no y dependancy. The effect is that the x dependancy appears for all v. That is, lines appear in the spectrum which is much weaker than the points for the correct version

The frequency values for u can then be read off as u=8 and u=2.

The spectral peaks are lines at u=±8 and u=±2.

d. State the sampling theorem.

The sampling theorem states that if a signal f(x) is band limited with a band limit W, then f(x) can be exactly recovered so long as it is sampled at an interval in x, Δx, that is no more than 1/(2w)

ie. $\Delta x <= 1/(2W)$

e. Give a mathematical statement of the convolution theorem.

The convolution theorem states that if f and g are functions for which F and G are the corresponding Fourier transforms. Thus

$f \longleftrightarrow F$ and $g \longleftrightarrow G$.

If f*g is the convolution of f and g then $f*g \longleftrightarrow FG$.

That is, the convolution of f and g is the transform pair with the product of F and G.

Qn 6.

a. Define the morphological operations of erosion and dilation, and opening and closing in black-and-white images.

In the definitions below, A is the image and B is the structuring element.
Reflection of A is represented by A^R
Translation of A by x is represented by A_x

Erosion $A \ominus B = \{ x \mid B_x \subseteq A \}$

Dilation $A \oplus B = \{ x \mid B^R{}_x \cap A \neq \phi \}$

Opening $A \circ B = (A \ominus B) \oplus B$

Closing $A \cdot B = (A \oplus B) \ominus B$

b. State two methods of emphasising edges in an image.

Pick two from:

Sobel or other convolution mask that models differencing,
Laplacian of Gaussian (Mexican Hat)
Difference of Gaussian
High frequency emphasis
Vector connecting geometric centre with intensity centre of gravity
Graph search with suitable cost function
Snakes/Active contours
Radon transform with suitable delta function (not very convincing as Hough is usually applied to edge enhanced image)
Shift whole image one pixel and subtract from original
Morphological gradient

c. What is a 4-connected region?

A 4-connected region is a set of pixels such that each pixel of the region has at least one 4-neighbour that also belongs to the region. As a consequence it is possible to make a journey from any pixel of the region to any other pixel, staying in the region for each step of the journey and making only steps from a pixel to one of its 4-neighbours.

d. Draw a diagram to show the decomposition of an image as a quad-tree and indicate a systematic labelling of the components.

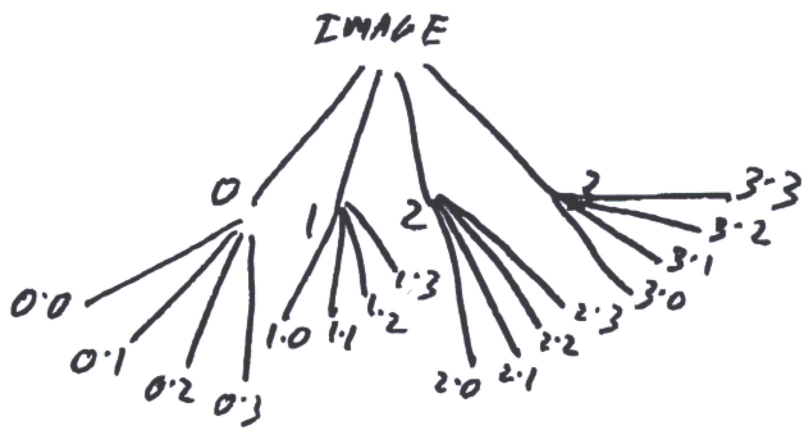
Labelled diagram to show decomposition as a quad-tree

Number subregions as:

0	1
3	2

Successive levels use a . notation, right hand end of label is more deeply nested.

0·0	0·1	1·0	1·1
0·3	0·2	1·3	1·2
3·0	3·1	2·0	2·1
3·3	3·2	2·3	2·2



e. Write pseudo-code for a procedure that extracts regions by the 'split and merge' method

```
Set region list to empty;
REPEAT
   nochange:=TRUE;

   (* consider merge *)
   For each region label L where L.0, L.1, L.2, L.3
      are in the list DO
   BEGIN
      IF uniform(L.0 union L.1 union L.2 union
                  L.3)
      THEN BEGIN
               delete L.0; delete L.1;
               delete L.2; delete L.3;
               append L;
               nochange:=FALSE;
           END;
   END;

   (* consider split *)
   For each region label L where L is in the list DO
   BEGIN
      IF NOT uniform(L)
      THEN BEGIN
               delete L;
               append L.0; append L.1;
               append L.2; append L.3;
               nochange:=FALSE;
           END;
   END;

UNTIL nochange;
```

Qn 7

a. Define the Fourier descriptor of a shape.

The Fourier descriptor of a shape is the Fourier transform of the sequence that is obtained by treating coordinates of the boundary pixels as complex numbers and following one circuit of the contour. It is conventional to normalise the contour length so that it represents a angular range of 2PI, and also to use m <= N coefficients from the transform over N points.

b. To what extent do regions in an image represent geometrical surfaces of an object?

Regions in an image are sets of adjacent pixels that share some property, typically a brightness within a small range. Surfaces of an object will become regions in an image when the illumination and reflectance (including colour) of the surfaces happen to produce pixels that same similar intensity values. This will not occur where there are changes in the surface marking even though geometrically the structure would still be described as a surface.

Edges, which in geometrical terms are discontinuities in the surface normal direction, will tend to lead to abrupt changes in the intensity of light reflected into the camera. This may not occur at every edge, and so boundaries between geometric surfaces may not appear as changes in region in the image.

Another sort of edge is created by a view of two parallel surfaces that present a discontinuity of viewing distance. These may not be separable in the reflected light image.

c. Draw a detailed diagram that shows the structure of a neural net and sketch a typical activation function.