

1. Theme: Bezier Curves (25 points)

- a) Given the three control points \mathbf{p}_0 , \mathbf{p}_1 and \mathbf{p}_2 . Show how to derive the quadric Bezier curve $\mathbf{P}^2(t)$ using the three control points. Show the intermediate steps. **(6 points)**
- b) Describe the de Casteljau algorithm in your own words and provide a figure that illustrates your description. **(7 points)**
- c) Write pseudo code for the de Casteljau algorithm. **(7 points)**
- d) What is the convex hull property? Is this a property of Bezier curves? **(3 points)**
- e) Can every nth-degree polynomial curve be written in the form of a Bezier curve? **(2 point)**

2. Theme: Illumination (25 points)

- a) Describe the Phong illumination model. Describe all components and how they are computed. **(10 points)**
- b) Given a polygon (triangle) comprised of the three vertices \mathbf{v}_{i-1} , \mathbf{v}_i and \mathbf{v}_{i+1} , provide a formula for computing the normal vector \mathbf{n} of the triangle. **(5 points)**
- c) Describe how to derive the intensity $I(x_s, y_s)$ for a pixel centered at x_s, y_s given the three vertices \mathbf{v}_1 , \mathbf{v}_2 and \mathbf{v}_3 using Gouraud shading where $\mathbf{v}_1 = (x_1, y_1)$, $\mathbf{v}_2 = (x_2, y_2)$ and $\mathbf{v}_3 = (x_3, y_3)$. **(7 points)**
- d) Use the formulas derived in c) to compute $I(2.5, 2.5)$, given $\mathbf{v}_1 = (1.0, 2.0)$, $\mathbf{v}_2 = (4.0, 1.0)$, and $\mathbf{v}_3 = (3, 4)$ where $I(\mathbf{v}_1)=2.0$, $I(\mathbf{v}_2)=3.0$ and $I(\mathbf{v}_3)= 4.0$ **(3 points)**

3. Theme: Texturing (25 points)

- a) Describe environment mapping. **(7 points)**
- b) What is a view-dependent texture map? Discuss its pros and cons. **(6 points)**
- c) What is mip-mapping and what problem does it solve? **(6 points)**
- d) What are procedural textures? Give examples (with formulas) for two common 3D procedural textures of your choosing. **(6 points)**

4. Theme: Ray Tracing (25 points)

- a) Explain the principles of ray tracing. **(8 points)**
- b) The 2D scene illustrated in figure 1 has two light sources, a partially transparent and reflective object and a mirror object. Draw all rays needed to compute the color of the eye ray. Note the angle of the eye ray. **(4 points)**
- c) Write pseudo code for the data structures required for ray tracing. **(5 points)**
- d) Write pseudo code for the recursive ray-tracing algorithm. **(8 points)**

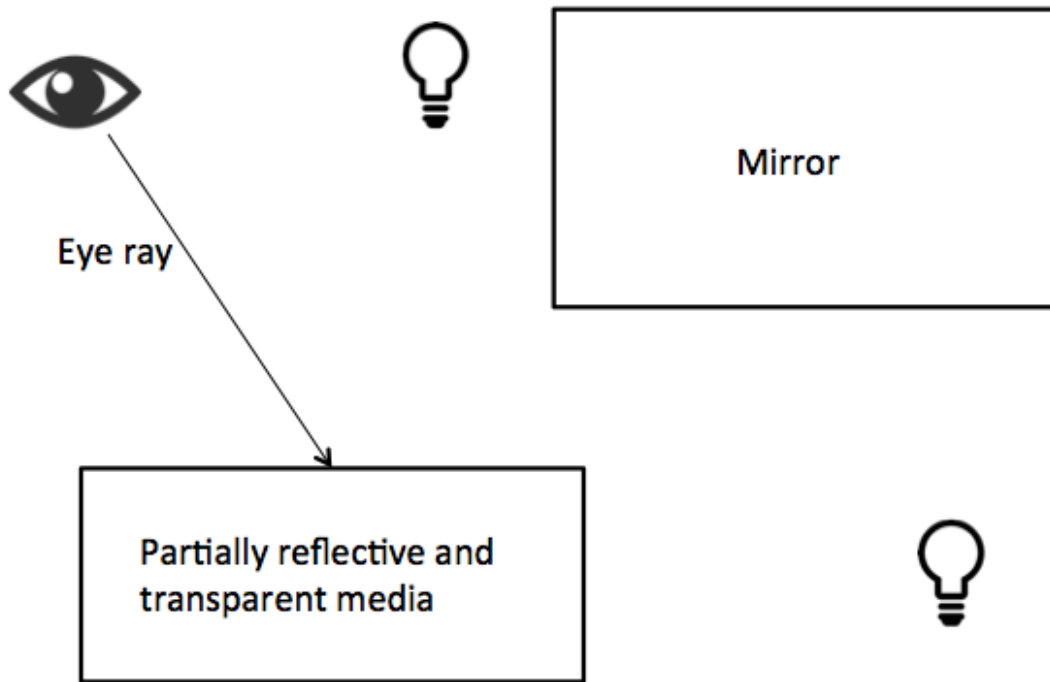


Figure 1: Draw all rays needed to compute the color of the eye ray.