LF eksamen 3/12 2010 TKT4126 Mekanikk

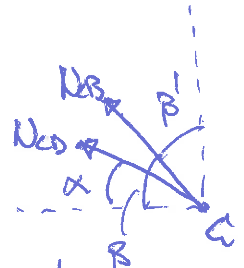
Oppg. 1 a) $K=4,\ s=5$ og $r=3 \Rightarrow 2K = s+r \Rightarrow$ Statisk bestemt.

Reaksjoner:

$\sum M_A = 0: \quad C_y \cdot 2L - FL = 0 \Rightarrow C_y = \frac{F}{2} = \underline{2kN}$

$\sum F_x = 0: \quad \underline{A_x = F = 4kN}, \quad \sum F_y = 0 \Rightarrow \underline{A_y = C_y}$

b) Stavkrefter:



$\alpha = 30, \quad \beta' = 45$

$\beta = 90 - 45 = 45$

Knutepkt C:

$\sum F_x = 0: \quad N_{CD}\cos\alpha + N_{CB}\cos\beta = 0$

$\Rightarrow \underline{N_{CD} = -\frac{N_{CB}\,2}{\sqrt{2}\sqrt{3}}} \Leftarrow \cos\alpha = \frac{\sqrt{3}}{2},\ \cos\beta = \frac{1}{\sqrt{2}}$

$\sum F_y = 0:$

$C_y + N_{CD}\sin\alpha + N_{CB}\sin\beta = 0$

m/ $C_y = F/2$ og $N_{CD} = -\frac{N_{CB}\,2}{\sqrt{2}\sqrt{3}}$, $\sin\alpha = \frac{1}{2}$, $\sin\beta = \frac{1}{\sqrt{2}}$

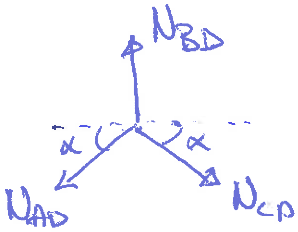
$\Rightarrow \frac{F}{2} + N_{CB}\left(\frac{1}{\sqrt{2}} - \frac{1}{\cancel{2}}\frac{\cancel{2}}{\sqrt{2}\sqrt{3}}\right) = 0$

$\Rightarrow N_{CB}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\right) = -\frac{F}{2}$

$N_{CB} = \frac{-\sqrt{6}}{\sqrt{3}-1}\cdot\frac{F}{2} = \underline{-6.7\,kN}$

$N_{CD} = \frac{-N_{CB}\,2}{\sqrt{6}} = \frac{F}{2}\frac{\sqrt{6}}{\sqrt{3}-1}\cdot\frac{2}{\sqrt{6}} = \frac{F}{\sqrt{3}-1} = \underline{5{,}46\,kN}$

Kp. D:



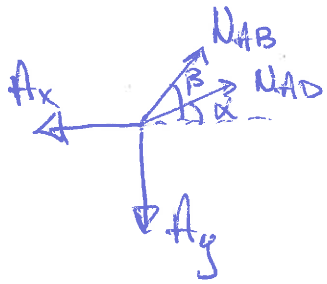
$\Sigma F_x = 0: \quad N_{AD}\cos\alpha = N_{CD}\cos\alpha$

$$\Rightarrow N_{AD} = N_{CD} = \frac{F}{\sqrt{3}-1} = \underline{5{,}46\,kN}$$

$\Sigma F_y = 0:$

$$N_{BD} = 2\cdot N_{AD}\cdot \sin\alpha = N_{AD} = N_{CD} = \frac{F}{\sqrt{3}-1} = \underline{5.46\,kN}$$

Kp. A:



$\Sigma F_y = 0: \quad N_{AB}\sin\beta + N_{AD}\sin\alpha = A_y = \frac{F}{2}$

$$\Rightarrow \frac{N_{AB}}{\sqrt{2}} + \frac{N_{AD}}{2} = \frac{F}{2}, \quad N_{AD} = \frac{F}{\sqrt{3}-1}$$

$$\Rightarrow \frac{N_{AB}}{\sqrt{2}} = \frac{1}{2}\left(F - N_{AD}\right) = \frac{F}{2}\left(1 - \frac{1}{\sqrt{3}-1}\right)$$

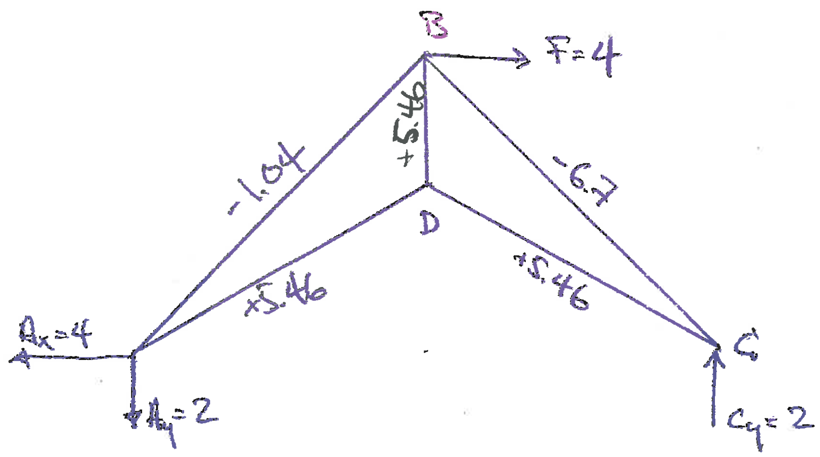
$$\Rightarrow N_{AB} = \frac{\sqrt{2}}{2} F \frac{\sqrt{3}-2}{\sqrt{3}-1} = -\frac{F}{\sqrt{2}}\frac{2-\sqrt{3}}{\sqrt{3}-1} = \underline{-1{,}04}$$

c) Frå b) i Kp. D:

$$\Sigma F_y = 0 \Rightarrow N_{BD} = 2 N_{AD}\sin\alpha$$

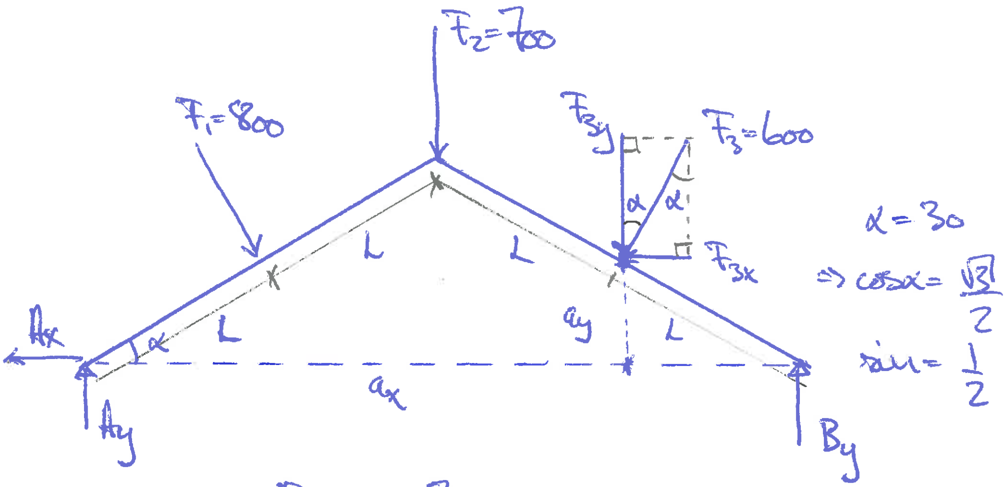
m/ $\alpha = 0 \Rightarrow \sin\alpha = 0 \Rightarrow \underline{N_{BD} = 0}$

dvs. N_{BD} blir ein nullstav når $\alpha = 0$.



Oppg. 2

a) Statisk bestemt? 3 LVL for bjelka = 3 ukjente ⇒ OK!



Dekomponer $F_3 = [F_{3x}, F_{3y}]$ m/ armer a_y og a_x:

$$a_x = 4L\cos\alpha - L\cos\alpha = 3L\cos\alpha, \quad a_y = L\sin\alpha$$

$$F_{3x} = F_3\sin\alpha, \quad F_{3y} = F_3\cos\alpha$$

$\sum M_A = 0$:

$$B_y 2\cdot 2L\cos\alpha - F_2 2L\cos\alpha - F_1 L - F_{3y} a_x + F_{3x} a_y = 0$$

$$\Rightarrow 4B_y\cos\alpha - 2F_2\cos\alpha - F_1 - 3F_3\cos^2\alpha + F_3\sin^2\alpha = 0$$

$$\Rightarrow B_y = \frac{2F_2\cos\alpha + F_1 + F_3(3\cos^2\alpha - \sin^2\alpha)}{4\cos\alpha}$$

$$= \frac{2F_2\sqrt{3}/2 + F_1 + F_3(3\cdot 3/4 - 1/4)}{4\cdot\sqrt{3}/2}$$

$$B_y = \frac{F_1 + \sqrt{3}F_2 + 2F_3}{2\sqrt{3}} = \underline{927\ \text{kN}}$$

Oppg. 2

a) (forts).

$$\sum F_y = 0: \quad A_y + B_y = F_1\cos\alpha + F_2 + F_3\cos\alpha$$

$$A_y = \frac{F_1\sqrt{3}}{2} + F_2 + \frac{F_3\sqrt{3}}{2} - \underbrace{\left(\frac{F_1 + \sqrt{3}F_2 + 2F_3}{2\sqrt{3}}\right)}_{=B_y}$$

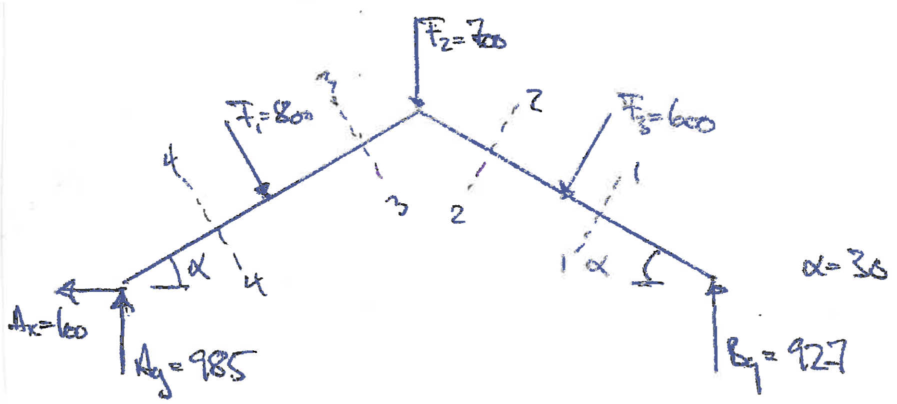
$$= \frac{3F_1 + 2\sqrt{3}F_2 + 3F_3 - F_1 - \sqrt{3}F_2 - 2F_3}{2\sqrt{3}}$$

$$A_y = \frac{2F_1 + \sqrt{3}F_2 + F_3}{2\sqrt{3}} = \underline{985\,\mathrm{kN}}$$

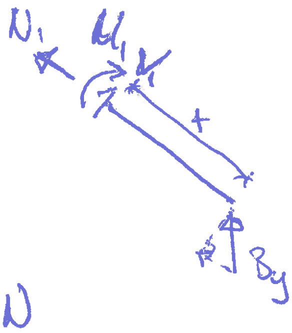
$$\sum F_x = 0: \quad A_x + F_3\sin\alpha - F_1\sin\alpha = 0$$

$$A_x = (F_1 - F_3)\sin\alpha = \frac{F_1 - F_3}{2} = \underline{100\,\mathrm{kN}}$$

b)



Snitt 1-1: $(0 \le x \le L)$



$$\sum F_{x'} = 0: \quad N_1 + B_y\sin\alpha = 0 \Rightarrow N_1 = -B_y/2$$

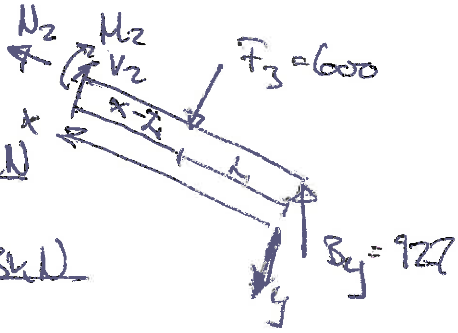
$$\underline{N_1 = -464\,\mathrm{kN}}$$

$$\sum F_{y'} = 0: \quad V_1 = -B_y\cos\alpha = -927\frac{\sqrt{3}}{2} = \underline{-803\,\mathrm{kN}}$$

$$\sum M_1 = 0: \quad M_1 = B_y\cos\alpha \cdot x = 803x \Rightarrow M_1(x=L) = 803$$

Oppg. 2 b) (forts.)

Snitt 2-2: $(L \le x \le 2L)$



$\Sigma F_x = 0$: $N_2 = N_1 = -B_y \sin\alpha = \underline{-464\,kN}$

$\Sigma F_y = 0$: $V_2 = F_3 - B_y \cos\alpha = \underline{-203\,kN}$

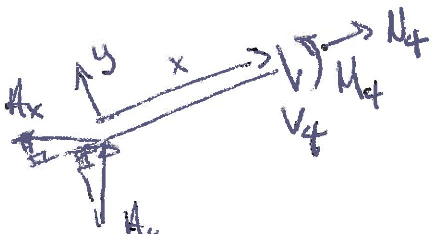
$\Sigma M_2 = 0$: $M_2 = B_y \cos\alpha \cdot x - F_3 (x - L)$

$= (B_y \cos\alpha - F_3)\,x + F_3 \cdot L \Rightarrow M_2(x=L) = M_1(x=L)$

$= 803$ dvs. OK.

$M_2(x=2L) = \underline{1.006\,MNm}$

Snitt 4-4: $(0 \le x \le L)$

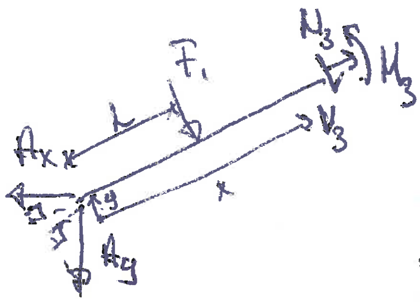


$\Sigma F_x = 0$: $N_4 = A_x \cos\alpha - A_y \sin\alpha = \underline{-406}$

$\Sigma F_y = 0$: $V_4 = A_y \cos\alpha + A_x \sin\alpha = \underline{903\,kN}$

$\Sigma M_4 = 0$: $M_4 = (A_y \cos\alpha + A_x \sin\alpha)\,x$, $\Rightarrow M_4(x=L) = 903\,kNm$

Snitt 3-3: $(L \le x \le 2L)$



$\Sigma F_x = 0$: $N_3 = N_4 = A_x \cos\alpha - A_y \sin\alpha$

$= \underline{-406\,kN}$

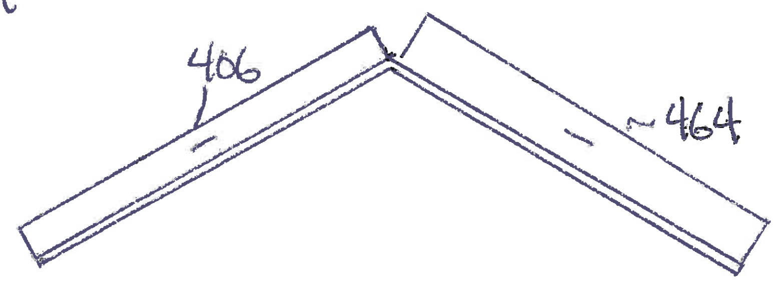
$\Sigma F_y = 0$: $V_3 = A_x \sin\alpha + A_y \cos\alpha - F_1 = \underline{103\,kN}$

$\Sigma M_3 = 0$: $M_3 = (A_x \sin\alpha + A_y \cos\alpha)\,x - F_1 (x - L)$

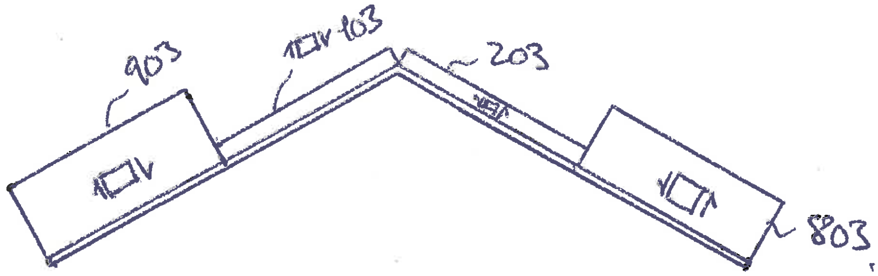
$\Rightarrow M_3(x=L) = M_4(x=L) = \underline{903\,kNm}$ ok!

$M_3(x=2L) = \underline{1.006\,MNm} = M_2(x=2L)$ ok!

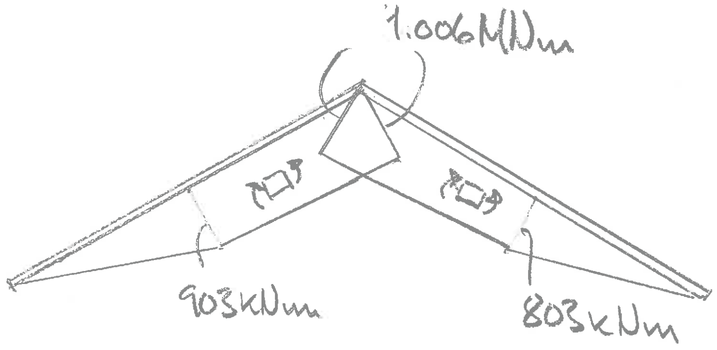
N-diagram



V-diagram



M-diagram



Oppg. 3

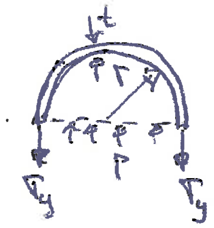
a) $\varepsilon_{45} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\cdot 45) + \frac{1}{2}\gamma_{xy}\sin(2\cdot 45) = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2}\gamma_{xy}$

$\Rightarrow \gamma_{xy} = 2\varepsilon_{45} - \varepsilon_x - \varepsilon_y = \underline{\underline{5.1\cdot 10^{-5}}}$

b) Likevekt gir $T = \tau_{xy} 2\pi r^2 t \;\wedge\; \tau_{xy} = G\gamma_{xy}$

$\Rightarrow G = \frac{T}{2\pi\gamma_{xy} r^2 t} = 37.448\,GPa$

c)



$2rpL = 2\sigma_y tL \Rightarrow \sigma_y = p\frac{r}{t} = \underline{\underline{101\,MPa}}$

$\pi r^2 p = 2\pi r t\sigma_x \Rightarrow \sigma_x = \frac{\sigma_y}{2} = p\frac{r}{2t} = \underline{\underline{50.5\,MPa}}$

d) Frå def. av skjærmodul og Hookes lov for planspenningstilstand:

$G = \frac{E}{2(1+\nu)} \;\wedge\; \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$

$\Rightarrow E = 2G(1+\nu) \;\wedge\; \sigma_y = \frac{2G(1+\nu)}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$

$\Rightarrow \sigma_y = \frac{2G}{1-\nu}(\varepsilon_y + \nu\varepsilon_x) \Leftarrow 1-\nu^2 = (1+\nu)(1-\nu)$

$2G(\varepsilon_y + \nu\varepsilon_x) = \sigma_y(1-\nu) \Rightarrow \nu(\sigma_y + 2G\varepsilon_x) = \sigma_y - 2G\varepsilon_y$

$\underline{\nu = \frac{\sigma_y - 2G\varepsilon_y}{\sigma_y + 2G\varepsilon_x}} \;\wedge\; E = 2G(1+\nu) = 2G\frac{\sigma_y + 2G\varepsilon_x + \sigma_y - 2G\varepsilon_y}{\sigma_y + 2G\varepsilon_x}$

$= 4G\frac{\sigma_y - G(\varepsilon_y - \varepsilon_x)}{\sigma_y + 2G\varepsilon_x}$

$\nu = 0.3494 \approx \underline{0.35}$,
$E = 101.06\,GPa \approx \underline{101\,GPa}$.

$\Rightarrow$ Røyret er laga av kopper.