Oppg 1

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17/12 2011

a) Statisk bestemt? 3 Ukj & 3 Likninger ⇒ OK!

b) $\Sigma F_x = 0 \Rightarrow \underline{A_x = 0}$ 1) $q = 1{,}5\,kN$

$a = 2m,\ b = 4m$

$$\Sigma M_c = 0: \quad A_y b - q(b-a)\frac{(b-a)}{2} = 0$$

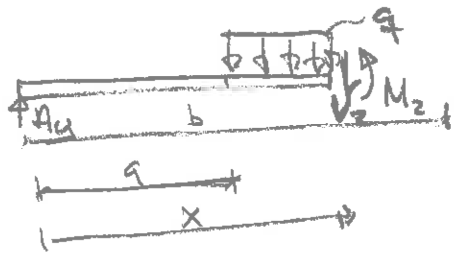
$$\underline{A_y = q\frac{(b-a)^2}{2b} = \frac{1{,}5 \cdot 2^2}{2 \cdot 4} = 0{,}75\,kN} \qquad 2)$$

$$\Sigma F_y = 0: \quad A_y + C_y - q(b-a) = 0 \qquad 3)$$

$$2) \,\&\, 3) \Rightarrow C_y = q(b-a)\left(1 - \frac{b-a}{2b}\right)$$

$$C_y = \frac{q(a+b)(b-a)}{2b}$$

b) Snitt 2-2:
(venstre)



$$\sum F_y = 0: \Rightarrow -V_2 - q(x-a) + A_y = 0$$

$$V_2(x) = A_y - q(x-a)$$

$$\sum M_2 = 0 \cdot \Rightarrow -A_y x + q\frac{(x-a)^2}{2} + M_2 = 0$$

$$M_2(x) = A_y x - q\frac{(x-a)^2}{2}$$

Sjekkverdiar

$$V_2(a) = A_y - q(a-a) = \underline{\frac{q(b-a)^2}{2b}} - 0 = A_y = \underline{0.75\text{ kN}}$$

$$V_2(b) = \frac{q(b-a)^2}{2b} - q(b-a) = q(b-a)\left(\frac{b-a}{2b} - 1\right)$$

$$= -q\frac{(b-a)(a+b)}{2b} = \underline{-C_y} = \frac{1{,}5 \cdot 2 \cdot 6}{2 \cdot 4} = -\frac{9}{4} = \underline{-2.25}$$

$$M_2(a) = A_y \cdot a - 0 = \underline{\frac{q(b-a)^2 a}{2b}}$$

$$M_2(b) = A_y b - \frac{q(b-a)^2}{2} = \frac{q(b-a)^2}{2b} \cdot b - \frac{q(b-a)^2}{2} = 0$$

$V_2 = 0$ ($M_2 = M_{max}$):

$$q(x-a) = A_y \Rightarrow x = a + A_y/q$$

$$= 2 + 0.75/1.5 = \underline{2.5}$$

$$M_2(x = 2.25) = 0.75 \cdot 2.5 - 1.5 \cdot \frac{(2.5-2)^2}{2} \approx 1.7$$

$=$

b) Snitt 2-2
(høyre)

$\sum F_y = 0: \quad V_2 + C_y - q x_2 = 0 \quad , \quad x_2 = b - x$

$$V_2 = q(b-x) - q\frac{(b-a)(b+a)}{2b}$$

$$V_2(a) = q(b-a)\left(1 - \frac{b+a}{2b}\right) = q\frac{(b-a)^2}{2b} \quad \text{ok!}$$

$$V_2(b) = -q\frac{(b-a)(a+b)}{2b} = -C_y$$

$\sum M_2 = 0: \quad C_y x_2 - q\frac{x_2^2}{2} - M_2 = 0$

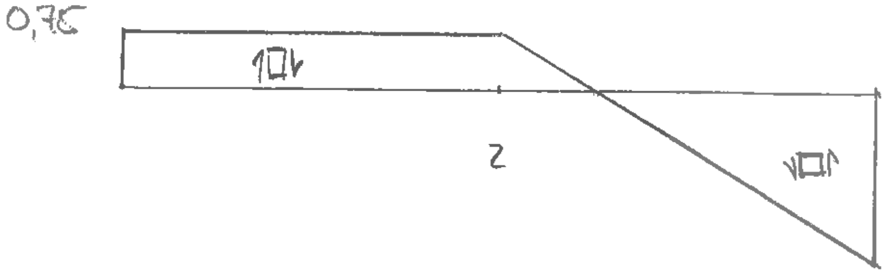
$$M_2(x) = q\frac{(b-x)^2}{2} - q\frac{(b-a)(a+b)}{2b}(b-x)$$

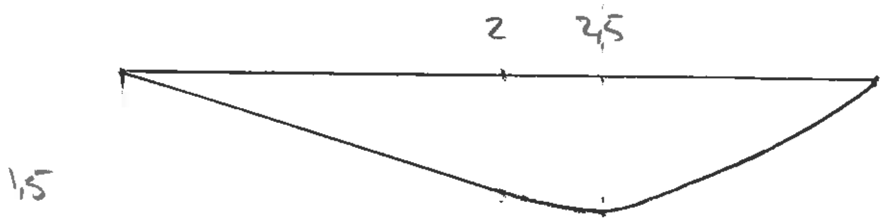
$$M_2(a) = \frac{q(b-a)^2}{2} - \frac{q(b-a)(a+b)(b-a)}{2b}$$

$$= \frac{q(b-a)^2}{2}\left(1 - \frac{a+b}{b}\right)$$

$$= \frac{q(b-a)^2 a}{2b}$$

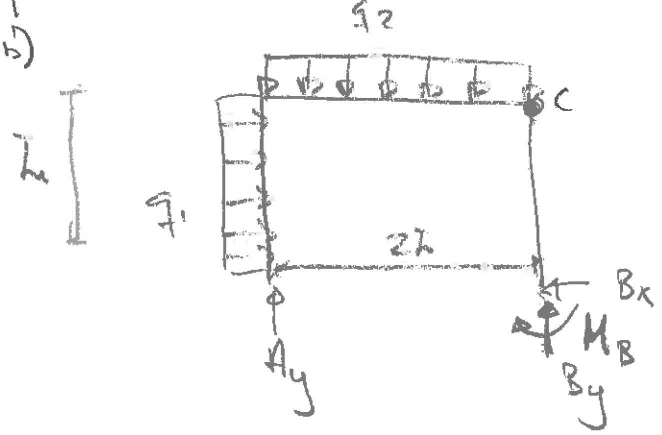
V :





Oppg. 2 (alt.) a) 3NL · 2e = 6 = # ukjende ($A_y, B_x, B_y, M_B, C_x, C_y$)

b)



$\Sigma F_x = 0$: $\underline{B_x = q_1 L}$ 1)

$\Sigma F_y = 0$: $A_y + B_y = 2q_2 L$ 2)

$\Sigma M_c = 0$: (Ramme) $A_y \cdot 2L - q_1 L \cdot \frac{L}{2} - q_2 2L \cdot L = 0$

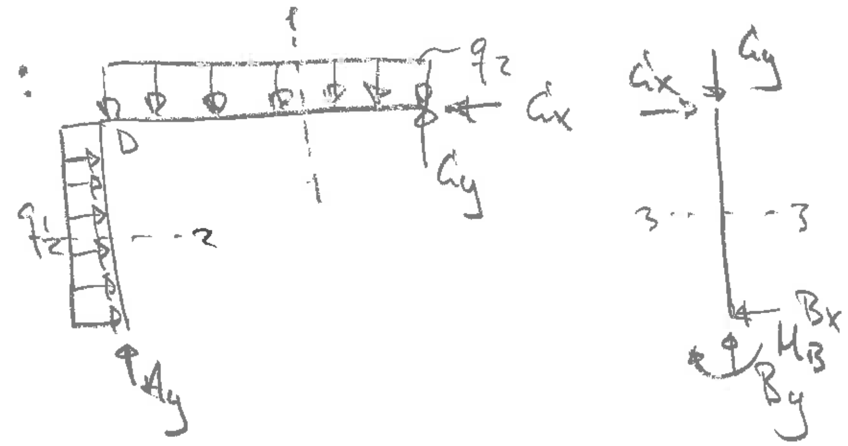
$$\underline{A_y = \left(\frac{q_1}{4} + q_2\right)L}$$ 3)

2) & 3) $\Rightarrow$ $\underline{B_y = \left(q_2 - \frac{q_1}{4}\right)L}$

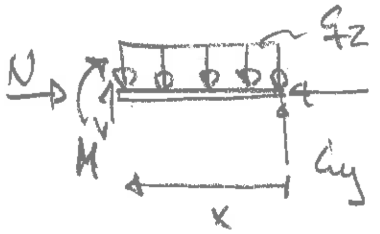
$\Sigma M_c = 0$: (Bjelke) $B_x L + M_B = 0 \Rightarrow M_B = -B_x L = \underline{-q_1 L^2}$

$\underline{C_x = B_x} \wedge \underline{C_y = B_y}$

FLD:



c) Snitt 1-1 (høyre) $0 \le x \le 2L$



$C_y = (q_2 - \frac{q_1}{4}) L$

$\Sigma F_x = 0$: $N = C_x = q_1 L$

$\Sigma F_y = 0$: $V = q_2 x - C_y = \underline{q_2 (x - L) + \frac{q_1}{4} L}$

$V(0) = (\frac{q_1}{4} - q_2) L$, $V(2L) = (q_2 + \frac{q_1}{4}) L$

$V = 0 \Rightarrow q_2 (L - x) = \frac{q_1 L}{4}$

$\Rightarrow x = L \left(1 - \frac{q_1}{4 q_2}\right)$

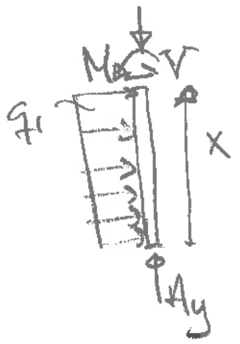
$\Sigma M_{s_1} = 0$:

$M(x) = C_y x - q_2 \frac{x^2}{2} = (q_2 - \frac{q_1}{4}) L x - q_2 \frac{x^2}{2}$

$M(0) = 0$

$M(2L) = -\frac{q_1}{2} L^2$

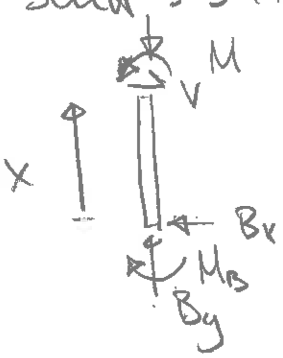
Snitt 2-2 (AD): $\Sigma F_x = 0 \Rightarrow N = A_y = (\frac{q_1}{4} + q_2) L$



$\Sigma F_y = 0 \Rightarrow V(x) = -q_1 x$, $V(0) = 0$, $V(L) = -q_1 L$

$\Sigma M_{s_2} = 0 \Rightarrow M(x) = -q_1 \frac{x^2}{2}$, $M(0) = 0$, $M(L) = -\frac{q_1}{2} L^2$

Snitt 3-3 (BC): $\Sigma F_x = 0 \Rightarrow N = B_y = (q_2 - \frac{q_1}{4}) L$



$\Sigma F_y = 0$: $V = B_x = q_1 L$

$\Sigma M_s = 0$: $M(x) + B_x x - M_B = 0$

$M(x) = B_x x + M_B = q_1 L x - q_1 L^2$

$= q_1 L (x - L)$

$M(0) = -q_1 L^2$, $M(L) = 0$

c) Forenkling med $q_1 = q$ ʌ $q_2 = 2q$

Snitt 1-1: $N = qL$
$(0 \le x \le 2L)$

$$V = q_2(x-L) + \frac{q_1}{4}L = q\left[2(x-L) + \frac{L}{4}\right]$$

$$V(0) = -\frac{7}{4}qL, \quad V(2L) = \frac{9}{4}qL$$

$$V = 0 \Rightarrow x = L\left(1 - \frac{q_1}{4q_2}\right) = \frac{7}{8}L$$

$$M(x) = \left(q_2 - \frac{q_1}{4}\right)Lx - q_2\frac{x^2}{2} = \left(2 - \frac{1}{4}\right)qLx - qx^2$$

$$= \frac{7}{4}qLx - qx^2$$

$$M(0) = 0, \quad M(2L) = \frac{7}{2}qL^2 - 4qL^2 = \underline{-\frac{q}{2}L^2}$$

$$M(x = \tfrac{7}{8}L) = \left(\frac{7}{8}\right)^2 qL^2 \approx \underline{0.77\,qL^2}$$

Snitt 2-2: $N = A_y = \left(\frac{q_1}{4} + q_2\right) = \frac{9}{4}qL$
(AD)

$$V(x) = -qx, \quad V(0) = 0, \quad V(L) = -qL$$

$$M(x) = -q\frac{x^2}{2}, \quad M(0) = 0, \quad M(L) = -q\frac{L^2}{2}$$

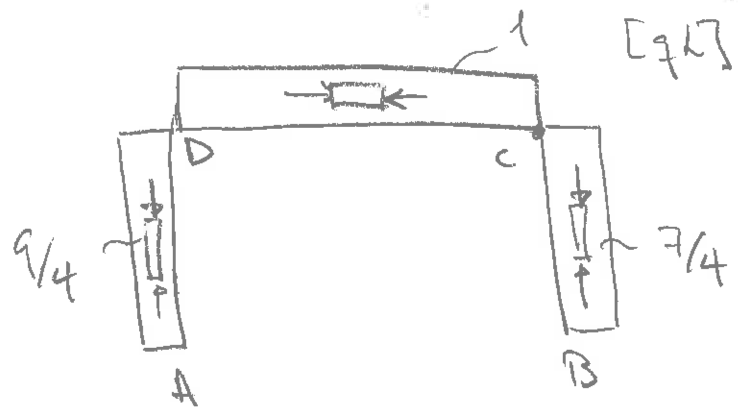
Snitt 3-3: $N = \left(q_2 - \frac{q_1}{4}\right)L = \left(2 - \frac{1}{4}\right)qL = \frac{7}{4}qL$
(BC)

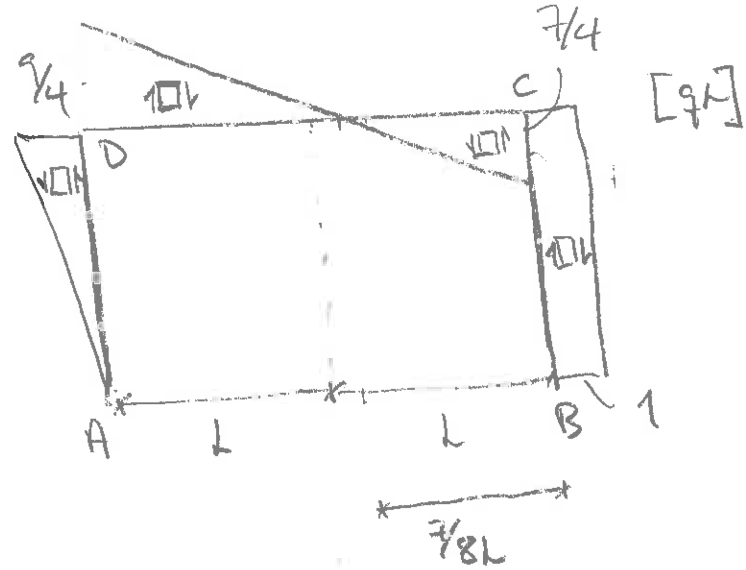
$$V = q_1 L = qL$$

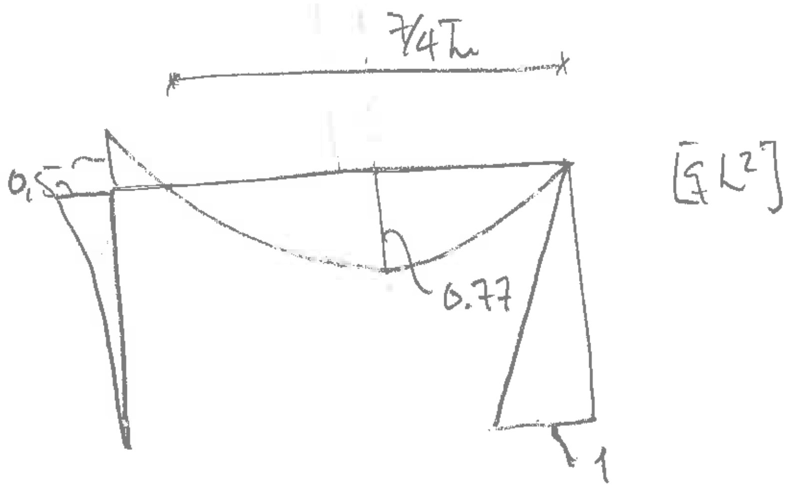
$$M(x) = q_1 L(x-L) = \underline{qL(x-L)}$$

$$M(0) = -qL^2, \quad M(L) = 0$$

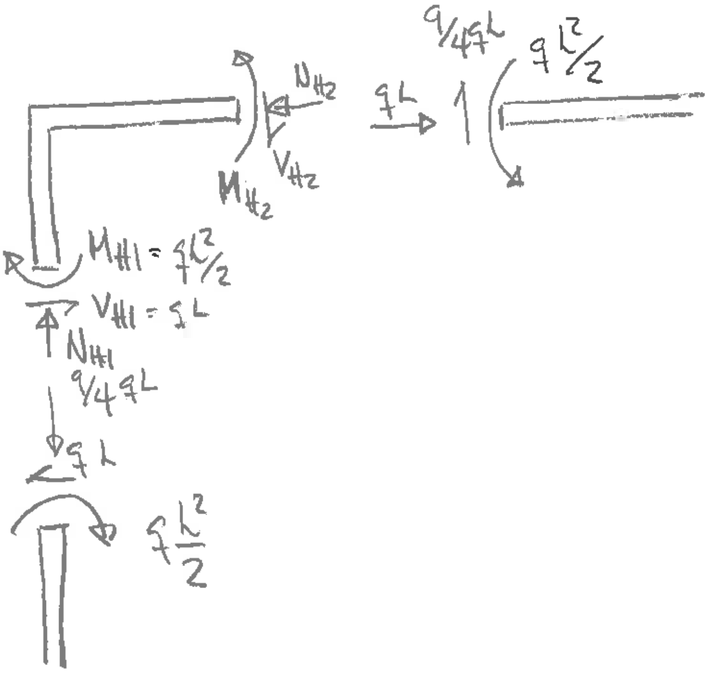
N







d)



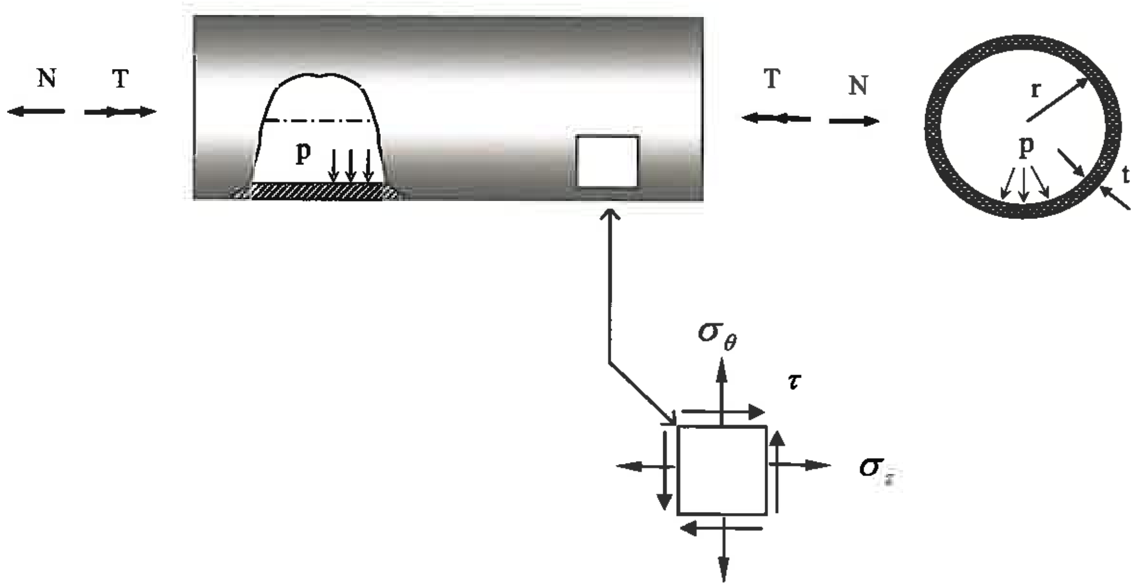
Likevekt for hjørnet:

$\Sigma M = 0: \quad M_{H1} = M_{H2}$,

$\Sigma F_x: \quad N_{H2} = V_{H1} = qL$

$\Sigma F_y: \quad N_{H1} = V_{H2} = \frac{9}{4} qL$

Oppgave ~~2~~ 3



a) Koordinatspenninger: $\tau = \frac{T}{I_p} r = \frac{T}{2\pi r^2 t} = \frac{50kNm}{2\pi 100^2 mm^2 10mm} = 79.6\,MPa$

$$\sigma_\theta = \frac{r}{t} p = \frac{100mm}{10mm} 10MPa = \underline{\underline{100\,MPa}}$$

$$\sigma_z = \frac{\sigma_\vartheta}{2} + \frac{N}{A} = \frac{r}{2t} p + \frac{N}{2\pi r t} = \frac{100\,MPa}{2} + \frac{200\,kN}{2\pi 100mm 10mm} = (50 + 31.8)MPa \approx \underline{\underline{81.8\,MPa}}$$

Hovedspenninger:

$$\sigma_{1,2} = \frac{\sigma_z + \sigma_\theta}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau^2} = \frac{81.8 + 100}{2} \pm \sqrt{\left(\frac{81.8 - 100}{2}\right)^2 + 79.6^2}$$

$$= 90.9 \pm 80.5 \qquad \underline{\underline{\sigma_1 = 171.4\,MPa}} \qquad \underline{\underline{\sigma_2 = 10.4\,MPa}}$$

$$\underline{\underline{\sigma_3 = 0}}$$

Hovedspenningsretning:

$$\Phi_1 = \arctan\frac{\tau}{\sigma_1 - \sigma_z} = \arctan\frac{\sigma_1 - \sigma_\vartheta}{\tau} = \arctan\frac{171.4 - 100}{79.6} \approx \underline{\underline{48.3°}}$$

$$\Phi_2 = \Phi_1 + \frac{\pi}{2} = \underline{\underline{138.3°}}$$

b) Maksimal skjærspenning:

$$\tau_{maks} = \tfrac{1}{2}(\sigma_{maks} - \sigma_{\min}) = \tfrac{1}{2}(\sigma_1 - 0) = \tfrac{1}{2} \cdot 171.4 MPa = \underline{\underline{85.5 MPa}}$$

c) Torsjonsmoment ved flytning:

Mises: $\sigma_z^2 + \sigma_\theta^2 - \sigma_z \sigma_\theta + 3\tau^2 = \sigma_F^2$

$$\tau = \sqrt{\frac{\sigma_F^2 - \sigma_z^2 - \sigma_\theta^2 + \sigma_z \sigma_\theta}{3}} = \sqrt{\frac{250^2 - 81.8^2 - 100^2 + 81.8 \cdot 100}{3}} MPa = 134.2\, MPa$$

$T = 2\pi r^2 t \cdot \tau = 2\pi \cdot 100^2\, mm^2 \cdot 10 mm \cdot 134.2\, MPa = \underline{\underline{84.325\, kNm}}$ ved flytning