NTNU – Trondheim
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **TMA4110/TMA4115 Matematikk 3**

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Examination date:

Examination time (from–to):

Permitted examination support material: C: Simple calculator (Casio fx-82ES PLUS, Citizen SR-270X or Citizen SR-270X College, Hewlett Packard HP30S), Rottmann: *Matematisk formelsamling*

Other information:
Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the 12 problem parts has the same weight when grading.

Language: English

Number of pages: 2

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1

a) Solve the equation $2\bar{z} - z = 1 - 6i$ and draw the solution on the complex plane.

b) Find all solutions of the equation $z^5 = 1 + i$, writing your answer in polar form.

Problem 2 Consider the equation

$$y'' - 9y = q(x).$$

a) Find the general solution of this equation when $q(x) = e^{3x}$.

b) Given that $y(x) = e^x \cos x$ is a solution, determine $q(x)$. For this $q(x)$ find the solution satisfying the initial conditions $y(0) = 0, y'(0) = 1$.

Problem 3 Find the general solution of the equation

$$x'' - 2x' + x = \frac{2e^t}{1+t^2}.$$

(Hint $\int (1+t^2)^{-1} dt = \arctan t$.)

Problem 4 Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & t & -7 \\ 0 & 1 & 1-t \end{pmatrix}.$$

a) For which values of t is A invertible?

b) Compute the inverse of A for $t = 0$.

Problem 5 Consider the following vectors in $\mathbb{R}^4$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 3 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 2 \\ -1 \\ -1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 6 \\ -2 \\ -2 \\ -2 \end{pmatrix}.$$

a) Are these vectors linearly independent? Find a basis for $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b) Find a vector $\mathbf{u} \neq \mathbf{0}$ in $\mathbb{R}^4$ which is orthogonal to $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Problem 6 Find the least squares line $y = mx + c$ that best fits the data points $\{(0,0),(1,-2),(2,1),(3,4),(4,2)\}$.

Problem 7 In the Spring in Trondheim, weather changes every hour. Each given hour, the weather can be sunny, rainy or snowy. The esteemed Institute for Baseless Estimations measured the following weather patterns.

- If the weather is sunny, there is 30 % chance that it will become rainy, and 30 % chance that it will become snowy the next hour.
- If it is rainy, there is 30 % chance that it will become sunny and 20 % chance that it will become snowy.
- If it is snowy, there is 20 % chance that it will stay snowy and 20 % chance that it will become rainy.

One day (after many hours of this pattern), you open your curtains. What is the most likely weather that you might see outside? (Give the probability for observing any of the three weather conditions.)

Problem 8 Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation that is one-to-one. Prove that if $\mathbf{v}_1, ..., \mathbf{v}_k$ are linearly independent vectors in $\mathbb{R}^n$ then the vectors $T(\mathbf{v}_1), ..., T(\mathbf{v}_k)$ are linearly independent in $\mathbb{R}^m$.