Norwegian University of Science and Technology
Department of Mathematical Sciences

Contacts during the exam:
Toke Meier Carlsen (46249940)

Exam in TMA4110 Calculus 3

English
Thursday December 13, 2012
Time: 09:00 – 13:00
Grades ready by January 13, 2013

Permitted aids (Code C): Specified, simple calculator (HP 30S or Citizen SR-270X)
Rottmann: *Matematisk formelsamling*

All answers must be justified, and your calculations should be detailed enough to clearly indicate your line of argument. Each of the 8 problems has the same weight.

Problem 1 Show that $z_1 = 1 + \sqrt{3}i$ is a zero of the polynomial $P(z) = z^5 - 2z^4 + 4z^3 - 8z^2 + 16z - 32$ and find the 4 other zeros of P.

Problem 2 Find the general solution to the differential equation $y'' + 2y' + 5y = 2\cos t + 4\sin t$.

Problem 3 Find the general solution to the system

$$\begin{aligned} 3x_1 - 6x_2 + 6x_3 &= -15 \\ x_1 + x_2 + 4x_3 &= 10. \end{aligned}$$

Problem 4 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be an invertible linear transformation such that $T(x_1, x_2, x_3) = (x_2 + 2x_3, x_1 + 3x_3, 4x_1 - 3x_2 + 8x_3)$. Find a formula for T^{-1}.

Problem 5 Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$. Find orthonormal bases for the column space $\mathrm{Col}(A)$, the row space $\mathrm{Row}(A)$, and the null space $\mathrm{Nul}(A)$.

Problem 6 Let $P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. Let $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots$ be the Markov chain defined by $\mathbf{x}_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$ and $\mathbf{x}_{i+1} = P\mathbf{x}_i$ for $i = 0, 1, 2, \ldots$.

Find the steady-state vector for P and an explicit formula for $\mathbf{x}_i$.

Problem 7 Find the solution of the system

$$\begin{aligned} x_1' &= x_1 + 3x_2 + 3x_3 \\ x_2' &= -3x_1 - 5x_2 - 3x_3 \\ x_3' &= 3x_1 + 3x_2 + x_3 \end{aligned}$$

that satisfies $x_1(0) = 1$, $x_2(0) = -1$ and $x_3(0) = 2$.

Problem 8 Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points (1,3), (2,5), (4,7) and (5,9).