Norwegian University of Science and Technology
Department of Mathematical Sciences

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EXAM IN TMA4115 MATHEMATICS 3
English
Monday 4th June 2012
Time: 9-13

Examination Aids (code C): Simple calculator (HP30S or Citizen SR-270X)
Rottman: *Matematisk formelsamling*

Sensur: 25th June 2012

Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the 12 parts (1,2a,2b,3,4,5a,5b,6,7a,7b,8a,8b) has the same weight.

Problem 1 Solve $w^2 = (-1 + i\sqrt{3})/2$.

Find all solutions of the equation $z^4 + z^2 + 1 = 0$ and draw them in the complex plane. Write the solutions in the form $x + iy$.

Problem 2

a) Find a particular solution of $y'' - 4y' + y = te^t + t$.

b) Find the solution of $y'' - 4y' + y = te^t + t$, where $y'(0) = y(0) = 0$.

Problem 3 Let a be a real number. Find the general solution of $y'' + ay = \cos x$.

Problem 4 Find the least squares solution of

$$\begin{aligned} -y + z &= 1 \\ 2y - z &= 0 \\ x + y + 3z &= 0 \\ -x + 3y + z &= 0 \end{aligned}.$$

Problem 5 Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x + y + z \\ -x + 3y + z \\ 2x - z \\ y + 4z \end{bmatrix}$.

a) Find a matrix A such that $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

b) Find dim Null(A) and a basis for Col(A). Is T one-to-one (injective)? Is T onto (surjective)?

Problem 6 Let A be a 4×4 matrix. Let $B = \begin{bmatrix} 2 & 1 & 4 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Assume that $\det(AB) = 4$. What is $\det(A)$?

Show that the equation $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has only the solution $x_1 = x_2 = x_3 = x_4 = 0$.

Problem 7

a) Find all the eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix}$.

b) Find a basis for each eigenspace of A. Is A diagonalisable?

Problem 8 Let $A = \begin{bmatrix} -2 & -5 \\ 5 & -2 \end{bmatrix}$.

a) Find the complex eigenvalues of A and the corresponding eigenvectors in $\mathbb{C}^2$.

b) Find the solution of the system of differential equations $\vec{y}\,'(t) = A\vec{y}(t)$ satisfying $\vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

The answer should be written in the form $\vec{y}(t) = e^{\lambda t}\begin{bmatrix} a\cos(\omega t) + b\sin(\omega t) \\ c\cos(\omega t) + d\sin(\omega t) \end{bmatrix}$.