Norwegian University of Science and Technology
Department of Mathematical Sciences

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Exam in TMA4110/TMA4115 Matematikk 3

English
11th August 2010
Time: 09:00 - 13:00

Examination aids (code C): Simple calculator (Hewlett Packard HP30S or Citizen SR-270X)
Rottmann: *Matematiske formelsamling*

Explanation should be given for all answers.

Problem 1 Find all solutions of the equation $z^2 + i\bar{z} - 1/4 = 0$.

Problem 2

a) For which values of the parameters a and b is $y = xe^x$ a solution of the equation $y'' + ay' + by = 0$?

b) An object has mass m and equation of motion $my'' + 4y' + y = 0$. For which m is the motion overdamped?

Problem 3

a) Solve the initial-value-problem $y'' - 3y' + 2y = 0; \quad y(0) = 1; \quad y'(0) = 2$.

b) Find the general solution of $y'' - 3y' + 2y = e^x - 5\sin x$.

Problem 4

a) Find two linearly independent solutions y_1, y_2 of the equation

$$y'' - 6x^{-1}y' + 12x^{-2}y = 0,$$

and compute the Wronskian $W(y_1, y_2)$.

b) Find the general solution of $y'' - 6x^{-1}y' + 12x^{-2}y = x^4$.

Problem 5 Let

$$A = \begin{bmatrix} 0 & 1 & 2 & -3 \\ 1 & 2 & -1 & 0 \\ 2 & 5 & 0 & -3 \end{bmatrix}$$

Find a basis for the column space, row space, and null space of the matrix A.

Problem 6 For which values of the parameter a are the vectors $\mathbf{v}_1 = (1, -3, a)$, $\mathbf{v}_2 = (0, 1, a)$ and $\mathbf{v}_3 = (a, 2, 0)$ linearly dependent?

Problem 7 Given the matrix

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & -1 \\ -1 & 6 & 5 \end{bmatrix}$$

a) Solve the equation $Ax = 0$.

b) Find the eigenvalues and eigenvectors of A.

Problem 8 A conic section is given by the equation

$$3x^2 + 8xy - 3y^2 = 10.$$

Find a new coordinate system in which the equation is in its simplest possible form (standard form). Decide which type of conic section it is, sketch it in the xy-plane, and draw on the axes for the new coordinate system.

Problem 9 A diagonalisable matrix A satisfies $A^4 = A$, show that $A^2 = A$.