NTNU

Department of Mathematical Sciences

Examination paper for **TMA4110 Matematikk 3**

Academic contact during examination: Eugenia Malinnikova, Andrew Stacey
Phone: 73550257, 73590154

Examination date: 1 December, 2010
Examination time (from–to): 09:00-13:00
Permitted examination support material: C: Simple calculator (Hewlett Packard HP30S or Citizen SR-270X), Rottmann: *Matematiske formelsamling*

Language: English
Number of pages: 2
Number pages enclosed: 0

Checked by:

Date Signature

Problem 1 Find all solutions of the equation $z^4 = \frac{-5+i\sqrt{3}}{2+i\sqrt{3}}$, writing your answer in Cartesian (normal), and draw the solutions on the complex plane.

Problem 2

a) Find the general solution of $y'' + y' - 2y = 0$.

b) Find the solution of $y'' + y' - 2y = 10\cos t + 1 - 2t^2$ with initial conditions $y(0) = 11,\ y'(0) = 3$.

Problem 3 Let

$$A = \begin{bmatrix} 1 & -2 & 2 & -4 & 3 \\ -2 & 4 & 0 & -4 & -5 \\ 4 & -8 & 3 & -1 & 7 \\ 3 & -6 & 1 & 3 & 0 \end{bmatrix}.$$

a) Find a basis for the column space $\mathrm{Col}(A)$ and a basis for the null space $\mathrm{Null}(A)$.

b) Find an orthogonal basis for the row space of the matrix A.

c) Let T be the linear transformation with matrix A. Is T one-to-one? Is it onto? Justify your answers.

Problem 4 Let P_2 be the space of all polynomials of degree less than or equal to two. What is the dimension of P_2?
Let $p_1(t) = t$, $p_2(t) = t(t-1)$ and $p_3(t) = (t-1)(t-2)$. Is $\{p_1, p_2, p_3\}$ a basis for P_2? Justify your answer.

Problem 5 In Sommerby the rental company has three locations for renting out boats: Market, Island, and Camping. The pattern of returns to the rental locations is the following: for boats rented at Market, one-quarter is returned to Market, one-half to Island, and one-quarter to Camping; half of the boats rented at Island are returned to Market and half to Camping; for boats rented at Camping one-sixth is returned to Market, one-half to Island, and one-third to Camping. Find the stochastic matrix P that describes how the distribution of boats changes. Find the steady-state vector for P.

Problem 6 Find the solution of the system

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 3x_1 + 2x_2 \end{aligned}$$

that satisfies the initial conditions $x_1(0) = 1$ and $x_2(0) = 1$.

Problem 7 Find the least squares line $y = mx + c$ that best fits the data points $\{(0, 3), (1, 3), (2, 6), (3, -3), (4, 1), (5, -1)\}$.