1 CIRCUIT ANALYSIS AND POWER CALCULATIONS

Consider the voltage and current waveforms taken from a single-phase circuit in Figure 1.

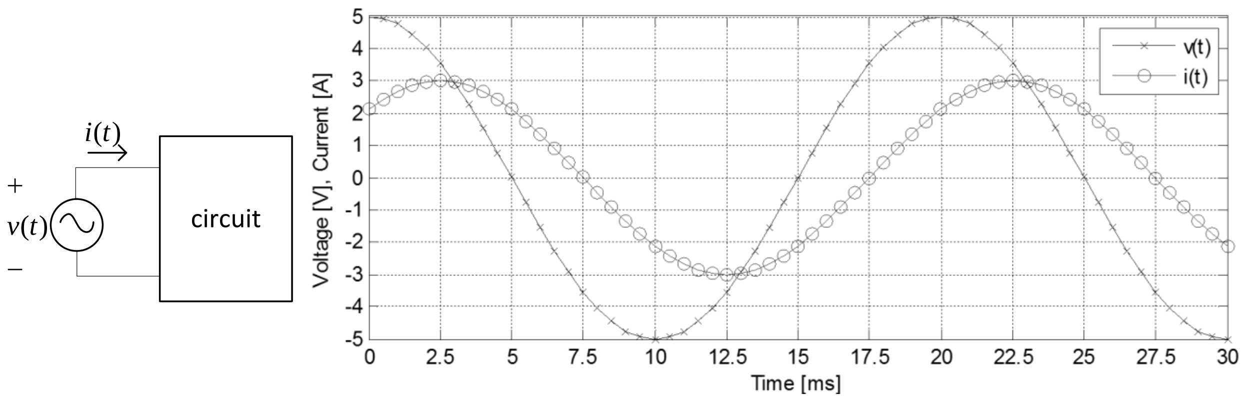
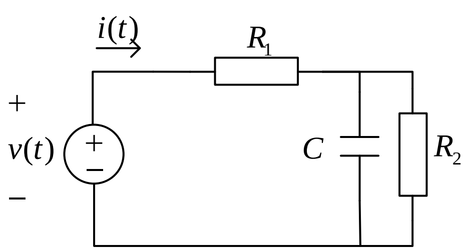


Figure 1: Single-phase circuit and time-domain waveforms of voltage and current

a) Show that the phase angle difference between the voltage and current waveform is $\gamma = 45^o$
b) Find an expression for the instantaneous power $p(t)$ **consumed** by the circuit. You don't need to simplify the expression.
c) Find the active power *P* and the reactive power *Q* **consumed** by the circuit *(as usual, both P and Q should be average values).* Is the circuit capacitive or inductive? Explain why.
d) Find the equivalent impedance Z_{eq} of the circuit.

2 STEP RESPONSES OF RC-CIRCUIT

Consider the following circuit and numerical values:



$R_1 = 1\,\Omega$
$R_2 = 2\,\Omega$
$C = 1\,F$

In problem a) and b) we assume that the voltage across the capacitor is zero at $t = 0$

a) Show that the transfer function between $i(t)$ and $v(t)$ is $H(s) = \dfrac{V(s)}{I(s)} = \dfrac{2s+3}{2s+1}$

In problem b), we apply a voltage step at $t = 0$, i.e. $v(t) = \begin{cases} 0\,V & , \ t < 0 \\ 1\,V & , \ t \geq 0 \end{cases}$

b) Based on the transfer function from a), find $i(t)$ for $t \geq 0$

In problem c) we assume that $v(t) = 100\cos(3t)$, and that the circuit has reached stationary/steady-state conditions.

c) Since the circuit operates in stationary conditions the current can be written as $i(t) = A\cos(3t+\varphi)$. Find the numerical values for *A [Ampere]* and φ *[radians]*.

3 MOTORS AND DRIVE SYSTEMS

Part 1: Induction motor

a) Figure 2 shows a typical inductor motor curve relating the mechanical speed ω_{mek} to the mechanical torque T_{mek}. Disregard mechanical gear and magnetic poles. Redraw the curve into your answer, and indicate the following items in the figure:
   I. Synchronous speed ω_s
   II. The speed at which the slip *s* equals 0.
   III. The speed at which the slip *s* equals 1.
   IV. The normal operating range
   V. The starting torque T_{start}

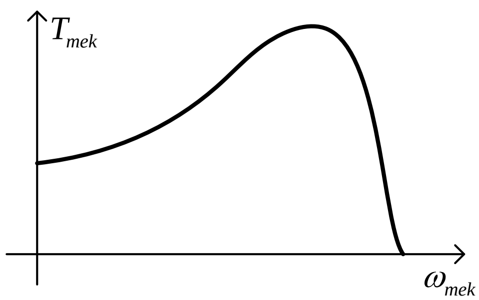


Figure 2: Induction motor mechanical torque versus mechanical speed

b) What is the torque of the machine when the mechanical speed equals synchronous speed?

Part 2: Wind turbine system

A wind turbine generator is directly connected to a power grid with fixed frequency $f_{grid} = 50\ Hz$ as shown in Figure 3. The system does not have any mechanical gear, but the generator is equipped with an unknown number of poles. ω_{mek} is the mechanical speed of the turbine and generator.

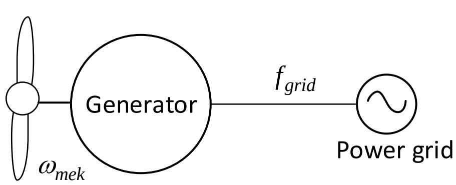


Figure 3: Wind turbine generator connected to a power grid

c) Assume $\omega_{mek} = 7.85$ rad/s. How many poles is the generator equipped with?

In Figure 4 the relation between mechanical speed ω_{mek} and the produced power P_{wind} is presented for three different wind speeds $v = 6, 8, 10$ m/s.

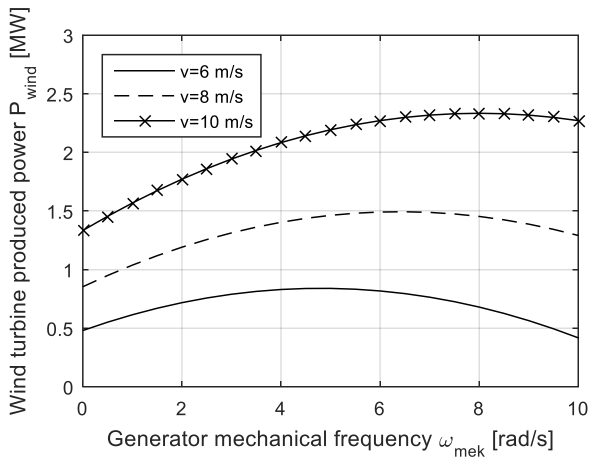


Figure 4: Relation between wind turbine mechanical speed ω_{mek} and produced power P_{wind}

d) Assume $\omega_{mek} = 7.85$ rad/s and wind speed $v = 6$ m/s. Explain why the wind turbine configuration in Figure 3 is not optimal if the goal is to produce as much power as possible.
e) Sketch a wind turbine configuration that is able to extract the maximum available power at every wind speed. What kind of converter configuration should be used?

4 DC-MOTOR CONTROLLED BY A DC-DC CONVERTER

Consider the DC-motor controlled by a DC-DC converter in Figure 5.

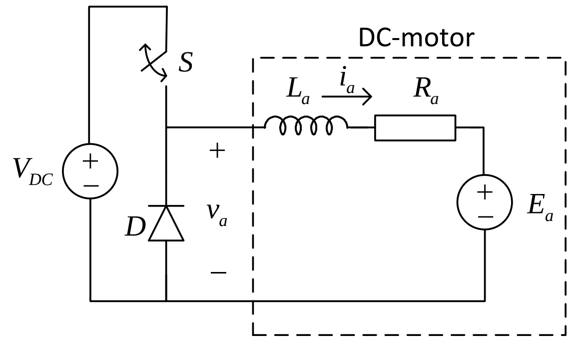
We can use the average voltage V_a when calculating torque and speed of the DC-motor by using the DC-motor formulas (appendix). The term *no-load* is defined as the condition where the machine is producing zero torque, i.e. $T = 0$

a) Assume the average terminal voltage $V_a = 7.5\,V$, and $R_a = 0.1\,\Omega$. What is the no-load speed of the DC-motor when the flux constant is equal to $K\varphi = 0.1 \frac{rad}{s} \cdot \frac{1}{V}$?

The DC-DC converter consists of an *ideal* switch *S* and an *ideal* diode *D*. The variable *k* is called *duty-cycle:* $k = \frac{t_{on}}{T_s}$, where t_{on} is the time period where the switch is closed (ON), and T_s is the period.

b) Consider the waveform for $v_a(t)$ presented in Figure 6. Find the expression of the average voltage $V_a = \frac{1}{T_s}\int_0^{T_s} v_a(t)dt$ as a function of the duty-cycle *k* and the DC-voltage V_{dc}

***NB:** Note the difference between the duty cycle k and the flux constant K, and also the difference between torque T and period T_s*



Swich closed: $v_a = V_{dc}$

Switch open: $v_a = 0$

Figure 5: DC-motor controlled by a DC-DC converter

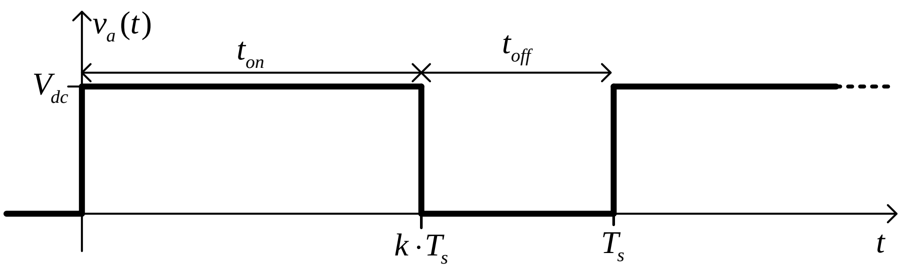


Figure 6: Waveform of the voltage applied to the DC-motor terminals (v_a)

c) Draw the two separate circuit diagrams corresponding to *closed switch* S and *open switch* S. Indicate the path of current flow in the two diagrams. Finally, show that the differential equation for i_a can be written as:

$$\frac{di_a}{dt} = \begin{cases} \dfrac{V_{dc} - R_a i_a - E_a}{L_a}, & \text{ON-state (S is closed)} \\ \dfrac{-R_a i_a - E_a}{L_a}, & \text{OFF-state (S is open)} \end{cases}$$

Hint: Assume i_a is always positive, then the diode will always be on when the switch S is open.

Use the following numerical values in problem d):

$V_{dc} = 15\,V \quad R_a = 0\,\Omega \quad L_a = 1\,mH \quad T_s = 2\,ms \quad k = 0.5 \quad E_a = 7.5\,V$

d) Assume $i_a(0) = 7.5\,A$. Calculate $i_a(t)$ for $0 \le t \le T_s$. Make a sketch of the waveform.

NB: *Note that we have assumed $R_a = 0$ in order to simplify the calculations.*

APPENDIX: FORMULAS

Inductor and capacitor

$$v_L = L\frac{di_L}{dt} \;,\; i_C = C\frac{dv_C}{dt} \;,\; X_L = j\omega L \;,\; X_C = \frac{1}{j\omega C}$$

Phasors and complex power

$$X\cos(\omega t+\theta) \Leftrightarrow Xe^{j\theta} \quad , \quad S = VI^* = P + jQ$$

Electromagnetism:

$$\varepsilon = N\frac{d\varphi}{dt} \;,\; NI = \Re\varphi \,,\; \Re = \frac{l}{\mu A} \quad , \quad \varphi = BA$$

Trigonometrics

$$\cos(2x) = 1 - 2\sin^2(x)$$
$$= 2\cos^2(x) - 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Three-phase

$$|V_{LL}| = \sqrt{3}|V_{ph}|$$

Electrical machines

$$f_{el} = \frac{p}{2} f_{mek}$$

DC-machine

$$E_a = K\varphi\omega \qquad T = K\varphi I_a$$

Induction (asynchronous) machine

$$\omega_{mek} = (1-s)\omega_s$$

Mechanics

$$P = T\omega \quad P = F\cdot v \quad v = \omega r \quad E_k = \frac{1}{2}mv^2$$

Laplace transforms

Konstant: $\mathcal{L}(K\cdot f(t)) = K\cdot F(s)$

Sprangrespons: $\mathcal{L}(u(t)) = \frac{1}{s}$

Eksponential: $\mathcal{L}\left(e^{at}\right) = \frac{1}{s-a}$

s-shift: $\mathcal{L}\left\{e^{-at}\cdot f(t)\right\} = F(s+a)$

Sinus: $\mathcal{L}\left\{\sin(\omega t)\right\} = \frac{\omega}{s^2+\omega^2}$

Cosinus: $\mathcal{L}\left\{\cos(\omega t)\right\} = \frac{s}{s^2+\omega^2}$

Dempa sinus: $\mathcal{L}\left\{e^{-at}\sin(\omega t)\right\} = \frac{\omega}{(s+a)^2+\omega^2}$

Dempa cosinus: $\mathcal{L}\left\{e^{-at}\cos(\omega t)\right\} = \frac{s+a}{(s+a)^2+\omega^2}$