Solution – TTK4240 December 2017

1 DC MOTOR

The separately excited dc machine having the magnetization curve shown in Figure 1 is operating as a motor at a speed of 1500 rpm with a developed (shaft) power of 10 hp (1hp=735 Watt) and I_F = 2.5 A. The armature resistance is R_A=0.3 Ω and the field resistance is R_F = 50 Ω.

a) Explain shortly why the magnetization curve in Figure 1 is curved at high field current. You can disregard this phenomenon in your subsequent calculations.

Answer: *The magnetization curve starts to bend at high field current due to saturation in the iron core. When the iron is not saturated, the induced voltage is proportional to the field current. When the iron saturates, it becomes more and more difficult to increase the induced voltage. Eventually, the iron behaves like air (or vacuum). For standard iron this happens around B=2 Tesla.*

Students writing "the curve bends due to iron saturation" will get full score in most cases.

b) Find the machine speed in radians per second

Answer:

$$\omega = \frac{n \cdot 2\pi}{60} = \frac{1500 \cdot 2\pi}{60} = 157\ rad/s$$

c) In this operating condition, find the developed torque, the armature current I_A and the voltage V_T applied to the armature circuit.

Answer:

NOTE: During the exam, it was falsely informed that the speed n=1200 rpm in the magnetization curve should be 1500 rpm. Then the problem becomes easier to solve, as the induced voltage can be read directly from the figure. In the following solution, both approaches will be shown.

Assuming the magnetization curve is given for 1500 rpm:

Then, the induced voltage can be read directly as $E_a = 145V$.

The armature current can be found based on the shaft power and induced voltage as:

$$I_a = \frac{P_{shaft}}{E_a} = \frac{735 \cdot 10}{145} = 50.69\ A$$

The induced torque is $T_{dev} = \frac{P_{shaft}}{\omega_m} = \frac{7350}{50\pi} = 46.79\ Nm$

Finally, the terminal voltage is $V_T = E_a + R_a I_a = 145 + 50.69 \cdot 0.3 = 160.21V$

Assuming the magnetization curve is given for 1200 rpm:

The method is identical, the only difference is that we need to find the induced voltage by the proportional relation:

$$\frac{E_{a2}}{E_{a1}} = \frac{n_2}{n_1} \Rightarrow E_{a2} = 145 \cdot \frac{1500}{1200} = 181.25V$$

Using the formulas above, the following results are obtained:

$$I_a = \frac{P_{shaft}}{E_a} = \frac{735 \cdot 10}{181.25} = 40.55\ A$$

$$T_{dev} = \frac{P_{shaft}}{\omega_m} = \frac{7350}{50\pi} = 46.79\ Nm$$

$$V_T = E_a + R_a I_a = 145 + 50.69 \cdot 0.3 = 193.42\,V$$

d) The field current is still I_F = 2.5 A. Assume the terminal voltage is V_T =127.5 V and the power supplied to the motor terminals is 3.2 kW. What is the speed of the machine?

Answer:

Assuming the magnetization curve is given for 1500 rpm:

Since we know the terminal voltage and the power supplied to the terminals, we can find the induced voltage:

$$E_A = V_T - R_a I_a = V_T - R_a \frac{P_T}{V_T} = 127.5 - 0.3\frac{3200}{127.5} = 119.97\,V$$

NOTE: Some students subtracted the field winding losses $P_F = R_F I_F^2 = 50 \cdot 2.5^2 = 312.5\,W$, hence the power supplied to the armature winding is $3200 - 312.5 = 2887.5\,W$. This approach will give full score, and is maybe even more correct than the solution below. The problem description could be more clear to avoid this ambiguity.

NOTE 2: Many students assumed that $3200 = E_a I_a$. This is not entirely correct since the power supplied to the terminals is not equal to the shaft power. These students got a small point reduction

When the induced voltage is known, we can calculate the speed of the machine by the proportional relation:

$$\frac{E_{A1}}{E_{A2}} = \frac{n_1}{n_2} \Rightarrow n_2 = 1500 \cdot \frac{119.97}{145} = 1241.1\,\text{rpm}$$

Assuming the magnetization curve is given for 1200 rpm:

The induced voltage is the same, we only need to modify the proportional relation:

$$\frac{E_{A1}}{E_{A2}} = \frac{n_1}{n_2} \Rightarrow n_2 = 1200 \cdot \frac{119.97}{145} = 992.9\text{rpm}$$

NOTE 3: Rounding errors are not punished. If the answer is given in Hz or rad/s, this is of course not punished either.

2 MAGNETIC CIRCUIT

For the core shown in Figure 2, the reluctances of all three paths between points *a* and *b* are equal. $\mathcal{R}1 = \mathcal{R}2 = \mathcal{R}3 = 106$ (A·turns)/Wb Assume that all of the flux is confined to the core.

a) Do the fluxes produced by i_1 and i_2 aid or oppose one another in path 1? In path 2? In path 3? Explain why.

Answer: *By applying the right hand rule, we find that both coils produce a flux in the clockwise direction (assuming current is positive in the defined direction). Hence, the fluxes aid each other in path 1 and 2, while they will oppose each other in path 3.*

b) If a dot is placed on the top end of coil 1, which end of coil 2 should carry a dot? Explain why

Answer:

According to the dot convention, the dots are placed such that currents entering the dotted terminals produce aiding magnetic flux. By inspecting our core, we see that when both currents are entering the upper terminals of the coil, both fluxes will flow in the same (clockwise) direction. The dot on coil 2 should therefore be placed on the upper end.

This subproblem was one of the most difficult on the exam. In order to get any points on this subproblem, it is necessary to look either at the direction of fluxes, or on how the coils are wound to the core. Many students gave an answer only based on the defined direction of voltages and currents. This is incorrect, as the dot would move if the coil is wound in the opposite direction around the core.

c) Should the mutual term for the voltages in the following equations carry a plus sign or a minus sign? Explain why

Answer*: This problem can be answered in two ways:*

1. *If both currents are entering the dot (or leaving the dot), the mutual inductance terms will have positive sign.*
2. *Since both currents with their defined direction produce a flux that is aiding the flux in the other coil, the mutual inductance will have positive sign.*

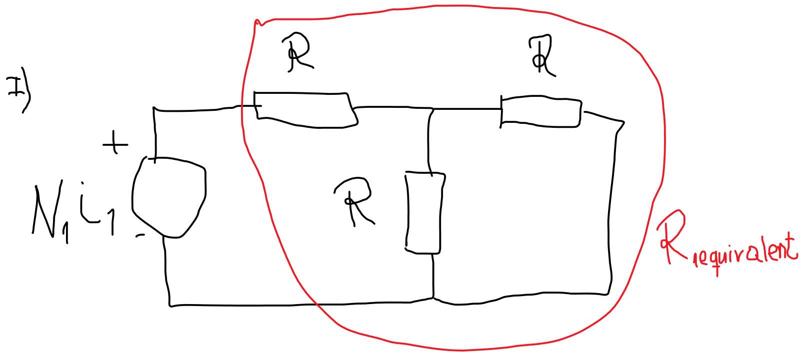
Comment*: Many students answered "negative sign" in this problem since they placed the dot on the lower terminal in problem 2b. They will still get full score on 2c if their reasoning is correct.*

d) Determine the values of L_1, L_2, and M.

Answer:

NOTE: Unfortunately the reluctances was specified as 106 At/Wb instead of 10^-6 At/Wb. As a consequence, the resulting inductances become extremely high. Many students commented on this, and during the grading this has been taken into account.

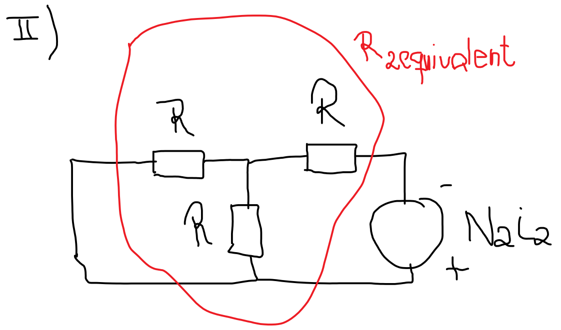
To solve this part of the problem, the superposition principle is applied. First, coil 1 is considered to be source while coil 2 is short circuited, and L_1 can be calculated considering the totality of flux1. Then, coil 1 is short-circuited and coil 2 is considered to be the source and L_2 can be calculated considering the totality of flux2.



$$L_1 = \frac{N_1^2}{\mathcal{R}_{1equivalent}}$$

$$\mathcal{R}_{1equivalent} = \mathcal{R} + \frac{\mathcal{R}^2}{\mathcal{R}+\mathcal{R}} = \frac{3}{2}\mathcal{R}$$

$$L_1 = \frac{N_1^2}{\mathcal{R}_{1equivalent}} = \frac{100^2}{1.5 \cdot 106} = 62.89\ H$$



$$L_2 = \frac{N_2^2}{\mathcal{R}_{2equivalent}}$$

$$\mathcal{R}_{2equivalent} = \mathcal{R} + \frac{\mathcal{R}^2}{\mathcal{R}+\mathcal{R}} = \frac{3}{2}\mathcal{R}$$

$$L_2 = \frac{N_2^2}{\mathcal{R}_{2equivalent}} = \frac{300^2}{1.5 \cdot 106} = 566\ H$$

The mutual inductance is defined as the flux linkage in one coil that is induced by the current from the other coil:

$M = \frac{N_1\varphi_1}{i_2}$*, where we assume that* $i_1=0$ *(principle of superposition)*

By inspecting the magnetic circuit in II) above, it is clear that the flux induced from coil 2 is divided in two equal branches, one going to path 1 and one to path 3.

$$\varphi_2 = \frac{N_2 i_2}{1.5R} \quad , \quad \varphi_1 = \frac{\varphi_2}{2} \qquad \Rightarrow \varphi_1 = \frac{N_2 i_2}{3R}$$

Where φ_1 *is the flux through coil 1 and* φ_2 *is the flux through coil 2.*

We can then find the mutual inductance as:

$$M = \frac{N_1 N_2 i_2}{3R i_2} = \frac{N_1 N_2}{3R} = \frac{100 \cdot 300}{3 \cdot 106} = 94.33\ H$$

We can also solve the problem in other ways, e.g. by $M = \frac{N_2\varphi_2}{i_1}$ *and assuming* $i_2 = 0$.

3 WIND TURBINE CONTROL AND GRID CONNECTION

The wind turbine is connected through the grid by a generator, rectifier and inverter. The mechanical rotational speed of the generator is ω_{gen}, while the electrical frequency of the generator voltages is f_{gen}. The maximum available power from the wind is P_{wind}, while the actually produced power is P_{gen}. The generator is a synchronous generator with 60 poles.

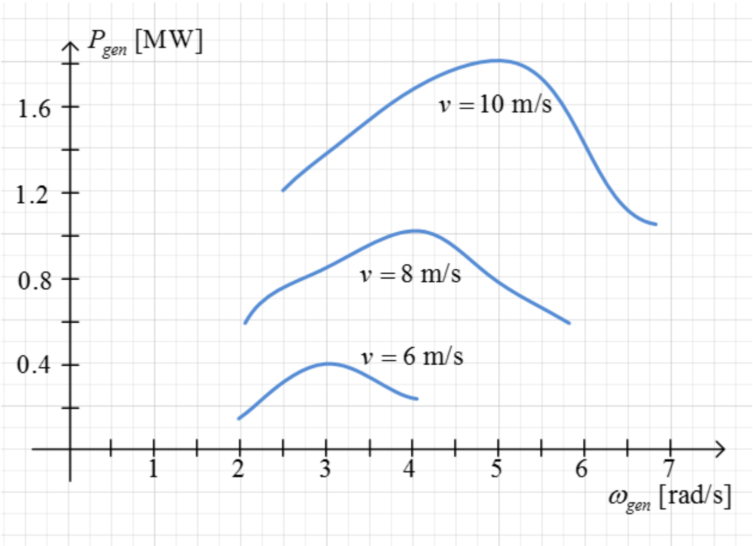


Figure 4: Wind power production curve as a function of generator speed (for three different wind speeds v)

The curves in Figure shows the wind power production P_{gen} as a function of generator mechanical speed ω_{gen}.

a) If the wind speed is 10 m/s, what is the produced power if the generator electrical frequency is $f_{gen} = 18\,\text{Hz}$?

Answer: *The mechanical frequency is then* $\omega_{gen} = 2\pi \cdot \frac{18}{30} = 3.8\,\text{rad/s}$. *Reading from the curve, the produced power is then approximately* $P_{gen} = 1.6\,\text{MW}$. *If a student makes correct calculation and read between 1.5 and 1.7 MW from the figure, he will get full score.*

b) Use Figure to discuss the best strategy to control ω_{gen}. Also, explain the concept of Maximum Power-Point Tracking (MPPT).

Answer: *We need to change the rotational speed if we want to extract the maximum amount of power. The peak point of each wind speed occurs at different rotational speed, therefore a control algorithm is required in order to always maximize power. Such control algorithm is called Maximum Power Point Tracking (MPPT). The control algorithm can be based either on measured wind speed, or it can employ various techniques to always track the optimal rotational speed.*

Most students got full score or almost full score on this problem. It is possible to elaborate more, e.g. by additional drawings, but this is not required.

The maximum available power from the wind can be expressed by the formula $P_{wind} = kv^3$, where k is a constant and v is the wind speed. Assume $k = 3480$.

c) Assume the turbine is controlled based on MPPT. What is the efficiency of the wind turbine for the three wind speeds in Figure ? Efficiency is expressed as (produced power)/(available power).

<u>Answer:</u> *The three maximum power points can be read from the figure, and the available power can be calculated from the formula above. The efficiency is then calculated in the table below:*

	v=6 m/s	v=8 m/s	v=10 m/s
Produced power	0.4 MW	1.0 MW	1.8 MW
Available power	0.7517 MW	1.7818 MW	3.48 MW
efficiency	53.21 %	*56.1 %*	51.7 %

Most students got full score on this problem. Erroneous readings in the range of +- 0.1 MW has not been punished as long as the reasoning is correct.

d) The turbine produces $P_{gen} = 1.5\,MW$. Neglect all losses in the generator, rectifier and inverter. The maximum allowed line current i_a is 350 A (RMS). Phase-to-phase grid voltage is $3000\,V$ (RMS). What is the maximum possible reactive power that the inverter can supply to the grid? You can assume a symmetrical three-phase system, hence the per phase equivalent can be used.

<u>Answer</u>**:** *We first find the maximum possible apparent power*

$$S_{\max} = V_{LL} \cdot I_{\max} \cdot \sqrt{3} = 3000 \cdot 350 \cdot \sqrt{3} = 1.819\ \text{MW}$$

Note that we need to multiply with $\sqrt{3}$ since we have the line-to-line voltage and the phase current. Alternatively, we could divide the line-to-line voltage with $\sqrt{3}$ to get the phase-to-neutral voltage, and then multiply with 3 phases to get the three-phase power.

Since we know the active power, the maximum reactive power can be calculated as:

$$Q_{\max} = \sqrt{S_{\max}^2 - P^2} = \sqrt{1.819^2 - 1.5^2} = 1.028\ \text{MVAr}$$

This reactive power can be either inductive or capacitive, it is not relevant.

It is considered a severe <u>mistake</u> in this subproblem to apply the factor $\sqrt{2}$ or $\sqrt{3}$ in an incorrect way. It is also a very severe <u>mistake</u> to subtract apparent and active power directly as

$$Q_{\max} = S_{\max} - P = 1.819 - 1.5$$

4 PHASOR TRANSFORM

a) Write the corresponding differential equations that models the RLC circuit in Figure 5. The source voltage is $v(t) = V_o cos(\omega t + \alpha_v)$

Answer: *By applying KVL, we can obtain the following equation:*

$$v(t) = V_o \cos\left(\omega t + \alpha_v\right) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,\mathrm{d}t$$

Alternatively, we can differentiate the equation to remove the integral:

$$\frac{dv(t)}{dt} = -\omega V_0 \sin\left(\omega t + \alpha_v\right) = L\frac{d^2 i(t)}{dt} + R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Both of the above equations give full score.

b) Transform the differential equations into phasor domain, keep in mind that $\cos\left(\omega t + \theta\right) = \mathrm{Re}\left(e^{j\theta} \cdot e^{j\omega t}\right),\ \cos\left(\theta\right) = \mathrm{Re}\left(e^{j\theta}\right)$

Answer: *This problem was not well formulated, and this has been taken into account during grading. The intention was to transform the differential equation into phasor domain through fundamental definitions. However, many students transformed the equation into phasor domain directly using either Laplace or the well-known relations* $L\frac{di}{dt} \Rightarrow j\omega L$, $C\frac{dV}{dt} \Rightarrow \frac{1}{j\omega C}$. *As a consequence, we have been relatively kind with points on this subproblem, and it has also been given relatively low weight.*

<u>*Solution using fundamental definitions:*</u>

Asuming linearity, the response of the circuit, $i(t)$, should have the same form as its excitation, $v(t)$

$$v(t) = V_o \cos(\omega t + \alpha_v) \xrightarrow{PhasorTransformed} V = V_o e^{j\alpha_v}$$

$$i(t) = I_o \cos(\omega t + \alpha_i) \xrightarrow{PhasorTransformed} I = I_o e^{j\alpha i}$$

V_o and α_v are known

I_o and α_i are unknown and it is the solution we search

Using the above defined phasors the differential equation in (2) can be written as:

$$\Re\left\{\underbrace{j\omega \cdot V_o e^{j\alpha_v}}_{j\omega \cdot V} \cdot e^{j\omega t}\right\} = L\Re\left\{\underbrace{-\omega^2 \cdot I_o e^{j\alpha i}}_{-\omega^2 \cdot I} \cdot e^{j\omega t}\right\} + R \cdot \Re\left\{\underbrace{j\omega \cdot I_o e^{j\alpha i}}_{j\omega \cdot I} \cdot e^{j\omega t}\right\} + \frac{1}{C}\Re\left\{\underbrace{I_o e^{j\alpha i}}_{I} \cdot e^{j\omega t}\right\}$$

Writing the algebraic expression that results after the Phasor Transformation of the above equation we get

$$j\omega \cdot V = -L\omega^2 \cdot I + j\omega R \cdot I + \frac{1}{C} \cdot I$$

$$V = j\omega L \cdot I + R \cdot I + \frac{1}{j\omega C} \cdot I$$

Solution using Laplace

KVL in Laplace domain gives:

$V(s) = RI(s) + sLI(s) + \frac{1}{sC} I(s)$, where initial conditions are disregarded since we are only interested in the resulting phasor domain equivalent. Replacing $s = j\omega$ gives:

$$V_o e^{j\alpha_v} = RI_o e^{j\alpha_i} + j\omega L I_o e^{j\alpha_i} + \frac{1}{j\omega C} I_o e^{j\alpha_i}$$

Both approaches will give full score as long as each step in the derivation is correct. Also, students that found incorrect differential equation in 4a can still get full score if their derivation in 4b is correct (følgefeil is never punished).

In the above equations the phasors are based on peak amplitudes (not RMS). Students that use RMS-phasors also get full score.

Based on the analysis of the expression obtained in b) demonstrate the following:

c) Demonstrate that the impedance of the series RLC circuit is $Z = R + j(\omega L - \frac{1}{\omega C})$ and that the current phasor is $I = \frac{V}{\left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right)}$

Answer:

By using the expression in b), the impedance can be found as:

$$\frac{V_o e^{j\alpha_v}}{I_0 e^{j\alpha_i}} = \frac{V}{I} = Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

And the current can be found similarly:

$$I = \frac{V}{Z} = \frac{V}{R + j\omega L + \frac{1}{j\omega C}} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Many students found these expressions even with wrong answers on problem 4a and 4b, and will then get full score. We have been relatively kind with points on this subproblem.

d) Write the expression of the current in time domain, i(t), as a function of the current phasor parameters found in part c)

Answer: We already know the current in phasor domain:

$$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} e^{j \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$

This can be converted into time-domain directly using the inverse phasor transform:

$$i(t) = \mathrm{Re}\left(Ie^{j\omega t}\right) = \mathrm{Re}\left(\frac{V_o e^{j\alpha_v}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} e^{-j\arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)} e^{j\omega t}\right)$$

$$= \mathrm{Re}\left(\frac{V_o e^{j\omega t + j\alpha_v - j\arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right) = \frac{V_o \cos\left(\omega t + \alpha_v - \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

It is also OK to write the expression directly from the phasor equation without using the formal definition with $\mathrm{Re}(Ie^{j\omega t})$ *. Students with correct answer get full score.*

The following mistakes are considered minor, and will only give a small point reduction:

- *Missing* α_v *in the final expression*
- *Sign error in one of the angles*

The following mistakes are more severe and will give a medium point reduction

- *Missing* ωt *in the final answer*
- *Incorrect use of* $\sqrt{2}$ *in the final answer*

Students that include j in the final answer will get a large point reduction since it does not make sense to have a complex number in a time-domain expression.