1 RLC CIRCUIT

Figure 1 shows a RLC-circuit. The input voltage is $v_{in}(t)$, while the output voltage $v_o(t)$ is defined as the voltage across the LC-branch.

Assume $v_{in}(t) = 10\cos(\omega t)$ throughout the entire problem. $R = 1\Omega$ and $L = 1\,mH$, while *C* is unknown.

We will first assume this circuit is used as a filter with input $v_{in}(t)$ and output $v_o(t)$, and $H(s) = \frac{V_o(s)}{V_{in}(s)}$

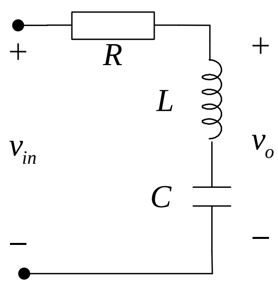


Figure 1 - RLC-circuit

a) What kind of filter does $H(s)$ represent? (lowpass, highpass, band-pass or band-stop)? Give reasons for your answer.

b) We want $v_o(t)$ to have as small amplitude as possible when the frequency is 250 Hz. Find the value of *C* that minimizes the amplitude of $v_o(t)$ at this frequency. What is the value of the minimum amplitude?

Assume now the frequency is 50 Hz. If you did not solve *b),* use $C = 1\,mF$ in problems c) and d).

c) How much active and reactive power is supplied from the voltage source? Is the circuit inductive or capacitive (give reasons for your answer)?

Assume now we connect another impedance Z_{load} in parallel to the LC branch as shown in Figure 2.

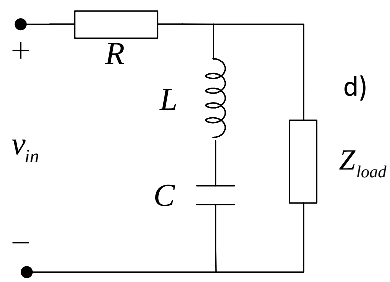
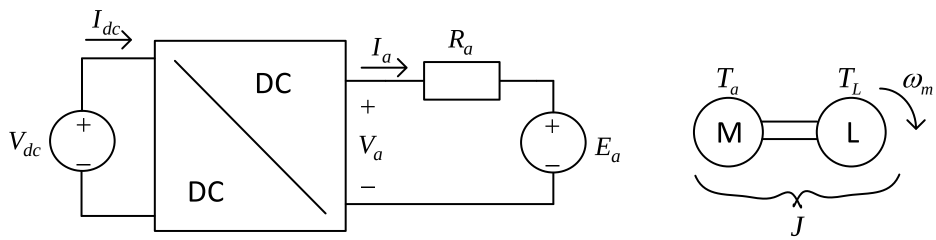


Figure 2 – Adding load to Figure 1

d) Find the value of Z_{load} that maximizes the active power consumption in Z_{load}.

2 DC-MOTOR DRIVEN BY BATTERY AND DC/DC CONVERTER



The figure shows a battery with voltage V_{dc} connected to a DC-motor with resistance R_a and induced voltage E_a through a DC/DC-converter. The DC/DC-converter control system can set the required duty-cycle *D* defined as $D = \frac{V_a}{V_{dc}}$. The mechanical load has a load torque T_L.

a) The field winding is not shown in the figure. Make a complete schematic of a shunt-excited DC-machine, including the field winding resistance R_f and field winding inductance L_f.

Assume in b) and c) that $T_L = k_L \omega_m$, where ω_m is the rotational speed of the shaft. The system has no mechanical gearbox. In problems b) and c), assume the motor operates in steady-state, i.e. no acceleration. Some useful formulas for the DC-motor is included in appendix.

Numerical values: $R_a = 0.1\,\Omega$, $k\varphi = 0.3\,\frac{Vs}{rad}$, $k_L = 0.1\,\frac{Nms}{rad}$ and $V_{dc} = 100\,V$

b) What duty-cycle is needed in order to have a shaft speed of $\omega_m = 200$ rad/s ?

c) Find an expression for the motor speed ω_m expressed by k_L, D, V_{dc}, k, R_a, as well as the motor constant *k* and the flux φ (these are the only variables you should include in the expression)

Assume now that the load torque is zero, $T_L = 0$. Assume also the system has a total moment of inertia equal to J. At $t = 0$, the DC/DC-converter makes a step in duty cycle from $D = 0$ to $D = D_0$.

d) Find the expression for the motor speed $\omega_m(t)$ for $t \ge 0$ (only variables, no numerical values).

3 CIRCUIT CALCULATIONS INCLUDING IRON CORE

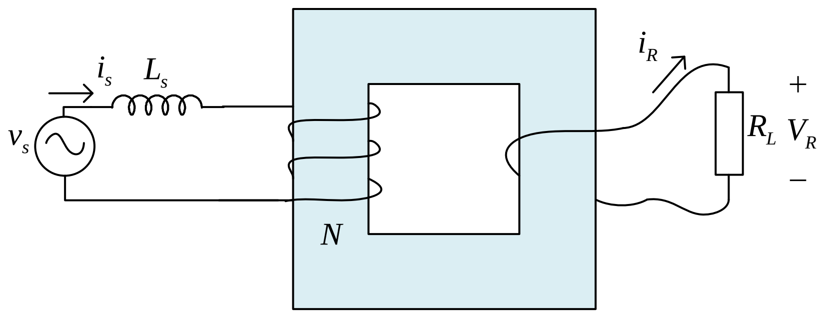


Figure 3 – Voltage source, iron core and load resistance

A resistor is connected to a voltage source through an iron core as presented in Figure 3. The core has $N = 3$ windings on the voltage source side, and a single winding at the load resistor side. We make the following assumptions:

- All induced magnetic field is kept within the core
- The iron is ideal, i.e. $\mu_r = \infty$
- The core will not saturate

a) Explain why we can treat the iron core as an ideal transformer with ratio *N*.
b) If i_s is positive, is i_R positive or negative? Explain why.

If the frequency is $f = 50\ Hz$, $v_s = 10\ V\ (RMS)$ and $R_L = 0.5\ \Omega$, the current i_s has an RMS-value of $2\ \text{A}$.

c) Find the numerical value of the inductor *L*
d) Derive an expression for the power consumed by the resistor R_L. No numerical values, only with symbols (v_s, L_s, N, R_L, f)

4 Diode Rectifier

Figure 4 shows a diode rectifier where the AC-voltage is a triangular wave. In problem a) and b), the DC-side is a constant current source I_{dc} , while in problem c) and d) it is represented by a resistor R.

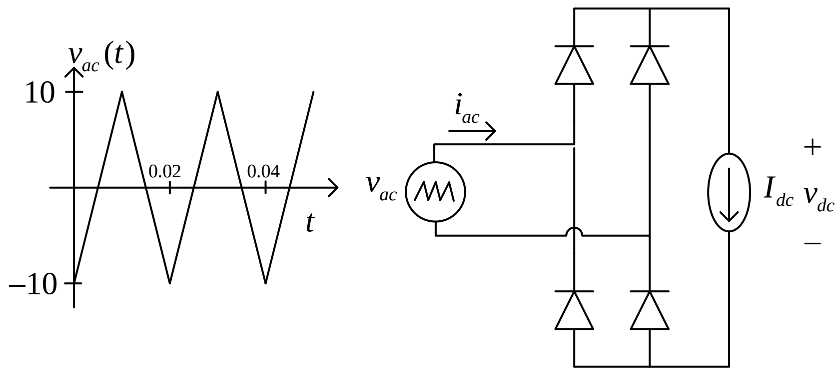


Figure 4: Diode rectifier supplied by a triangular voltage source. Constant DC-current

Assume $I_{dc} = 5A$

a) Find and sketch the DC-side voltage $v_{dc}(t)$ and the AC-side current $i_{ac}(t)$ for $0 \le t \le 0.04\,s$.

b) What is the average power delivered from the AC voltage source?

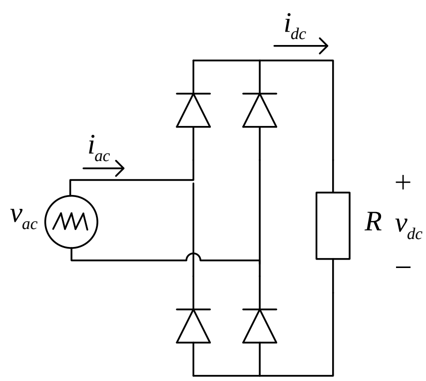


Figure 5 - Diode rectifier supplied by a triangular voltage source. Resistive DC-load

Now the constant current source is replaced with a resistor $R = 2\,\Omega$ as shown in Figure 5.

c) Find and sketch the DC-side current $i_{dc}(t)$ and AC-side current $i_{ac}(t)$ for $0 \le t \le 0.04\,s$.

d) What is the average power delivered from the AC voltage source?

APPENDIX: FORMULAS

Inductance and capacitance

$$v_L = L\frac{di_L}{dt} \ , \ i_C = C\frac{dv_C}{dt} \ , \ X_L = \omega L \ , \ X_C = \frac{1}{\omega C}$$

Phasors and complex power

$$X\cos(\omega t + \theta) \Leftrightarrow Xe^{j\theta} \quad , \quad S = VI^* = P + jQ$$

Electromagnetism:

$$\varepsilon = N\frac{d\varphi}{dt}, NI = \Re\varphi, \Re = \frac{l}{\mu_0\mu_r A} ,$$

$$\varphi = BA, \mu_0 = 4\pi \cdot 10^{-7}$$

Trigonometrics

$$\cos(2x) = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Three-phase

$$|V_{LL}| = \sqrt{3}|V_{ph}|$$

Electrical machines

$$f_{el} = \frac{p}{2} f_{mech}$$

Cut-off frequency

$$|H(j\omega_c)| = \frac{|H_{\max}|}{\sqrt{2}}$$

DC-machine

$$E_a = K\varphi\omega \qquad T_a = K\varphi I_a$$

Induction (asynchronous) machine

$$\omega_{mech} = (1 - s)\omega_s$$

DC-DC converters

Buck (step-down): $D = \frac{V_{out}}{V_{in}}$

Boost (step-up): $\frac{V_{out}}{V_{in}} = \frac{1}{1-D}$

Inverters

Single-phase: $m = \frac{\sqrt{2}|V_{ac}|}{V_{dc}}$

Three-phase: $m = \frac{2\sqrt{2}|V_{LL}|}{\sqrt{3}V_{dc}}$

Mechanics

$$P = T\omega \quad P = F \cdot v \quad v = \omega r \quad E_k = \frac{1}{2}mv^2$$

$$T_{mot} - T_{load} = J\frac{d\omega}{dt}$$

Laplace transforms

Constant: $\mathcal{L}(K \cdot f(t)) = K \cdot F(s)$

Step response: $\mathcal{L}(u(t)) = \frac{1}{s}$

Exponential: $\mathcal{L}(e^{at}) = \frac{1}{s-a}$

s-shift: $\mathcal{L}\{e^{-at} \cdot f(t)\} = F(s+a)$

Sine: $\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$

Cosine: $\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$

Damped sine: $\mathcal{L}\{e^{-at}\sin(\omega t)\} = \frac{\omega}{(s+a)^2+\omega^2}$

Damped cosine: $\mathcal{L}\{e^{-at}\cos(\omega t)\} = \frac{s+a}{(s+a)^2+\omega^2}$