LF Kont 2016 - TTK4240

Disclaimer – dette LF er ikkje kvalitetssikra. Meld gjerne ifrå om feil

Oppgave 1: Kretsanalyse

a) Thevenin-impedansen er gitt av:

$$Z_{th} = j\omega L + R \,||\, \frac{1}{j\omega C} = j\omega L + \frac{R}{1 + j\omega CR}$$

Thevenin-spenningen er gitt av spenningen over ZL når ZL er uendelig stor. Dermed går det ingen strøm i spolen, og Vth blir lik spenningen over R:

$$V_{th} = V_R \cdot \frac{R}{R + \frac{1}{j\omega C}}$$

b) Maksimal overføring av gjennomsnittlig effekt (=aktiv effekt) oppnås når lastimedansen er den komplekskonjugerte av thevenininimpedansen: $Z_L = Z_{th}^*$. Setter inn tallverdier:

$$Z_{th} = j4000 + \frac{4000}{1+j} = j4000 + \frac{4000(1-j)}{(1+j)(1-j)} = j4000 + 2000 - j2000 = 2000 + j2000\,\Omega$$

Lastimpedansen blir da:

$$Z_L = Z_{th}^* = 2000 - j2000\,\Omega$$

c) Thevenin-spenningen finnes ved å sette inn tallverdier i a)

$$V_{th} = 10\frac{4000}{4000 + \frac{1}{j8000 * 31.25 * 10^{-9}}} = 10\frac{j}{1+j} = \frac{10}{\sqrt{2}}e^{j90 - j45} = \frac{10}{\sqrt{2}}e^{j45}$$

Finner så tidsuttrykket:

$$V_{th}(t) = \frac{10}{\sqrt{2}}\cos\left(8000t + 45 \cdot \frac{\pi}{180}\right)$$

d) Total impedans blir $Z_{tot} = Z_{th} + Z_L = 4000\,\Omega$, så vi kan finne strømmen gjennom lasten som:

$$I_L = \frac{V_{th}}{R} = \frac{\left(\frac{10}{\sqrt{2}}\right)\Big/\sqrt{2}}{4000} = \frac{1}{800}\,A$$ (NB: Må dele igjen på rot(2) pga. RMS)

Finner effekten i lasten som: $P_L = 2000 \cdot \left(\frac{1}{800}\right)^2 = 0.003125\ W$

Filter question

Geir Kulia

August 2016

1 English

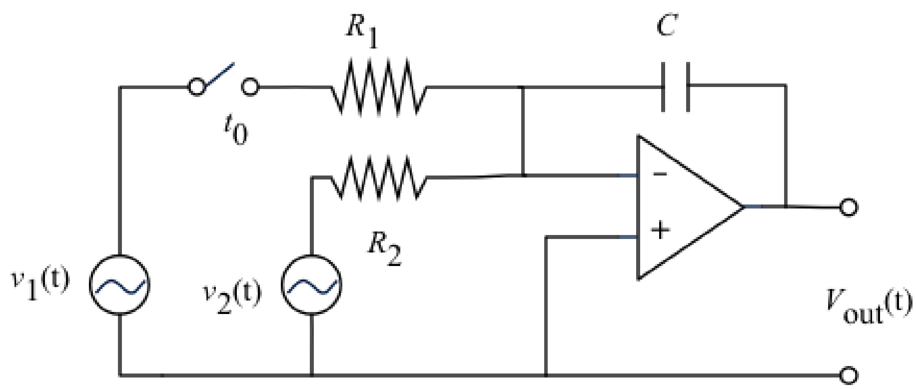


Figure 1: The circuit describes an active filter with two inputs. At $t = t_0$, the swich is turned on, conecting $v_1(t)$ to the filter.

Question 1

What are the characteristics of an ideal Op-amp?

Answer (3 min)

The IC Op-amp comes so close to ideal performance that it is useful to state the characteristics of an ideal amplifier without regard to what is inside the package.

Infinite voltage gain
Infinite input impedance
Zero output impedance
Infinite bandwidth
Zero input offset voltage (i.e., exactly zero out if zero in).

Question 2 (2 min)

Given arbitrary $v_1(t)$ and $v_2(t)$, find a mathematical expression of $v_{out}(t)$ in fig. 1, assuming no saturation.

Answer

For any ideal op-amp, we have that

$$v_+ = v_- = 0 \tag{1}$$

This gives us that

$$i_1(t) + i_2(t) = -i_c(t) \tag{2}$$

$$\frac{v_1(t) \cdot u(t - t_0)}{R_1} + \frac{v_2(t)}{R_2} = i_c(t) \tag{3}$$

And for the capacitor C the current is given by

$$i_c = C\frac{dv_{\text{out}}(t)}{dt} \tag{4}$$

$$v_{\text{out}}(t) = \frac{1}{C}\int_0^t i_c(t)dt \tag{5}$$

where $v_{\text{out}} = v_c$. This results in

$$v_{\text{out}}(t) = -\frac{1}{C}\int_0^t (\frac{v_1(t) \cdot u(t - t_0)}{R_1} + \frac{v_2(t)}{R_2})dt \tag{6}$$

Question 3

Given that $v_1(t) = v_2(t) = 1$ V, $C = 1$ μC, and $t_0 = 5$ ms choose any R_1 and R_2. Sketch $v_{\text{out}}(t)$ in the interval -5 ms $< t <$ 20 ms. Assume no saturation.

Answer

Choose , $R_1 = R_2 = 1$ KΩ.

$$v_{\text{out}}(t) = -\frac{1}{RC}\int_0^t \big(v_1(t) \cdot u(t - t_0) + v_2(t)\big)dt \tag{7}$$

$$v_{\text{out}}(t) = -1K \cdot \big((t - 5ms)u(t - 5ms) + t\big) \tag{8}$$

Question 4

Given that $R_1 = R_2 = R$. The op-amp achives saturation when -15 V $> v_{\text{out}} > 15$ V. Find the value of R so that the op-amp achieve saturation at 15 ms. Sketch $v_{\text{out}}(t)$ in the interval -5 ms $< t <$ 20 ms. Comment on the result.

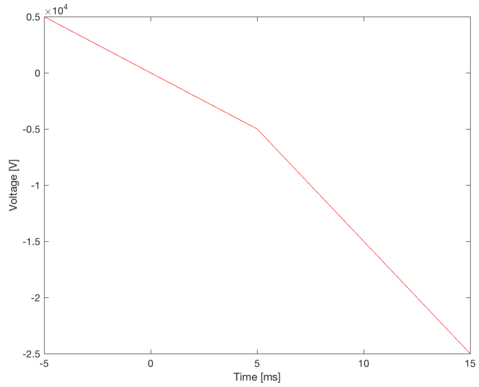


Figure 2: Solution to Problem 3

Answer

$$v_{\text{out}} = -\frac{2t - t_0}{RC}, t > t_0 \tag{9}$$

$$v_{\text{out}} = -15 \text{ V} \tag{10}$$

$$R = -\frac{2t - t_0}{C \cdot v_{\text{out}}}, t > t_0 \tag{11}$$

$$R = -\frac{10 \text{ ms} - 5 \text{ ms}}{10^{-6} \text{ C} \cdot -15 \text{ V}} \tag{12}$$

$$R = \frac{5 \cdot 10^{-3} \cdot 10^{6} \text{ s}}{15 \text{ VC}} \tag{13}$$

$$R = \frac{1}{3} \text{ K}\Omega \simeq 333\ \Omega (14)$$

Oppgave 3: Effektberegninger i motordrift

Figur 1 viser en skisse av en motordrift bestående av kraftforsyning, inverter, motor og last. Vi antar at motoren opererer uten gir, slik at turtallet til lasten er lik turtallet til motoren.

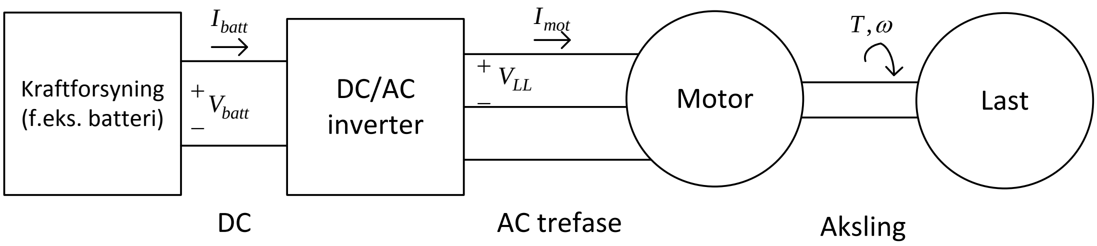
Følgende informasjon er oppgitt:

Batterispenning: *600 V (DC)*
Motorspenning: *400 V (linje-linje, RMS)*
Nominell effektfaktor: $\cos\varphi = 0.9$
Antall poler til motor: *4*
Nominelt turtall til last: $n_{mek,nom} = 1500$ o/min

Lastkarakteristikk: $T_{load} = 400 \cdot \left(\frac{\omega_{mek}}{\omega_{mek,nom}} \right)^2$ *Nm*

Anta at lasten driftes på nominelt turtall.

a) I nominelle forhold, vis at effekt overført til last blir lik $P_{nom} = 62.83\ kW$
b) Hva blir frekvensen til motorspenningen, i Hz?
c) Se bort fra alle tap i systemet. Hva blir batteristrømmen I_{batt} og strømmen i motoren $|I_{mot}|$?
d) Lastens effektbehov (målt i watt) reduseres så til det halve. Hva blir den nye frekvensen til motorspenningen?



Figur 1: Skisse av motordrift

Anta at lasten driftes på nominelt turtall.

e) I nominelle forhold, vis at effekt overført til last blir lik $P_{nom} = 62.83\ kW$

Svar: Regner først om lastens nominelle turtall til rad/s:

$$\omega_{mek,nom} = 1500 \left[\frac{\text{o}}{\text{min}} \right] \cdot \frac{1}{60} \left[\frac{\text{min}}{s} \right] \cdot 2\pi \left[\frac{\text{rad}}{\text{o}} \right] = 157.08 \text{ rad/s}$$

Ser at momentet blir lik $T_{load,nom} = 400 \cdot \left(\frac{\omega_{nom}}{\omega_{nom}} \right)^2 = 400\ Nm$, og effekten blir da lik:

$$P_{nom} = \omega_{mek,nom} T_{nom} = 157.08 \cdot 400 = 62.83\ kW$$

f) Hva blir frekvensen til motorspenningen, i Hz?

Denne finnes direkte fra antall poler siden vi ikke har mekanisk gir:

$$f_{el} = \frac{p}{2} f_{mek} = \frac{4}{2} \cdot 1500 \frac{\text{o}}{\text{min}} \cdot \frac{1}{60} \frac{\text{min}}{\text{s}} = 50\ Hz$$

g) Se bort fra alle tap i systemet. Hva blir batteristrømmen I_{batt} og strømmen i motoren $|I_{mot}|$?

Svar: Siden den samme effekten leveres fra batteriet har vi at

$$P_{nom} = V_{batt} I_{batt} \Rightarrow I_{batt} = \frac{P_{nom}}{V_{batt}} = \frac{62830}{600} = 104.7\ A$$

For å finne strømmen i motoren må vi ta hensyn til at dette er en trefasemotor. Finner først fasespenningen:

$$|V_{ph}| = \frac{|V_{LL}|}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\,V$$

Effekten per fase blir:

$$P_{ph} = \frac{P_{nom}}{3} = \frac{62.83}{3} = 20.94\ kW$$

Før vi kan finne strømmen må vi finne den tilsynelatende effekten. Effektfaktoren er $\cos\varphi = 0.9$, og vi har at $P = |S|\cos\varphi$. Dermed blir tilsynelatende effekt per fase lik:

$$|S_{ph}| = \frac{20.94}{0.9} = 23.27\ kVA$$

Dette gir motorstrømmen:

$$|I_{mot}| = \frac{|S_{ph}|}{|V_{ph}|} = \frac{23.27 \cdot 10^3}{230.94} = 100.76\ A$$

Lastens effektbehov reduseres så til det halve

h) Lastens effektbehov (målt i watt) reduseres så til det halve. Hva blir den nye frekvensen til motorspenningen?

Det nye effektbehovet blir dermed

$$P_L = \frac{P_{nom}}{2} = \frac{62.83}{2} = 31.415\,kW$$

Kan dermed finne det nye mekaniske turtallet:

$$P_L = T_{load}\,\omega_{mek} = 400\left(\frac{\omega_{mek}}{\omega_{mek,nom}}\right)^2 \omega_{mek}$$

$$\Rightarrow \omega_{mek} = \sqrt[3]{\frac{P_L \cdot \omega_{mek,nom}^2}{400}} = \sqrt[3]{\frac{31415 \cdot 157.08^2}{400}} = 124.67\ \text{rad/s}$$

Dette gir følgende frekvens til motorspenningen:

$$f_{el} = \frac{p}{2} f_{mek} = \frac{p}{2}\frac{\omega_{mek}}{2\pi} = \frac{4}{2}\frac{1}{2\pi} \cdot 124.67 = 39.68 \text{ Hz}$$

Oppgave 4: Elektromagnetisme:

The iron cores as shown in Figure 1 and 2 have a cross section of 2 cm by 2 cm and a relative permeability of 1000. The coil has 500 turns. Note that iron core in Figure 1 does not have any air gap and iron core in Figure 2 has two air gaps with a length of 1 cm and 0.5 cm, respectively.

1. Determine the current required to establish a flux density of 0.6 T in the center of the coil for iron core 1 (Figure 1).

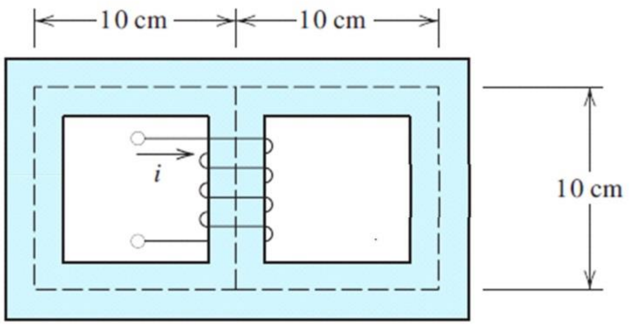


Fig 1: Iron Core 1

2. Determine the current required to establish a flux density of 0.6 T in the center of the coil for iron core 2 (Figure 2).
3. Find the flux density in each air gap of iron core 2.
4. Determine the inductance of the coil for both iron cores.

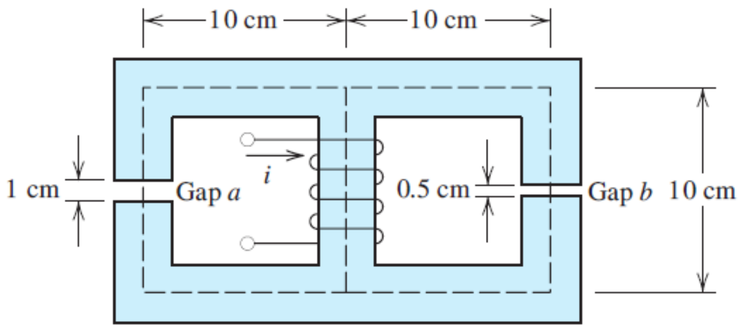


Fig 2: Iron Core 2

Formula:

$$\varepsilon = N\frac{d\varphi}{dt} \ , \ NI = \Re\varphi \, , \ \Re = \frac{l}{\mu A} \ , \ \varphi = BA \ , \ L = \frac{N^2}{\Re}$$

$$\Re_{sereis} = \Re_1 + \Re_2 + \ldots$$
$$\frac{1}{\Re_{parallel}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots$$

Solution:

1. Determine the current required to establish a flux density of 0.6 T in the center of the coil for iron core 1 (Figure 1).

First, we compute the reluctances of the three paths. For the center path, we have

$$\Re_{middle} = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} = \frac{10x10^{-2}}{1000x4\pi x10^{-7}x4x10^{-4}} = 1.9894x10^5 \text{ A}.turns/Wb$$

Since all the dimensions are same for both left and right path, reluctance will be the same for right and left path.

$$\Re_{left} = \Re_{right} = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} = \frac{30x10^{-2}}{1000x4\pi x10^{-7}x4x10^{-4}} = 5.9683x10^5 \text{ A}.turns/Wb$$

Total reluctance

$$\Re_{total} = \Re_{middle} + \frac{\Re_{left}\Re_{right}}{\Re_{left} + \Re_{right}} = 1.9894x10^5 + \frac{5.9683x10^5}{2} = 4.9736x10^5 \text{ A}.turns/Wb$$

We have flux density in the center leg is 0.6 T. Total flux,

$$\varphi = BA = 0.6x4x10^{-4} = 240\mu Wb$$

Current required to established that flux

$$NI = \Re\varphi, \Rightarrow I = \frac{\Re\varphi}{N} = \frac{4.9736x10^5x240x10^{-6}}{500} = 0.2387 \text{ A}$$

2. Determine the current required to establish a flux density of 0.6 T in the center of the coil for iron core 2 (Figure 2).

Reluctance of the center path remains the same; however, the reluctance of the left and right will not be the same because of the air gaps.

Comment: Students that do not compensate for fringing by increasing the air gap area get full score.

We first calculate the area of the left air gap

A_{gap}=(2+1)x(2+1)=9x10^{-4} m^2

$$\Re_{left} = \Re_{gap} + \Re_{core} = \frac{l}{\mu_0 A_{gap}} + \frac{l}{\mu_r \mu_0 A_{core}}$$
$$= \frac{1x10^{-2}}{4\pi x10^{-7}x9x10^{-4}} + \frac{29x10^{-2}}{1000x4\pi x10^{-7}x4x10^{-4}} = 9.420x10^6 \text{ A}.turns/Wb$$

Similarly, we calculate the area of the right air gap

A_{gap}=(2+0.5)x(2+0.5)=6.25x10^{-4} m^2

$$\Re_{right} = \Re_{gap} + \Re_{core} = \frac{l}{\mu_0 A_{gap}} + \frac{l}{\mu_r \mu_0 A_{core}}$$
$$= \frac{0.5x10^{-2}}{4\pi x10^{-7} x6.25x10^{-4}} + \frac{29.5x10^{-2}}{1000x4\pi x10^{-7} x4x10^{-4}} = 6.953x10^6 \text{ A.}turns/Wb$$

Total reluctance

$$\Re_{total} = \Re_{middle} + \frac{\Re_{left}\Re_{right}}{\Re_{left} + \Re_{right}} = 4.199x10^6 \text{ A.}turns/Wb$$

We have flux density in the center leg is 0.6 T.

Total flux in the center leg,

$$\varphi_{total} = BA = 0.6x4x10^{-4} = 240\mu Wb$$

Current required to established the flux

$$NI = \Re\varphi, \Rightarrow I = \frac{\Re\varphi}{N} = \frac{4.199x10^6 x240x10^{-6}}{500} = 2.0155 \text{ A}$$

3. Find the flux density in each air gap of iron core 2.

Fluxes are analogous to currents. Thus, we use the current-division principle to determine the flux in the left-hand and right-hand paths, resulting in

$$\varphi_{left} = \varphi_{total}\frac{\Re_{right}}{\Re_{left} + \Re_{right}} = 240x10^{-6}\frac{6.953x10^6}{9.420x10^6 + 6.953x10^6} = 101.919 \ \mu\text{Wb}$$
$$\varphi_{right} = \varphi_{total}\frac{\Re_{left}}{\Re_{left} + \Re_{right}} = 240x10^{-6}\frac{9.420x10^6}{9.420x10^6 + 6.953x10^6} = 138.081 \ \mu\text{Wb}$$

Now, we find the flux densities in the gaps by dividing the fluxes by the areas:

$$B_{left} = \frac{\varphi_{left}}{A_{left}} = \frac{101.919\mu\text{Wb}}{9x10^{-4}} = 0.113\text{T } T$$
$$B_{right} = \frac{\varphi_{right}}{A_{right}} = \frac{138.081 \ \mu\text{Wb}}{6.25x10^{-4}} = 0.221 \ T$$

4. Determine the inductance of the coil for both iron cores.

Inductance of iron core 1, $L = \frac{N^2}{\Re} = \frac{500^2}{4.9736x10^5} = 0.5027\,H$

Inductance of iron core 2, $L = \frac{N^2}{\Re} = \frac{500^2}{4.199x10^6} = 0.0595\,H$