NORWEGIAN UNIVERSITY
OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Contact during examination:
Name: Magne H. Johnsen
Tel.: 930 25 534

# EXAMINATION IN COURSE TTT4120 DIGITAL SIGNAL PROCESSING

Date: Wednesday, August 12th 2009
Time: 09.00 - 13.00

Permitted aids: D–No calculators allowed.
No printed or handwritten materials allowed.

## INFORMATION

- The examination includes 4 problems.
  - Problem 1 deals with basic properties of systems/filters.
  - Problem 2 deals with multirate systems.
  - Problem 3 deals with filter structures.
  - Problem 4 deals with stationary processes and parametric estimation.
- The weight of each subproblem is given in parenthesis. The total number of points are 100.
- **The solution steps should be evident and all answers should be justified!**
- Some basic formulas can be found in the appendix.
- The teacher will go around twice, around 10.00 and around 11.45.

## Problem 1 (6+8+6+5=25 points)

**1a)** Which properties have to be fulfilled in order to describe a system by its unit pulse response $h(n)$?

Define the two properties stability and causality in terms of $h(n)$.

**1b)** Define the z-transform $H(z)$ in terms of $h(n)$, $n = -\infty, \infty$.

What is meant by the term "region of convergence" (ROC) of the transfer function $H(z)$?

Sketch ROC in the z-plane for respectively a causal and anti-causal system.

What area in the z-plane must be included in ROC if the system is to be stable? State the reason for your answer.

**1c)** Given a filter with the following transfer function

$$H(z) = \frac{1 - bz^{-1}}{1 + az^{-1}} \tag{1}$$

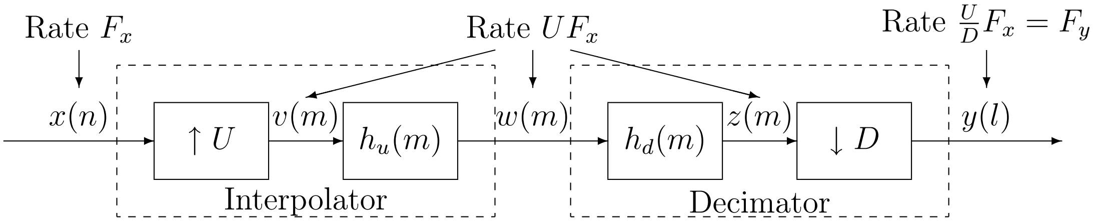
Given $a \geq 0$ and $b \geq 0$.

Give the possible values for the filter coeffisients $\{a, b\}$ and the corresponding ROC when the system is stable, causal and has minimum phase.

**1d)** Derive the expression for the unit pulse response $h(n)$.

## Problem 2 **(7+8+8=25 points)**

Given the following block diagram for a general samplingrate converter. Assume ideal filters.




**2a)** Give the expression for $w(m)$ in terms of $x(n)$ and $h_u(m)$.

Give the expression for $y(l)$ in terms of $w(m)$ and $h_d(m)$.

Give the expression for $y(l)$, in terms of $x(n)$ and $h(m) = h_u(m) * h_d(m)$.

$$x(n) = x_a(nT_x) = \sin(2\pi F_1 nT_x) + 2\sin(2\pi F_2 nT_x) \tag{2}$$

In the remaining part of problem 2 the input signal is given by equation 2 where $F_1 = 100$ Hz, $F_2 = 200$ Hz and $F_x = \frac{1}{T_x} = 1000$ Hz

**2b)** Sketch $|X(f)|$ for $f \in [0, 0.5]$ where $f = \frac{F}{F_x}$.

Sketch $|Y(f)|$ for $f \in [0, 0.5]$ where $f = \frac{F}{F_y}$ and give the cut-off frequency for the filter $h(m)$ in Hz (real frequency) for $\frac{U}{D} = \frac{1}{2}$

Sketch $|Y(f)|$ for $f \in [0, 0.5]$ where $f = \frac{F}{F_y}$ and give the cut-off frequency for the filter $h(m)$ in Hz (real frequency) for $\frac{U}{D} = \frac{2}{1}$

**2c)** Sketch $|Y(f)|$ for $f \in [0, 0.5]$ where $f = \frac{F}{F_y}$ and give the cut-off frequency for the filter $h(m)$ in Hz (real frequency) for $\frac{U}{D} = \frac{2}{3}$

Sketch $|Y(f)|$ for $f \in [0, 0.5]$ where $f = \frac{F}{F_y}$ and give the cut-off frequency for the filter $h(m)$ in Hz (real frequency) for $\frac{U}{D} = \frac{1}{3}$
Comment on the result.

## Problem 3 **(6+8+5+6=25 points)**

**3a)** Given the following stable filter $H_1(z)$.

$$H_1(z) = \frac{z^{-1} - a}{1 - az^{-1}} \tag{3}$$

Show that the filter is allpass.

Discuss the possibility of that the filter can have minimum phase.

In the remaining part of problem 3 the filter coeffisient is set to $a = \frac{1}{2}$.

**3b)** Given a stable, causal filter $H(z)$ on the form

$$H(z) = H_1(z)H_2(z) \tag{4}$$

where $H_1(z)$ is given in problem 3a (with $a = \frac{1}{2}$) and $H_2(z)$ is given as

$$H_2(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \tag{5}$$

Show that $H(z)$ can be transformed to the following parallel form

$$H(z) = H_3(z) + H_4(z) = \frac{\frac{3}{4}}{1 - \frac{1}{2}z^{-1}} + \frac{-\frac{5}{4}}{1 + \frac{1}{2}z^{-1}} \tag{6}$$

**3c)** Derive the unit pulse response of $H(z)$

**3d)** Sketch respectively the Direct Form 2 (DF2) and parallel structures for $H(z)$.

## Problem 4 **(7+6+4+8=25 points)**

White noise $w(n)$ with power $\sigma_w^2$ is input to a general causal, stable filter with unit pulse response $g(n) \quad n = 0, \infty$.

The autocorrelation function $\gamma_{yy}(m), \quad m = -\infty, \infty$, and the power spectrum $\Gamma_{yy}(z)$ of the resulting output signal $y(n)$ are given by

$$\gamma_{yy}(m) = \begin{cases} \sigma_w^2 \sum_{n=0}^{\infty} g(n)g(n+m) & m \geq 0 \\ \gamma_{yy}(-m) & m < 0 \end{cases} \tag{7}$$

$$\Gamma_{yy}(z) = \sigma_w^2 \; G(z)G(z^{-1}) \tag{8}$$

In problem 4 the filters $H_1(z)$ and $H_3(z)$ are given by problem 3.

**4a)** Find the autocorrelation function of the output signal $y(n)$ when $G(z) = H_3(z)$.

**4b)** Find the autocorrelation function of the output signal $y(n)$ when $G(z) = H_1(z)$.

**4c)** Which kind of parametric process does the output signal $y(n)$ represent when white noise is input to respectively $H_1(z)$ and $H_3(z)$?

**4d)** Find the process parameters to the best AR[1]-model of the output signal $y(n)$ when the filter is given by respectively $H_1(z)$ and $H_3(z)$.

Give a short argument for the results.

# Some basic equations and formulaes.

**A. Sequences :**

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \Longleftrightarrow |\alpha| < 1$$
$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

**B. Linear convolution :**

$$y(n) = h(n) * x(n) = \sum_k h(k)x(n-k) = \sum_k x(k)h(n-k)$$
$$Y(z) = H(z)X(z) \Rightarrow Y(f) = H(f)X(f) \Rightarrow Y(k) = H(k)X(k) \quad k = 0, ..., N-1$$

**C. Transforms :**

$$H(z) = \sum_n h(n)z^{-n} \Rightarrow H(f) = \sum_n h(n)\ e^{-j2\pi nf}$$
$$H(k) = \sum_n h(n)\ e^{-j2\pi nk/N} \qquad k = 0, ..., N-1$$
$$h(n) = \sum_k H(k)\ e^{j2\pi nk/N} \qquad n = 0, ..., N-1$$

**D. The sampling theorem :**

$$x(n) = x(nT_s) = x_a(t)|_{t=nT_s} \qquad n = -\infty, ...., \infty$$

$$X(f) = X(F/F_s) = F_s \sum_k X_a[(f-k)F_s]$$

**E. Autocorrelation, energy spectrum and Parsevals theorem :**

Given a sequence $x(n)$ with finite energy $E_x$ :

Autocorrelation : $r_{xx}(m) = \sum_n x(n)x(n+m) \quad m = -\infty, ...., \infty$

Energy spectrum : $S_{xx}(z) = X(z)X(z^{-1}) \Rightarrow S_{xx}(f) = |X(f)|^2$

Parsevals theorem: $E_x = \sum_n x^2(n) = \int_0^{2\pi} |X(f)|^2 df = \int_0^{2\pi} S_{xx}(f) df$

**F. Autocorrelation, power spectrum and Wiener-Khintchin theorem :**

Given a stationary, ergodic sequence $x(n)$ with infinite energy :

Autocorrelation : $\gamma_{xx}(m) = E[x(n)x(n+m)] \; m = -\infty, ...., \infty$

Power spectrum: $\Gamma_{xx}(z) = Z[\gamma_{xx}(m)] \quad \Rightarrow$

Wiener-Khintchin : $\Gamma_{xx}(f) = DTFT[\gamma_{xx}(m) = \sum_m \gamma_{xx}(m) \; e^{-j2\pi mf}$

**G. The Yule-Walker and Normal equations where $a_0 = 1$ :**

Yule-Walker equations : $\sum_{k=0}^{P} a_k \gamma_{xx}(m-k) = \sigma_f^2 \; \delta(m) \quad m = 0, ..., P$

Normal equations: $\sum_{k=1}^{P} a_k \gamma_{xx}(m-k) = -\gamma_{xx}(m) \quad m = 1, ..., P$