NORWEGIAN UNIVERSITY
OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Contact during examination:
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**EXAMINATION IN COURSE**
**TTT4120 DIGITAL SIGNAL PROCESSING**

Date: Thursday, 19 August 2010
Time: 15.00 - 19.00

Permitted aids: D–No printed or hand-written material allowed.
Specified, simple calculator allowed.

**INFORMATION**

- The examination consists of 4 problems:
  - Problem 1 concerns analysis of digital filters.
  - Problem 2 concerns finite word length effects in filter structures.
  - Problem 3 concerns stocastical processes.
  - Problem 4 concerns sampling in frequency domain and DFT.

  The weight of each subproblem is given in parenthesis. Total number of points is 56.
- **The solution steps should be evident and all answers should be justified!**
- Some important formulae can be found in the appendix.
- The teacher will go around twice, around 16.00 and around 17.45.
- Grades will be announced by September 9th.

## Good luck!

## Problem 1 (3+6+9+4=22 points)

A discrete-time system with transfer function $H(z)$ consists of two subsystems connected in series. The first subsystem is anticausal and is given by its transfer function

$$H_1(z) = \frac{1}{1 - 3z^{-1}}.$$

The second subsystem is causal and is given by the following difference equation

$$2y(n) - y(n-1) = x(n) - 2x(n-1),$$

where $x(n)$ and $y(n)$ are the input and the output signal of the subsystem, respectively.

**1a)** Sketch the region of convergency (ROC) for the transfer functions of the subsystems $H_1(z)$ and $H_2(z)$, and their serial connection $H(z)$.

**1b)** Show that the unit pulse responses of the two subsystems are given by

$$\begin{aligned} h_1(n) &= \begin{cases} -3^n & n < 0 \\ 0 & n \geq 0 \end{cases} \\ h_2(n) &= \begin{cases} 0 & n < 0 \\ \frac{1}{2} & n = 0 \\ -3\left(\frac{1}{2}\right)^{n+1} & n > 0, \end{cases} \end{aligned}$$

and that the unit pulse response of the entire system is given by

$$h(n) = \begin{cases} -\frac{3^n}{5} & n < 0 \\ \frac{3}{5}\left(\frac{1}{2}\right)^{n+1} & n \geq 0. \end{cases}$$

**1c)** Find out whether the two subsystems and their serial connection are stable?

Are they FIR- or IIR systems?

Do they have a linear phase response?

Determine their type (lowpass, highpass, bandpass, bandstop or all-pass).

**1d)** Draw the direct form II structure of the subsystem $H_2(z)$.

Draw the cascade structure of the entire system $H(z)$.

## Problem 2 (2+3+4+4=13 points)

Figure 1 shows the parallel structure of the filter $H(z)$ from Problem 1.

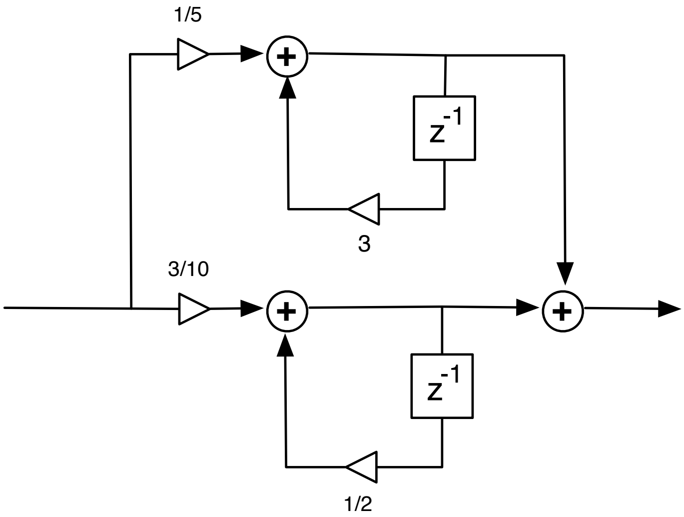



Figure 1: Implementation of a digital filter

All internal and external signals in the filter structure are represented by binary fixed-comma representation with 4 bits and the dynamic range $[-1, 1)$. Rounding is performed after each multiplication. Assume that the rounding error can be considered as uniformly distributed white noise.

**2a)** Derive an expresion for rounding error power $\sigma_e^2$ after a multiplication.

Find also the value of $\sigma_e^2$.

**2b)** Show that the signal power at the output of an LTI filter with unit sample response $g(n)$ is equal to

$$\sigma_q^2 = \sigma_e^2 \, r_{gg}(0)$$

when white noise $e(n)$ with power $\sigma_e^2$ is sent at the input.

**2c)** Find the total rounding error power at the output of the filter in Figure 1.

**2d)** Find the necessary scaling factor at the input of the filter in Figure 1 in order to avoid overflow.

## Problem 3 (1+4+5+2=12 points)

A stocastical process $x(n)$ is generated by sending white Gaussian noise $w(n)$ with varianse $\sigma_w^2 = 1$ through a system with transfer function $H(z) = 1 - 2z^{-1}$.

**3a)** Which type of prosess is $x(n)$?

**3b)** Find the autocorrelation function $\gamma_{xx}(l)$ and the power spectral density $\Gamma_{xx}(\omega)$ of the process.

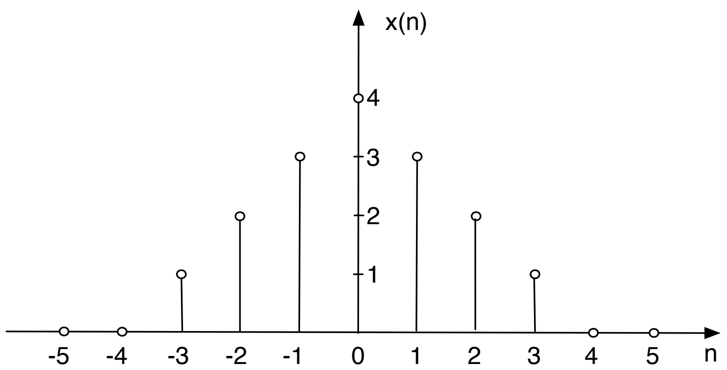
**3c)** We wish to model the process by an optimal AR(2) model.

Find the model parameters

**3d)** Find an expression for the power spectral density estimator $\hat{\Gamma}_{xx}(\omega)$ based on the optimal AR(2) model.

## Problem 4 (2+4+3=9 points)

A discrete-time signal $x(n)$ is given by the following figure.




**4a)** Find the spectrum $X(\omega)$ of the signal $x(n)$.

**4b)** We take samples of the spectrum $X(\omega)$ in the following way

$$X(k) = X(\omega)\big|_{\omega=\frac{2\pi k}{N}}.$$

Sketch the signal $y(n)$ which has spectrum given by $X(k)$.

How do we have to chose $N$ in order to be able to recover the signal $x(n)$ from the spectral samples $X(k)$?

**4c)** Explain the principle of the overlap-add method for filtering very long sequences through an FIR filter using DFT.