CHAPTER 8

INFORMATION ECONOMICS

In the neoclassical theory of consumer and firm behavior, consumers have perfect information about important features of the commodities they buy, such as their quality and durability. Firms have perfect information about the productivity of the inputs they demand. Because of this, it was possible to develop separately the theories of consumer demand and producer supply, and thereafter simply put them together by insisting on market-clearing prices.

One might hope that extending consumer and producer theory to include imperfect information would be as simple as incorporating decision making under uncertainty into those neoclassical models of consumer and producer behavior. One might then derive theories of demand and supply under imperfect information, and simply put the two together once again to construct a theory of market equilibrium. Unfortunately, this approach would only make sense if the sources of the uncertainty on both sides of the market were exogenous and so not under the control of any agent involved.

Of course, the quality and durability of a commodity, for example, are not exogenous features. They are characteristics that are ultimately chosen by the producer. If consumers cannot directly observe product quality before making a purchase, then it may well be in the interest of the producer to produce only low-quality items. Of course, knowing this, consumers will be able to *infer* that product quality must be low and they will act accordingly. Thus, we cannot develop an adequate equilibrium theory of value under imperfect information without taking explicit account of the relevant *strategic* opportunities available to the agents involved. Notably, these strategic opportunities are significantly related to the *distribution of information* across economic agents.

A situation in which different agents possess different information is said to be one of **asymmetric information**. As we shall see, the strategic opportunities that arise in the presence of asymmetric information typically lead to *inefficient market outcomes*, a form of **market failure**. Under asymmetric information, the First Welfare Theorem no longer holds generally.

Thus, the main theme to be explored in this chapter is the important effect of asymmetric information on the efficiency properties of market outcomes. In the interest of simplicity and clarity, we will develop this theme within the context of one specific market: the market for insurance. By working through the details in our models of the insurance market, you will gain insight into how theorists would model other markets with similar informational asymmetries. By the end, we hope to have stimulated you to look for analogies and applications in your own field of special interest.

8.1 ADVERSE SELECTION

8.1.1 INFORMATION AND THE EFFICIENCY OF MARKET OUTCOMES

Consider a market for auto insurance in which many insurance companies sell insurance to many consumers.

Consumers are identical except for the exogenous probability that they are involved in an accident. Indeed, suppose that for $i = 1, 2, \ldots, m$, consumer i's accident probability is $\pi_i \in [0, 1]$, and that the occurrence of accidents is independent across consumers.[1] Otherwise, consumers are identical. Each has initial wealth w, suffers a loss of L dollars if an accident occurs, and has a continuous, strictly increasing, strictly concave von Neumann-Morgenstern utility of wealth function $u(\cdot)$. Consumers behave so as to maximize expected utility.

Insurance companies are identical. Each offers for sale full insurance only. That is, for a price, they promise to pay consumers L dollars if they incur an accident and zero dollars otherwise. For the moment, we will suppose that this full insurance policy is a lumpy good—that fractional amounts can be neither purchased nor sold. We'll also suppose that the cost of providing insurance is zero. Thus, if the full insurance policy sells for p dollars and is purchased by consumer i, then the insurance company's expected profits from this sale are $p - \pi_i L$. Insurance companies will be assumed to maximize expected profits.

Symmetric Information

Consider the case in which each consumer's accident probability can be identified by the insurance companies. Thus, there is no asymmetry of information here. What is the competitive (Walrasian) outcome in this benchmark setting in which all information is public?

To understand the competitive outcome here, it is important to recognize that the price of any particular commodity may well depend on the "state of the world." For example, an umbrella in the state "rain" is a different commodity than an umbrella in the state "sunny." Consequently, these distinct commodities could command distinct prices.

The same holds true in this setting where a state specifies which subset of consumers have accidents. Because the state in which consumer i has an accident differs from that in which consumer j does, the commodity (policy) paying L dollars to consumer i when he has an accident differs from that paying L dollars to j when she does. Consequently, policies benefiting distinct consumers are in fact distinct commodities and may then command distinct prices.

So, let p_i denote the price of the policy paying L dollars to consumer i should he have an accident. For simplicity, let's refer to this as the ith policy. We wish then to determine, for each $i = 1, 2, \ldots, m$, the competitive equilibrium price p_i^* of policy i.

[1] Thus, think of an accident as "hitting a tree" as opposed to "hitting another car."

Let's first consider the supply of policy i. If p_i is less than $\pi_i L$, then selling such a policy will result in expected losses. Hence, the supply of policy i will be zero in this case. On the other hand, if p_i is greater than $\pi_i L$, positive expected profits can be earned, so the supply of such policies will be infinite. Finally, if $p_i = \pi_i L$, then insurance companies break even on each policy i sold and hence are willing to supply any number of such policies.

On the demand side, if p_i is less than $\pi_i L$, then consumer i, being risk-averse will demand at least one policy i. This follows from our analysis in Chapter 2 where we showed that risk-averse consumers strictly prefer to fully insure than not to insure at all whenever actuarially fair insurance is available (i.e., whenever $p_i = \pi_i L$). The same analysis shows that if p_i exceeds $\pi_i L$, consumer i will purchase at most one policy i. (Recall that fractional policies cannot be purchased.)

By putting demand and supply together, the only possibility for equilibrium is when $p_i = \pi_i L$. In this case, each consumer i demands exactly one policy i and it is supplied by exactly one insurance company (any one will do). All other insurance companies are content to supply zero units of policy i because at price $p_i = \pi_i L$ all would earn zero expected profits.

We conclude that when information is freely available to all, there is a unique competitive equilibrium. In it, $p_i^* = \pi_i L$ for every policy $i = 1, 2, \ldots, m$. Note that in this competitive equilibrium, all insurance companies earn zero expected profits, and all consumers are fully insured.

We wish to argue that the competitive outcome is Pareto efficient—no consumer or insurance company can be made better off without making some other consumer or insurance company worse off. By constructing an appropriate pure exchange economy, one can come to this conclusion by appealing to the First Welfare Theorem. You are invited to do so in Exercise 8.1. We shall give a direct argument here.

In this setting, an *allocation* is an assignment of wealth to consumers and insurance companies *in each state*. An allocation is *feasible* if in every state, the total wealth assigned is equal to the total consumer wealth.

We now argue that no feasible allocation Pareto dominates the competitive allocation. Suppose, by way of contradiction, that some feasible allocation does Pareto dominate the competitive one. Without loss of generality, we may assume that the competitive allocation is dominated by a feasible allocation in which each consumer's wealth is the same whether or not he has an accident. (See Exercise 8.5.) Consequently, the dominating outcome guarantees each consumer i wealth $\bar{w}_i$. For this allocation to dominate the competitive one, it must be the case that $\bar{w}_i \geq w - \pi_i L$ for each i.

Now, because each consumer's wealth is certain, we may assume without loss that according to the dominating allocation, there is no transfer of wealth between any two consumers in any state. (Again, see Exercise 8.5.) Therefore, each consumer's wealth is directly transferred only to (or from) insurance companies in every state.

Consider then a particular consumer, i, and the insurance companies who are providing i with insurance in the dominating allocation. In aggregate, their expected profits from consumer i are

$$(1 - \pi_i)(w - \bar{w}_i) + \pi_i(w - L - \bar{w}_i) = w - \pi_i L - \bar{w}_i, \tag{8.1}$$

because $\bar{w}_i - w$ (resp., $\bar{w}_i + L - w$) is the supplement to consumer i's wealth in states in which she does not have (resp., has) an accident, and the feasibility of the allocation implies that this additional wealth must be offset by a change in the aggregate wealth of insurance companies.

But we've already determined that the right-hand side of (8.1) is nonpositive. So, letting EP_i^j denote company j's expected profits from consumer i, we have shown that in the dominating allocation,

$$w - \pi_i L - \bar{w}_i = \sum_j EP_i^j \leq 0 \qquad \text{for every consumer } i. \tag{8.2}$$

But each insurance company must be earning nonnegative expected profits in the dominating allocation because each earns zero expected profits in the competitive allocation. Hence, we must also have

$$\sum_i EP_i^j \geq 0 \qquad \text{for every insurance company } j. \tag{8.3}$$

Summing (8.2) over i and (8.3) over j shows that each of the two inequalities must be equalities for every i and j. Consequently, each consumer's constant wealth and each firm's expected profits in the dominating allocation are identical to their competitive allocation counterparts. But this contradicts the definition of a dominating allocation and completes the argument that the competitive allocation is efficient.

Asymmetric Information and Adverse Selection

We now return to the more realistic setting in which insurance companies cannot identify consumers' accident probabilities. Although insurance companies can and do employ historical records of consumers to partially determine their accident probabilities, we will take a more extreme view for simplicity. Specifically, we shall suppose that insurance companies know only the distribution of accident probabilities among consumers and nothing else.

So let the nondegenerate interval $[\underline{\pi}, \bar{\pi}]$ contain the set of all consumer accident probabilities, and let F be a distribution function on $[\underline{\pi}, \bar{\pi}]$ representing the insurance companies' information. This specification allows either finitely many or a continuum of consumers. The possibility of allowing a continuum is convenient for examples. We'll also suppose that both $\underline{\pi}$ and $\bar{\pi}$ are in the support of F.[2] Therefore, for each $\pi \in [\underline{\pi}, \bar{\pi}]$, $F(\pi)$ denotes the fraction of consumers having accident probability less than or equal to π. Equivalently, $F(\pi)$ denotes the probability that any particular consumer has accident probability π or lower. Insurance companies are otherwise exactly as before. In particular, they each sell only full insurance.

The impact of asymmetric information is quite dramatic. Indeed, even though policies sold to different consumers can potentially command distinct prices, in equilibrium they will not. The reason is quite straightforward. To see it, suppose to the contrary that

[2] If there are finitely many consumers and therefore finitely many accident probabilities, this means simply that both $\underline{\pi}$ and $\bar{\pi}$ are given positive probability by F. More generally, it means that all nondegenerate intervals of the form $[\underline{\pi}, a)$ and $(b, \bar{\pi}]$ are given positive probability by F.

the equilibrium price paid by consumer i exceeds that paid by consumer j. Because both consumers are actually purchasing a policy, the expected profits on each sale must be nonnegative—otherwise, the insurance company supplying the money-losing policy would not be profit-maximizing. Consequently, because consumers i and j are identical to insurance companies from an accident probability point of view, the policy sold to consumer i must earn strictly positive expected profits. But then each insurance company would wish to supply an infinite amount of such a policy, which cannot be the case in equilibrium. This contradiction establishes the result: *There is a single equilibrium price of the full insurance policy for all consumers.*

Then let p denote this single price of the full insurance policy. We wish now to determine its equilibrium value, p^*.

Because positive expected profits result in infinite supply and negative expected profits result in zero supply, a natural guess would be to set $p^* = E(\pi)L$, where $E(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} \pi \, dF(\pi)$ is the expected accident probability. Such a price is intended to render insurance companies' expected profits equal to zero. But does it?

To see that it might not, note that this price might be so high that only those consumers with relatively high accident probabilities will choose to purchase insurance. Consequently, companies would be *underestimating* the expected accident probability by using the unconditional expectation, $E(\pi)$, rather than the expectation of the accident probability *conditional on those consumers actually willing to purchase the policy*. By underestimating this way, profits would be strictly negative on average. Thus to find p^* we must take this into account.

For any accident probability π, the consumer buys a policy for price p only if the expected utility from doing so exceeds the expected utility from remaining uninsured: that is, only if[3]

$$u(w-p) \geq \pi u(w-L) + (1-\pi)u(w).$$

Rearranging, and defining the function $h(p)$, we find that the policy will be purchased only if

$$\pi \geq \frac{u(w)-u(w-p)}{u(w)-u(w-L)} \equiv h(p).$$

Then we'll call p^* a *competitive equilibrium price under asymmetric information* if it satisfies the following condition:

$$p^* = E(\pi \mid \pi \geq h(p^*))L, \tag{8.4}$$

where the expression $E(\pi \mid \pi \geq h(p^*)) = \int_{h(p^*)}^{\bar{\pi}} \pi \, dF(\pi)$ is the expected accident probability conditional on $\pi \geq h(p^*)$.

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[3]For simplicity, we assume that a consumer who is indifferent between buying the policy or not does in fact buy it.

Note that a consumer with accident probability π will purchase the full insurance policy at price p as long as $\pi \geq h(p)$. Thus, condition (8.4) ensures that firms earn zero expected profits on each policy sold, conditional on the accident probabilities of consumers who actually purchase the policy. The supply of policies then can be set equal to the number demanded by consumers. Thus, the condition above does indeed describe an equilibrium.

An immediate concern is whether or not such an equilibrium exists. That is, does there necessarily exist a p^* satisfying (8.4)? The answer is yes, and here's why.

Let $g(p) = E(\pi \mid \pi \geq h(p))L$ for every $p \in [0, \bar{\pi}L]$, where $\bar{\pi}$ is the highest accident probability among all consumers. Note that the conditional expectation is well-defined because $h(p) \leq \bar{\pi}$ for every $p \in [0, \bar{\pi}L]$ (check this). In addition, because $E(\pi \mid \pi \geq h(p)) \in [0, \bar{\pi}]$, the function g maps the interval $[0, \bar{\pi}L]$ into itself. Finally, because h is strictly increasing in p, we know g is nondecreasing in p. Consequently, g is a nondecreasing function mapping a closed interval into itself. As you are invited to explore in the exercises, even though g need not be continuous, it must nonetheless have a fixed point $p^* \in [0, \bar{\pi}L]$.[4] By the definition of g, this fixed point is an equilibrium.

Having settled the existence question, we now turn to the properties of equilibria. First, there is no reason to expect a unique equilibrium here. Indeed, one can easily construct examples having multiple equilibria. But more importantly, equilibria need not be efficient here.

For example, consider the case in which F is uniformly distributed over $[\underline{\pi}, \bar{\pi}] = [0, 1]$. Then $g(p) = (1 + h(p))L/2$ is strictly increasing and strictly concave because $h(p)$ is. Consequently, there is a unique equilibrium price p^* and it satisfies $p^* = (1 + h(p^*))L/2$. But because $h(L) = 1$, we must then have $p^* = L$. However, when $p^* = L$, (10.4) tells us the expected probability of an accident for those who buy insurance must be $E(\pi \mid \pi \geq h(L)) = 1$. Thus, in equilibrium, all consumers will be uninsured except those who are certain to have an accident. But even these consumers have insurance only in a formal sense because they must pay the full amount of the loss, L, to obtain the policy. Thus, their wealth (and therefore their utility) remains the same as if they had not purchased the policy at all.

Clearly, this outcome is inefficient in the extreme. The competitive outcome with symmetric information gives every consumer (except those who are certain to have an accident) strictly higher utility, while also ensuring that every insurance company's expected profits are zero. Here, the asymmetry in information causes a significant market failure in the insurance market. Effectively, no trades take place and therefore opportunities for Pareto improvements go unrealized.

To understand why prices are unable to produce an efficient equilibrium here, consider a price at which expected profits are negative for insurance companies. Then, other things being equal, you might think that raising the price will tend to increase expected profits. But in insurance markets, other things will *not* remain equal. In general, whenever the price of insurance is increased, the expected utility a consumer receives from buying insurance falls, whereas the expected utility from not insuring remains the same. For some consumers, it will no longer be worthwhile to buy insurance, so they will quit doing so. But who continues to buy as the price increases? Only those for whom the expected loss from *not* doing so is greatest, and these are precisely the consumers with the highest accident probabilities. As

[4] Of course, if g is continuous, we can apply Brouwer's fixed-point theorem. However, you will show in an exercise that if there are finitely many consumers, g cannot be continuous.

a result, whenever the price of insurance rises, the pool of customers who continue to buy insurance becomes riskier on average.

This is an example of **adverse selection**, and it tends here to have a negative influence on expected profits. If, as in our example, the negative impact of adverse selection on expected profits outweighs the positive impact of higher insurance prices, there can fail to be any efficient equilibrium at all, and mutually beneficial potential trades between insurance companies and relatively low-risk consumers can fail to take place.

The lesson is clear. In the presence of asymmetric information and adverse selection, the competitive outcome need not be efficient. Indeed, it can be dramatically inefficient.

One of the advantages of free markets is their ability to "evolve." Thus, one might well imagine that the insurance market would somehow adjust to cope with adverse selection. In fact, real insurance markets do perform a good deal better than the one we just analyzed. The next section is devoted to explaining how this is accomplished.

8.1.2 SIGNALING

Consider yourself a low-risk consumer stuck in the inefficient equilibrium we've just described. The equilibrium price of insurance is so high that you've chosen not to purchase any. If only there were some way you could convince one of the insurance companies that you are a low risk. They would then be willing to sell you a policy for a price you would be willing to pay.

In fact, there often *will* be ways consumers can credibly communicate how risky they are to insurance companies, and we call this kind of behavior **signaling**. In real insurance markets, consumers can and do distinguish themselves from one another—and they do it by purchasing different types of policies. Although we ruled this out in our previous analysis by assuming only one type of policy, we can now adapt our analysis to allow it.

To keep things simple, we'll suppose there are only two possible accident probabilities, $\underline{\pi}$ and $\bar{\pi}$, where $0 < \underline{\pi} < \bar{\pi} < 1$. We'll assume that the fraction of consumers having accident probability $\underline{\pi}$ is $\alpha \in (0, 1)$. Consumers with accident probability $\underline{\pi}$ are called *low-risk* consumers, and those with accident probability $\bar{\pi}$ are called *high-risk* consumers.

To model the idea that consumers can attempt to distinguish themselves from others by choosing different policies, we shall take a game theoretic approach.

Consider then the following extensive form game, which we'll refer to as the **insurance signaling game**, involving two consumers (low-risk and high-risk) and a single insurance company:

- Nature moves first and determines which consumer will make a proposal to the insurance company. The low-risk consumer is chosen with probability α, and the high-risk consumer is chosen with probability $1 - \alpha$.

- The chosen consumer moves second. He chooses a *policy* (B, p), consisting of a *benefit* $B \geq 0$ the insurance company pays him if he has an accident, and a *premium* $0 \leq p \leq w$ he pays to the insurance company whether or not he has an accident.[5]

[5]Note the slight change in our use of the term *policy*. It now refers to a benefit–premium *pair*, (B, p), rather than simply the benefit. Restricting p to be no higher than w ensures that the consumer does not go bankrupt.

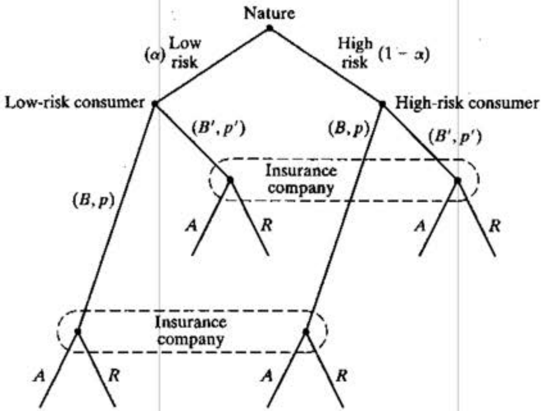


Figure 8.1. Insurance signaling game: a schematic diagram of the signaling extensive form game. The figure is complete except that it shows only two policy choices, (B, p) and (B', p'), available to the consumer when there are in fact infinitely many choices available.

- The insurance company moves last, not knowing which consumer was chosen by Nature, but knowing the chosen consumer's proposed policy. The insurance company either agrees to accept the terms of the consumer's policy or rejects them.

The extensive form of this game is shown in Fig. 8.1. When interpreting the game, think of the insurance company as being one of many competing companies, and think of the chosen consumer as a randomly selected member from the set of all consumers, of whom a fraction α are low-risk types and a fraction $1 - \alpha$ are high-risk types.

A pure strategy for the low-risk consumer is a specification of a policy $\psi_l = (B_l, p_l)$, and for the high-risk consumer, a policy $\psi_h = (B_h, p_h)$.

A pure strategy for the insurance company must specify one of two responses, either A (accept) or R (reject), for each potential policy proposed. Thus, a pure strategy for the insurance company is a response function, σ, where $\sigma(B, p) \in \{A, R\}$ for each policy (B, p). Note that σ depends only on the proposed policy and not on whether the consumer proposing it is low- or high-risk. This reflects the assumption that the insurance company does not know which risk type makes the proposal.

Once a policy is proposed, the insurance company formulates beliefs about the consumer's accident probability. Let probability $\beta(B, p)$ denote the insurance company's beliefs that the consumer who proposed policy (B, p) is the low-risk type.

We wish to determine the pure strategy sequential equilibria of this game.[6] There is, however, a purely technical difficulty with this. The definition of a sequential equilibrium

[6]See Chapter 7 for a discussion of sequential equilibrium. We have chosen to employ the sequential equilibrium concept here because we want to insist upon rational behavior on the part of the insurance company at each of its information sets, and further that consumers take this into account.

requires the game to be finite, but the game under consideration is not—the consumer can choose any one of a continuum of policies.

Now, the definition of a sequential equilibrium requires the game to be finite only because the consistency condition is not easily defined for infinite games. However, as you will demonstrate in an exercise, when the consumer's choice set is restricted to any finite set of policies, so that the game becomes finite, *every* assessment satisfying Bayes' rule also satisfies the consistency condition. Consequently, in every finite version of the insurance signaling game, an assessment is a sequential equilibrium if and only if it is sequentially rational and satisfies Bayes' rule.

With this in mind, we define a sequential equilibrium for the (infinite) insurance signaling game in terms of sequential rationality and Bayes' rule, alone, as follows.

DEFINITION 8.1 ***Signaling Game Pure Strategy Sequential Equilibrium***

The assessment $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ *is a pure strategy sequential equilibrium of the insurance signaling game if*

1. *given the insurance company's strategy,* $\sigma(\cdot)$, *proposing the policy* ψ_l *maximizes the low-risk consumer's expected utility, and proposing* ψ_h *maximizes the high-risk consumer's expected utility;*
2. *the insurance company's beliefs satisfy Bayes' rule. That is,*
   (a) $\beta(\psi) \in [0, 1]$, for all policies $\psi = (B, p)$,
   (b) if $\psi_l \neq \psi_h$, *then* $\beta(\psi_l) = 1$ *and* $\beta(\psi_h) = 0$,
   (c) if $\psi_l = \psi_h$, *then* $\beta(\psi_l) = \beta(\psi_h) = \alpha$;
3. *for every policy* $\psi = (B, p)$, *the insurance company's reaction,* $\sigma(\psi)$, *maximizes its expected profits given its beliefs* $\beta(B, p)$.

Conditions (1) and (3) ensure that the assessment is sequentially rational, whereas condition (2) ensures that the insurance company's beliefs satisfy Bayes' rule. Because we are restricting attention to pure strategies, Bayes' rule reduces to something rather simple. If the different risk types choose different policies in equilibrium, then on observing the low- (high-) risk consumer's policy, the insurance company infers that it faces the low- (high-) risk consumer. This is condition (2.b). If, however, the two risk types choose the same policy in equilibrium, then on observing this policy, the insurance company's beliefs remain unchanged and equal to its prior belief. This is condition (2.c).

The basic question is this: Can the low-risk consumer distinguish himself from the high-risk one here, and as a result achieve a more efficient outcome? It is not obvious that the answer is yes. For note that there is no direct connection between a consumer's risk type and the policy he proposes. That is, the act of purchasing less insurance does not decrease the probability that an accident will occur. In this sense, the signals used by consumers—the policies they propose—are *unproductive*.

However, despite this, the low-risk consumer can still attempt to signal that he is low risk by demonstrating his willingness to accept a decrease in the benefit for a smaller compensating premium reduction than would the high-risk consumer. Of course, for this kind of (unproductive) signaling to be effective, the risk types must display different marginal rates of substitution between benefit levels, B, and premiums, p. As we shall shortly demonstrate, this crucial difference in marginal rates of substitution is indeed present.

Analyzing the Game

To begin, it is convenient to define for each risk type the expected utility of a generic policy (B, p). So, let

$$u_l(B, p) = \underline{\pi} u(w - L + B - p) + (1 - \underline{\pi})u(w - p) \qquad \text{and}$$
$$u_h(B, p) = \bar{\pi} u(w - L + B - p) + (1 - \bar{\pi})u(w - p)$$

denote the expected utility of the policy (B, p) for the low- and high-risk consumer, respectively.

The following facts are easily established.

FACTS

(a) $u_l(B, p)$ and $u_h(B, p)$ are continuous, differentiable, strictly concave in (B, p), strictly increasing in B, and strictly decreasing in p,

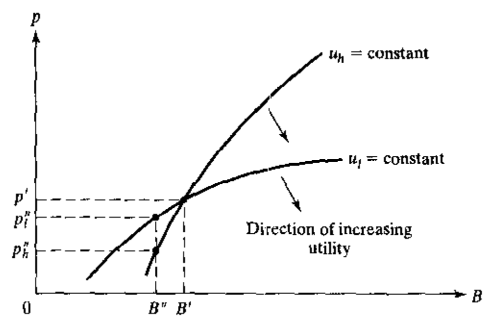
(b) $MRS_l(B, p)$ is greater than, equal to or less than $\underline{\pi}$ as B is less than, equal to, or greater than L. $MRS_h(B, p)$ is greater than, equal to, or less than $\bar{\pi}$ as B is less than, equal to, or greater than L.

(c) $MRS_l(B, p) < MRS_h(B, p)$ for all (B, p).

The last of these is often referred to as the **single-crossing property**. As its name suggests, it implies that indifference curves for the two consumer types intersect at most once. Equally important, it shows that the different risk types display different marginal rates of substitution when faced with the same policy.

Fig. 8.2 illustrates facts (a) and (c). In accordance with fact (c), the steep indifference curves belong to the high-risk consumer and the flatter ones to the low-risk consumer. The difference in their marginal rates of substitution indicates that beginning from a given policy (B', p'), the low-risk consumer is willing to accept a decrease in the benefit to B'' for a smaller compensating premium reduction than would the high-risk consumer. Here,

Figure 8.2. Single crossing property. Beginning from policy (B', p'), the benefit is reduced to B''. To keep the low-risk type just as well off, the price must be reduced to p_l''. It must be further reduced to p_h'' to keep the high-risk type just as well off.



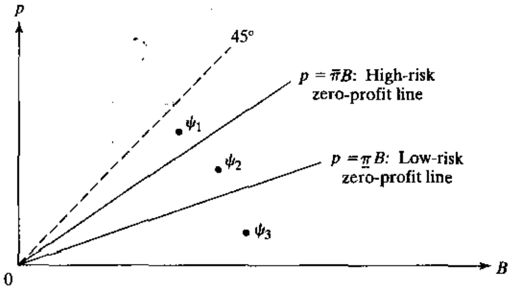


Figure 8.3. Zero-profit lines. Policy ψ_1 earns positive profits on both consumer types; ψ_2 earns positive profits on the low-risk consumer and negative profits on the high-risk consumer; ψ_3 earns negative profits on both consumer types.

reducing the benefit is less costly to the low-risk consumer because he is less likely to have an accident.

The insurance company maximizes expected profits. Now, in case it knows that the consumer is low-risk, it will accept any policy (B, p) satisfying $p > \underline{\pi} B$, because such a policy yields positive profits. Similarly, it will reject the policy if $p < \underline{\pi} B$. It is indifferent between accepting and rejecting the policy if $p = \underline{\pi} B$. If the insurance company knows the consumer is high-risk, then it accepts the policy (B, p) if $p > \bar{\pi} B$ and rejects it if $p < \bar{\pi} B$.

Fig. 8.3 illustrates the two zero-profit lines for the insurance company. The line $p = \underline{\pi} B$ contains those policies (B, p) yielding zero expected profits for the insurance company when the consumer is known to be low-risk. The line $p = \bar{\pi} B$ contains those policies yielding zero expected profits when the consumer is known to be high-risk. These two lines will play an important role in our analysis. Note that the low-risk zero profit line has slope $\underline{\pi}$, and the high-risk zero profit line has slope $\bar{\pi}$.

Now is a good time to think back to the competitive equilibrium for the case in which the insurance company can identify the risk types. There we showed that in the unique competitive equilibrium the price of full insurance, where $B = L$, is equal to $\underline{\pi} L$ for the low-risk consumer, and $\bar{\pi} L$ for the high-risk consumer. This outcome is depicted in Fig. 8.4. The insurance company earns zero profits on each consumer, each consumer purchases full insurance, and, by fact (b) above, each consumer's indifference curve is tangent to the insurance company's respective zero-profit line.

Returning to the game at hand, we begin characterizing its sequential equilibria by providing lower bounds on each of the consumers' expected utilities, conditional on having been chosen by Nature. Note that the most pessimistic belief the insurance company might have is that it faces the high-risk consumer. Consequently, both consumer-types' utilities ought to bounded below by the maximum utility they could obtain when the insurance company believes them to be the high-risk consumer. This is the content of the next lemma.

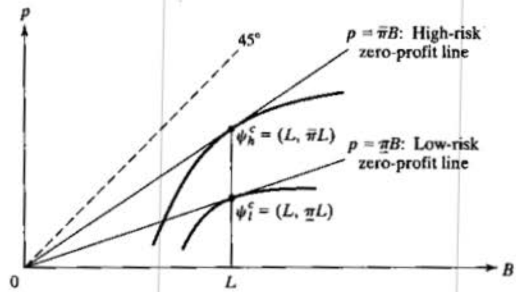


Figure 8.4. Competitive outcome, ψ_l^c and ψ_h^c denote the policies consumed by the low- and high-risk types in the competitive equilibrium when the insurance company can identify risk types. The competitive outcome is efficient.

LEMMA 8.1 *Let $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ be a sequential equilibrium, and let u_l^* and u_h^* denote the equilibrium utility of the low- and high-risk consumer, respectively, given that he has been chosen by Nature. Then*

1. $u_l^* \geq \tilde{u}_l$, *and*
2. $u_h^* \geq u_h^c$,

where $\tilde{u}_l \equiv \max_{(B,p)} u_l(B, p)$ s.t. $p = \bar{\pi} B \leq w$, and $u_h^c \equiv u_h(L, \bar{\pi} L)$ denotes the high-risk consumer's utility in the competitive equilibrium with full information.

Proof: Consider a policy (B, p) lying above the high-risk zero-profit line, so that $p > \bar{\pi} B$. We wish to argue that in equilibrium, the insurance company must accept this policy.

To see this, note that by accepting it, the company's expected profits given its beliefs $\beta(B, p)$ are

$$p - \{\beta(B, p)\underline{\pi} + (1 - \beta(B, p))\bar{\pi}\}B \geq p - \bar{\pi} B > 0.$$

Consequently, accepting is strictly better than rejecting the policy because rejecting results in zero profits. We conclude that all policies (B, p) above the high-risk zero-profit line are accepted by the insurance company.

Thus, for any policy satisfying $\bar{\pi} B < p \leq w$, the low-risk consumer, by proposing it, can guarantee utility $u_l(B, p)$, and the high-risk consumer can guarantee utility $u_h(B, p)$. Therefore, because each risk type maximizes expected utility in equilibrium, the following inequalities must hold for all policies satisfying $\bar{\pi} B < p \leq w$:

$$u_l^* \geq u_l(B, p) \quad \text{and} \tag{P.1}$$

$$u_h^* \geq u_h(B, p). \tag{P.2}$$

Continuity of u_l and u_h implies that (P.1) and (P.2) must in fact hold for all policies satisfying

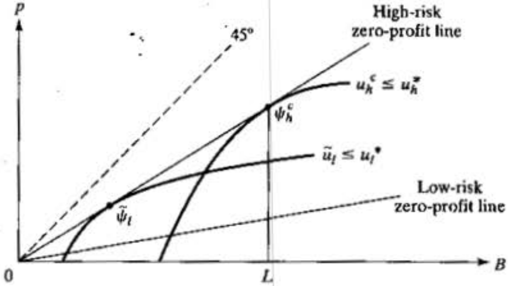


Figure 8.5. Lower bounds. Because all policies (B, p) above the high-risk zero-profit line are accepted by the insurance company in equilibrium, the low-risk consumer must obtain utility no smaller than $\tilde{u}_l = u_l(\tilde{\psi}_l)$ and the high-risk consumer utility no smaller than $u_h^c = u(\psi_h^c)$. Note that although in the figure $\tilde{\psi}_l \neq (0, 0)$, it is possible that $\tilde{\psi}_l = (0, 0)$.

the weak inequality $\bar{\pi} B \le p \le w$. Thus, (P.1) and (P.2) may be rewritten as

$$u_l^* \ge u_l(B, p) \qquad \text{for all} \qquad \bar{\pi} B \le p \le w, \tag{P.3}$$

$$u_h^* \ge u_h(B, p) \qquad \text{for all} \qquad \bar{\pi} B \le p \le w. \tag{P.4}$$

But (P.3) is equivalent to (1) because utility is decreasing in p, and (P.4) is equivalent to (2) because, among all no better than fair insurance policies, the full insurance one uniquely maximizes the high-risk consumer's utility. ∎

Fig. 8.5 illustrates Lemma 8.1. A consequence of the lemma that is evident from the figure is that the high-risk consumer must purchase insurance in equilibrium. This is because without insurance his utility would be $u_h(0, 0)$ which, by strict risk aversion, is strictly less than u_h^c, a lower bound on his equilibrium utility.

The same cannot be said for the low-risk consumer even though it appears so from Fig. 8.5. We have drawn Fig. 8.5 for the case in which $MRS_l(0, 0) > \bar{\pi}$, so that $u_l(0, 0) < \tilde{u}_l$. However, in the equally plausible case in which $MRS_l(0, 0) < \bar{\pi}$ we have $u_l(0, 0) \ge \tilde{u}_l$. In this latter case, the low-risk consumer may choose not to purchase insurance in equilibrium (by making a proposal that is rejected) without violating the conclusion of Lemma 8.1.

The preceding lemma applies to every sequential equilibrium. We now separate the set of equilibria into two kinds: separating and pooling.

An equilibrium is a **separating equilibrium** if the different types of consumers propose different policies. In this way, the consumers separate themselves from one another and can be identified by the insurance company by virtue of the chosen policy. In contrast, an equilibrium is a **pooling equilibrium** if both consumer types propose the same policy. Consequently, the consumer types cannot be identified by observing the policy they propose. In summary, we have the following definition.

DEFINITION 8.2 ***Separating and Pooling Signaling Equilibria***

A pure strategy sequential equilibrium $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ is separating if $\psi_l \neq \psi_h$, while it is pooling otherwise.

With only two possible types of consumers, a pure strategy sequential equilibrium is either separating or pooling. Thus, it is enough for us to characterize the sets of separating and pooling equilibria. We begin with the former.

Separating Equilibria

In a separating equilibrium, the two risk types will propose different policies if chosen by Nature, and on the basis of this the insurance company will be able to identify them. Of course, each risk type therefore could feign the identity of the other simply by behaving as the other would according to the equilibrium.[7] The key conceptual point to grasp, then, is that in a separating equilibrium, *it must not be in the interest of either type to mimic the behavior of the other*. Based on this idea, we can characterize the policies proposed and accepted in a separating pure strategy sequential equilibrium as follows.

THEOREM 8.1 ***Separating Equilibrium Characterization***

The policies $\psi_l = (B_l, p_l)$ and $\psi_h = (B_h, p_h)$ are proposed by the low- and high-risk consumer, respectively, and accepted by the insurance company in some separating equilibrium if and only if

1. $\psi_l \neq \psi_h = (L, \bar{\pi} L)$.
2. $p_l \geq \underline{\pi} B_l$.
3. $u_l(\psi_l) \geq \tilde{u}_l \equiv \max_{(B,p)} u_l(B, p) \quad s.t. \quad p = \bar{\pi} B \leq w$.
4. $u_h^c \equiv u_h(\psi_h) \geq u_h(\psi_l)$.

Proof: Suppose first that $\psi_l = (B_l, p_l)$ and $\psi_h = (L, \bar{\pi} L)$ satisfy (1) to (4). We must construct a strategy $\sigma(\cdot)$ and beliefs $\beta(\cdot)$ for the insurance company so that the assessment $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ is a sequential equilibrium. It then will be clearly separating. The following specifications will suffice:

$$\beta(B, p) = \begin{cases} 1, & \text{if } (B, p) = \psi_l, \\ 0, & \text{if } (B, p) \neq \psi_l. \end{cases}$$

$$\sigma(B, p) = \begin{cases} A, & \text{if } (B, p) = \psi_l, \quad \text{or} \quad p \geq \bar{\pi} B, \\ R, & \text{otherwise.} \end{cases}$$

[7] There are other ways to feign the identity of the other type. For example, the low-risk type might choose a proposal that neither type is supposed to choose in equilibrium, but one that would nonetheless induce the insurance company to believe that it faced the high-risk consumer.

According to the beliefs $\beta(\cdot)$, any policy proposed other than ψ_l induces the insurance company to believe that it faces the high-risk consumer with probability one. On the other hand, when the policy ψ_l is proposed, the insurance company is sure that it faces the low-risk consumer. Consequently, the insurance company's beliefs satisfy Bayes' rule.

In addition, given these beliefs, the insurance company's strategy maximizes its expected profits because, according to that strategy, the company accepts a policy if and only if it results in nonnegative expected profits.

For example, the proposal $\psi_l = (B_l, p_l)$ is accepted because, once proposed, it induces the insurance company to believe with probability one that it faces the low-risk consumer. Consequently, the insurance company's expected profits from accepting the policy are $p_l - \underline{\pi} B_l$, which, according to (2), is nonnegative. Similarly, the proposal $\psi_h = (L, \bar{\pi} L)$ is accepted because it induces the insurance company to believe with probability one that it faces the high-risk consumer. In that case, expected profits from accepting the policy are $\bar{\pi} L - \bar{\pi} L = 0$.

All other policy proposals (B, p) induce the insurance company to believe with probability one that it faces the high-risk consumer. Its expected profits from accepting such policies are then $p - \bar{\pi} B$. Thus, these policies are also accepted precisely when they yield nonnegative expected profits given the insurance company's beliefs.

We've shown that given any policy (p, B), the insurance company's strategy maximizes its expected profits given its beliefs. It remains to show that given the insurance company's strategy, both consumers are choosing policies that maximize their utility.

To complete this part of the proof, we'll show that no policy proposal yields the low-risk consumer more utility than ψ_l nor the high-risk consumer more than ψ_h. Note that because the insurance company accepts the policy (0, 0), and this policy is equivalent to a rejection by the insurance company (regardless of *which* policy was rejected), both consumers can maximize their utility by making a proposal that is accepted by the insurance company. We therefore may restrict our attention to the set of such policies that we'll denote by $\mathcal{A}$; i.e.,

$$\mathcal{A} = \{\psi_l\} \cup \{(B, p) \mid p \geq \bar{\pi} B\}.$$

Thus, it is enough to show that for all $(B, p) \in \mathcal{A}$ with $p \leq w$,

$$u_l(\psi_l) \geq u_l(B, p), \qquad \text{and} \tag{P.1}$$

$$u_h(\psi_h) \geq u_h(B, p). \tag{P.2}$$

But (P.1) follows from (3), and (P.2) follows from (1), (3), (4), and because $(L, \bar{\pi} L)$ is best for the high-risk consumer among all no better than fair policies.

We now consider the converse. So, suppose that $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ is a separating equilibrium in which the equilibrium policies are accepted by the insurance company. We must show that (1) to (4) hold. We take each in turn.

1. The definition of a separating equilibrium requires $\psi_l \neq \psi_h$. To see that $\psi_h \equiv (B_h, p_h) = (L, \bar{\pi} L)$, recall that Lemma 8.1 implies $u_h(\psi_h) = u_h(B_h, p_h) \geq u_h(L, \bar{\pi} L)$. Now because the insurance company accepts this proposal, it must earn nonnegative profits.

Hence, we must have $p_h \geq \bar{\pi} B_h$ because in a separating equilibrium, the insurance company's beliefs must place probability one on the high-risk consumer subsequent to the high-risk consumer's equilibrium proposal ψ_h. But as we've argued before, these two inequalities imply that $\psi_h = (L, \bar{\pi} L)$ (see, for example, Fig. 8.4).

2. Subsequent to the low-risk consumer's equilibrium proposal, (B_l, p_l), the insurance company places probability one on the low-risk consumer by Bayes' rule. Accepting the proposal then would yield the insurance company expected profits $p_l - \underline{\pi} B_l$. Because the insurance company accepts this proposal by hypothesis, this quantity must be nonnegative.

3. This follows from (1) of Lemma 8.1.

4. According to the insurance company's strategy, it accepts policy ψ_l. Because the high-risk consumer's equilibrium utility is $u_h(\psi_h)$, we must have $u_h(\psi_h) \geq u_h(\psi_l)$. ∎

Fig. 8.6 illustrates the policies that can arise in a separating equilibrium according to Theorem 8.1. The high-risk consumer obtains policy $\psi_h^c = (L, \bar{\pi} L)$ and the low-risk consumer obtains the policy $\psi_l = (B_l, p_l)$, which must lie somewhere in the shaded region.

Note the essential features of the set of low-risk policies. Each is above the low-risk zero-profit line to induce acceptance by the insurance company, above the high-risk consumer's indifference curve through his equilibrium policy to ensure that he has no incentive to mimic the low-risk consumer, and below the indifference curve giving utility $\tilde{u}_l$ to the low-risk consumer to ensure that he has no incentive to deviate and be identified as a high-risk consumer.

Theorem 8.1 restricts attention to those equilibria in which both consumers propose acceptable policies. Owing to Lemma 8.1, this is a restriction only on the low-risk consumer's policy proposal. When $MRS_l(0, 0) \leq \bar{\pi}$, there are separating equilibria in which the low-risk consumer's proposal is rejected in equilibrium. However, you are asked to show in an

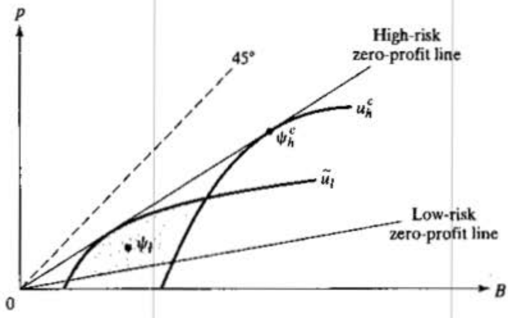


Figure 8.6. Potential separating equilibria. In a separating equilibrium in which both consumer types propose acceptable policies, the high-risk policy must be ψ_h^c and the low-risk policy, ψ_l, must be in the shaded region. Here, $MRS_l(0, 0) > \bar{\pi}$. A similar figure arises in the alternative case, noting that $MRS_l(0, 0) > \underline{\pi}$ always holds.

exercise that each of these is payoff equivalent to some separating equilibrium in which the low-risk consumer's policy proposal is accepted. Finally, one can show that the shaded region depicted in Fig. 8.6 is always nonempty, even when $MRS_l(0, 0) \leq \bar{\pi}$. This requires using the fact that $MRS_l(0, 0) > \underline{\pi}$. Consequently, a pure strategy separating equilibrium always exists.

Now that we have characterized the policies that can arise in a separating equilibrium, we can assess the impact of allowing policy proposals to act as signals about risk. Note that because separating equilibria always exist, allowing policy proposals to act as signals about risk is always effective in the sense that it does indeed make it possible for the low-risk type to distinguish himself from the high-risk type.

On the other hand, there need not be much improvement in terms of efficiency. For example, when $MRS_l(0, 0) \leq \bar{\pi}$, there is a separating equilibrium in which the low-risk consumer receives the (null) policy $(0, 0)$, and the high-risk consumer receives the policy $(L, \bar{\pi} L)$. That is, only the high-risk consumer is insured. Moreover, this remains an equilibrium outcome regardless of the probability that the consumer is high-risk![8] Thus, the presence of a bad apple—even with very low probability—can still spoil the outcome just as in the competitive equilibrium under asymmetric information wherein signaling was not possible.

Despite the existence of equilibria that are as inefficient as in the model without signaling, when signaling is present, there are always equilibria in which the low-risk consumer receives some insurance coverage. The one of these that is best for the low-risk consumer and worst for the insurance company provides the low-risk consumer with the policy labeled $\tilde{\psi}_l$ in Fig. 8.7.

Because the high-risk consumer obtains the same policy ψ_h^c in every separating equilibrium, and so receives the same utility, the equilibrium outcome $(\tilde{\psi}_l, \psi_h^c)$ is Pareto efficient among separating equilibria and it yields zero profits for the insurance company. This outcome is present in Fig. 8.7 regardless of the probability that the consumer is low-risk. Thus, even when the only competitive equilibrium under asymmetric information gives no insurance to the low-risk consumer (which occurs when α is sufficiently small), the low-risk consumer can obtain insurance, and market efficiency can be improved when signaling is possible.

We now turn our attention to the second category of equilibria.

Pooling Equilibria

Recall that an equilibrium is a pooling one if the two types of consumers propose the same policy. By doing so, the insurance company cannot distinguish between them. Consequently, the low-risk consumer will be treated somewhat more like the high-risk consumer and vice versa. It is fair to say that in such equilibria, the high-risk consumer is mimicking the low-risk one.

To characterize the set of pooling equilibria, let's first consider the behavior of the insurance company. If both consumers propose the same policy in equilibrium, then the insurance company learns nothing about the consumer's accident probability on hearing

[8]Or, according to our second interpretation, regardless of the proportion of high-risk consumers in the population.

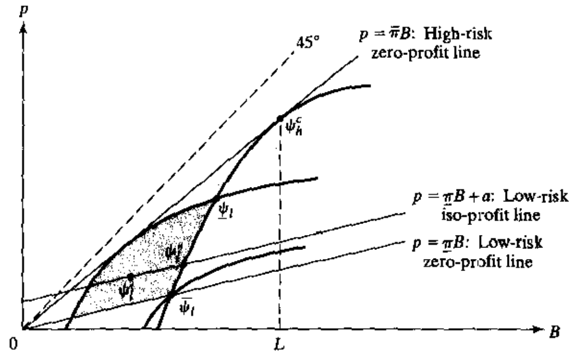


Figure 8.7. Separating equilibria. A pair of policies (ψ_l, ψ_h^c) is the outcome of a separating equilibrium if and only if $\psi_l \neq \psi_h$ and ψ_l is in the shaded region. Note that (ψ_l'', ψ_h^c) Pareto dominates (ψ_l', ψ_h^c). The high-risk consumer is indifferent between them as is the insurance company (ψ_l' and ψ_l'' are on the same low-risk iso-profit line, giving profits $a > 0$). But the low-risk consumer strictly prefers ψ_l'' to ψ_l' because by fact (b), $MRS_l(\psi_l') > \underline{\pi}$. Consequently, among separating equilibria, only those with ψ_l between $\bar{\psi}_l$ and ψ_l are not Pareto dominated by some other separating equilibrium.

the proposal. Consequently, if the proposal is (B, p), then accepting it would yield the insurance company expected profits equal to

$$p - (\alpha\underline{\pi} + (1-\alpha)\bar{\pi})B,$$

where, you recall, α is the probability that the consumer is low-risk.

Let

$$\hat{\pi} = \alpha\underline{\pi} + (1-\alpha)\bar{\pi}.$$

Then the policy will be accepted if $p > \hat{\pi}B$, rejected if $p < \hat{\pi}B$, and the insurance company will be indifferent between accepting and rejecting if $p = \hat{\pi}B$.

Owing to this, the set of policies (B, p) satisfying $p = \hat{\pi}B$ will play an important part in the analysis of pooling equilibria. Fig. 8.8 depicts the set of such policies. They lie on a ray through the origin called the *pooling zero-profit line*.

Now suppose that (B, p) is the pooling equilibrium proposal. According to Lemma 8.1, we must have

$$u_l(B, p) \geq \tilde{u}_l, \quad \text{and} \qquad (8.5)$$
$$u_h(B, p) \geq u_h^c.$$

Moreover, as the discussion following the lemma points out, this policy must be accepted

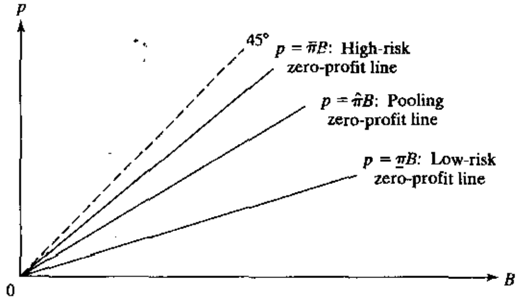


Figure 8.8. Pooling zero-profit line.

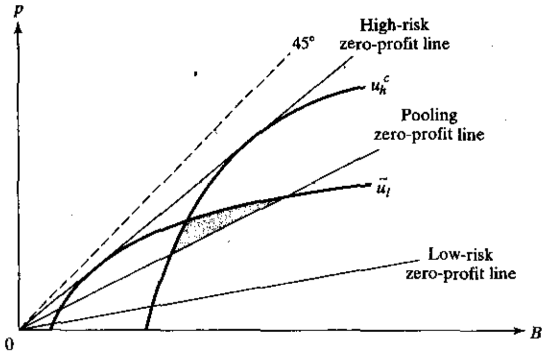


Figure 8.9. Pooling equilibria. The shaded region depicts the set of policies that can arise as pooling equilibria.

by the insurance company. Therefore, it must lie on or above the pooling zero-profit line, so we must have

$$p \geq \hat{\pi} B. \tag{8.6}$$

The policies satisfying the preceding three inequalities are depicted by the shaded region in Fig. 8.9. We now demonstrate that these are precisely the policies that can arise as pooling equilibrium outcomes.

THEOREM 8.2 ***Pooling Equilibrium Characterization***

The policy $\psi' = (B', p')$ is the outcome in some pooling equilibrium if and only if it satisfies inequalities (8.5) and (8.6).

Proof: The discussion preceding the statement of the theorem shows that (B', p') must satisfy (8.5) and (8.6) in order that ψ' be the outcome of some pooling equilibrium. It suffices therefore to prove the converse.

Suppose that $\psi' = (B', p')$ satisfies (8.5) and (8.6). We must define beliefs $\beta(\cdot)$ and a strategy $\sigma(\cdot)$ for the insurance company so that $(\psi', \psi', \sigma(\cdot), \beta(\cdot))$ constitutes a sequential equilibrium.

We follow the proof of Theorem 8.1 by choosing these functions as follows:

$$\beta(B, p) = \begin{cases} \alpha, & \text{if } (B, p) = \psi', \\ 0, & \text{if } (B, p) \neq \psi'. \end{cases}$$

$$\sigma(B, p) = \begin{cases} A, & \text{if } (B, p) = \psi', \quad \text{or} \quad p \geq \bar{\pi} B, \\ R, & \text{otherwise.} \end{cases}$$

Thus, just as in the proof of Theorem 8.1, the insurance company considers any deviation from the equilibrium proposal to have come from the high risk type. Consequently, it is profit-maximizing to accept a proposal $(B, p) \neq \psi'$ only if $p \geq \bar{\pi} B$, as $\sigma(\cdot)$ specifies.

On the other hand, when the equilibrium policy, ψ', is proposed, Bayes' rule requires the insurance company's beliefs to be unchanged because this proposal is made by both risk types. Because $\beta(\psi') = \alpha$, the beliefs do indeed satisfy Bayes' rule. And given these beliefs, it is profit-maximizing to accept the policy ψ', because by (8.6), it yields nonnegative expected profits.

Thus, the insurance company's beliefs satisfy Bayes' rule, and given these beliefs, it is maximizing expected profits subsequent to each policy proposal of the consumer. It remains to show that the two consumer types are maximizing their utility given the insurance company's strategy.

By proposing ψ', the consumer (high- or low-risk) obtains the policy ψ'. By deviating to $(B, p) \neq \psi'$, the consumer obtains the policy $(0, 0)$ if the insurance company rejects the proposal (i.e., if $p < \bar{\pi} B$), and obtains the policy (B, p) if it is accepted (i.e., if $p \geq \bar{\pi} B$). Thus, proposing ψ' is optimal for risk type $i = l, h$ if

$$u_i(\psi') \geq u_i(0, 0), \qquad \text{and}$$
$$u_i(\psi') \geq u_i(B, p) \qquad \text{for all} \quad \bar{\pi} B \leq p \leq w.$$

But these inequalities follow from (8.5) (see Fig. 8.9). Therefore, $(\psi', \psi', \sigma(\cdot), \beta(\cdot))$ is a sequential equilibrium. ∎

As Fig. 8.9 shows, there are potentially many pooling equilibria. It is instructive to consider how the set of pooling equilibria is affected by changes in the probability, α, that the consumer is low-risk.

As α falls, the shaded area in Fig. 8.9 shrinks because the slope of the pooling zero-profit line increases, while everything else in the figure remains fixed. Eventually, the shaded area disappears altogether. Thus, if the probability that the consumer is high-risk is sufficiently high, there are no pooling equilibria.

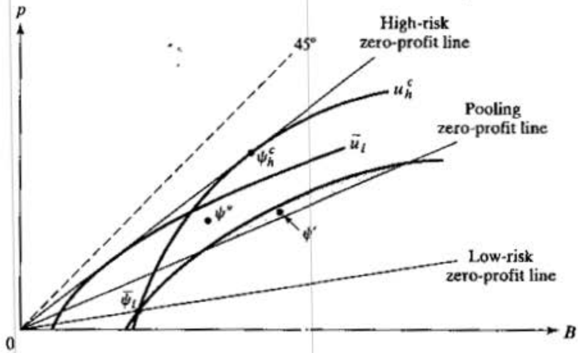


Figure 8.10. Pooling may dominate separation. The best separating equilibrium for consumers yields policies $\psi_l = \bar{\psi}_l$ and $\psi_h = \psi_h^c$. The pooling equilibrium outcome $\psi_l = \psi_h = \psi'$ in the shaded region is strictly preferred by both risk types. Other pooling equilibrium outcomes, such as $\psi_l = \psi_h = \psi''$, are not.

As α increases, the shaded region in Fig. 8.9 expands because the slope of the pooling zero-profit line decreases. Fig. 8.10 shows that when α is large enough, there are pooling equilibria that make both consumer types better off than they would be in every separating equilibrium—even the low-risk consumer. This is not so surprising for the high-risk consumer. The reason this is possible for the low-risk consumer is that it is costly for him to separate himself from the high-risk consumer.

Effective separation requires the low-risk consumer to choose a policy that the high-risk consumer does not prefer to ψ_h^c. This restricts the low-risk consumer's choice and certainly reduces his utility below that which he could obtain in the absence of the high-risk consumer. When α is sufficiently high, and the equilibrium is a pooling one, it is very much like the high-risk consumer is not present. The cost to the low-risk consumer of pooling is then simply a slightly inflated marginal cost per unit of benefit (i.e., $\hat{\pi}$), over and above that which he would pay if his risk type were known (i.e., $\underline{\pi}$). This cost vanishes as α tends to one. On the other hand, the cost of separating himself from the high-risk consumer is bounded away from zero.

The reader may have noticed that in the proofs of Theorems 8.1 and 8.2, there was a common, and not so appealing, component. In each case, when constructing an equilibrium assessment, the beliefs assigned to the insurance company were rather extreme.

Recall that in both proofs, the insurance company's beliefs were constructed so that every deviation from equilibrium was interpreted as having been proposed by the *high-risk* consumer. Although there is nothing formally incorrect about this, it is perhaps worth considering whether or not such beliefs are reasonable.

Let's be clear before proceeding further. The beliefs constructed in proofs of Theorems 8.1 and 8.2 are perfectly in line with our definition of a sequential equilibrium

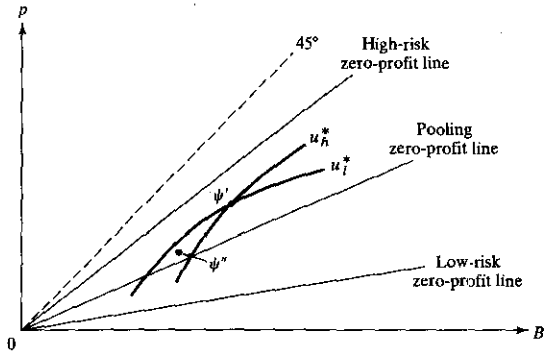


Figure 8.11. Are the firm's beliefs sensible? If ψ' is a pooling equilibrium outcome, then the proposal ψ'' is preferred only by the low-risk consumer. It also lies above the low-risk zero-profit line. Such a policy, ψ'', always exists because ψ' lies on or above the pooling zero-profit line, and $MRS_l(\psi') < MRS_h(\psi')$.

for the insurance signaling game. What we are about to discuss is whether or not we wish to place *additional* restrictions on the insurance company's beliefs.

A Refinement

Are the beliefs assigned to the insurance company in the proofs of Theorems 8.1 and 8.2 reasonable? To see that they might not be, consider a typical pooling equilibrium policy, ψ', depicted in Fig. 8.11.

According to the equilibrium constructed in the proof of Theorem 8.2, were the consumer to propose instead the policy ψ'', the insurance company would believe that the consumer had a high accident probability and would reject the proposal. But do such beliefs make sense in light of the equilibrium ψ'? Note that by proposing the equilibrium policy ψ', the low-risk consumer obtains utility u_l^* and the high-risk consumer obtains utility u_h^*. Moreover, $u_l^* < u_l(\psi'')$, and $u_h(\psi'') < u_h^*$. Therefore, whether the insurance company accepts or rejects the proposal ψ'', the high-risk consumer would be worse off making this proposal than making the equilibrium proposal ψ'. On the other hand, were the insurance company to accept the proposal ψ'', the low-risk consumer would be better off having made that proposal than having made the equilibrium proposal ψ'. Simply put, *only the low-risk consumer has any incentive at all in making the proposal ψ'', given that ψ' is the equilibrium proposal.*

With this in mind, it seems unreasonable for the insurance company to believe, after seeing the proposal ψ'', that it faces the high-risk consumer. Indeed, it is much more reasonable to insist that it instead believes it faces the low-risk consumer. Accordingly, we shall add the following restriction to the insurance company's beliefs. It applies to all sequential equilibria, not just pooling ones.

DEFINITION 8.3 ***(Cho and Kreps) An Intuitive Criterion***

A sequential equilibrium $(\psi_l, \bar{\psi}_h, \sigma(\cdot), \beta(\cdot))$, yielding equilibrium utilities u_l^* and u_h^* to the low- and high-risk consumer, respectively, satisfies the intuitive criterion if the following condition is satisfied for every policy $\psi \neq \psi_l$ or ψ_h:

If $u_i(\psi) > u_i^*$ and $u_j(\psi) < u_j^*$, then $\beta(\psi)$ places probability one on risk type i, so that

$$\beta(\psi) = \begin{cases} 1 & \text{if} \quad i = l, \\ 0 & \text{if} \quad i = h. \end{cases}$$

Restricting attention to sequential equilibria satisfying the intuitive criterion dramatically reduces the set of equilibrium policies. Indeed, we have the following.

THEOREM 8.3 ***Intuitive Criterion Equilibrium***

There is a unique policy pair (ψ_l, ψ_h) that can be supported by a sequential equilibrium satisfying the intuitive criterion. Moreover, this equilibrium is the best separating equilibrium for the low-risk consumer (i.e., $\psi_l = \bar{\psi}_l$, and $\psi_h = \psi_h^c$; see Fig. 8.7).

Proof: We first argue that there are no pooling equilibria satisfying the intuitive criterion. Actually, we've almost already done this in our discussion of Fig. 8.11 preceding Definition 8.3. There we argued that if ψ' were a pooling equilibrium outcome, then there would be a policy ψ'' that is preferred only by the low-risk type, which, in addition, lies strictly above the low-risk zero-profit line (see Fig. 8.11). Consequently, if the low-risk type makes this proposal and the intuitive criterion is satisfied, the insurance company must believe that it faces the low-risk consumer. Because ψ'' lies strictly above the low-risk zero-profit line, the insurance company must accept it (by sequential rationality). But this means that the low-risk consumer can improve his payoff by deviating from ψ' to ψ''. This contradiction establishes the claim: There are no pooling equilibria satisfying the intuitive criterion.

Suppose now that $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ is a separating equilibrium satisfying the intuitive criterion. Then, according to Lemma 8.1, the high-risk consumer's proposal must be accepted by the insurance company and his equilibrium utility, u_h^*, must be at least u_h^c (see Fig. 8.12).

Next, suppose by way of contradiction, that the low-risk consumer's equilibrium utility, u_l^*, satisfies $u_l^* < u_l(\bar{\psi}_l)$. Let $\bar{\psi}_l = (\bar{B}_l, \bar{p}_l)$ and consider the proposal $\psi_l^\varepsilon \equiv (\bar{B}_l - \varepsilon, \bar{p}_l + \varepsilon)$ for ε positive and small. Then due to the continuity of $u_l(\cdot)$, the following inequalities hold for ε small enough. (See Fig. 8.12.)

$$\begin{aligned} u_h^* \geq u_h^c &> u_h(\psi_l^\varepsilon), \\ u_l(\psi_l^\varepsilon) &> u_l^*, \\ \bar{p}_l + \varepsilon &> \underline{\pi}(\bar{B}_l - \varepsilon). \end{aligned}$$

The first two together with the intuitive criterion imply that on seeing the proposal ψ_l^ε, the insurance company believes that it faces the low-risk consumer. The third inequality

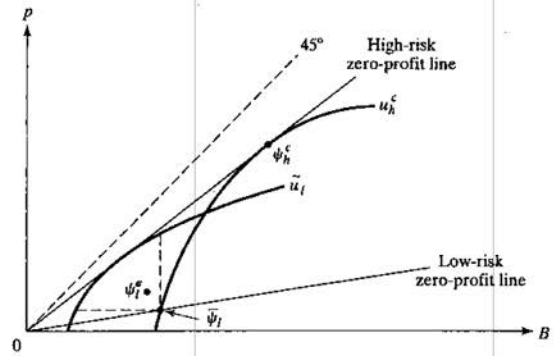


Figure 8.12. The low-risk consumer can obtain $\bar{\psi}_l$.

together with the sequential rationality property of the assessment imply that the insurance company must accept the proposal ψ_l^ε because it earns positive expected profits.

Hence, the low-risk consumer can achieve utility $u_l(\psi_l^\varepsilon) > u_l^*$ by proposing ψ_l^ε. But then u_l^* cannot be the low-risk consumer's equilibrium utility. This contradiction establishes that the low-risk consumer's equilibrium utility must be at least $u_l(\bar{\psi}_l)$. Thus, we've shown that the equilibrium utilities of the two consumer types must satisfy

$$u_l^* \geq u_l(\bar{\psi}_l), \qquad \text{and}$$
$$u_h^* \geq u_h\left(\psi_h^c\right).$$

Now, these inequalities imply that the proposals made by both consumer types are accepted by the insurance company. Consequently, the hypotheses of Theorem 8.1 are satisfied. But according to Theorem 8.1, these two inequalities can hold in a sequential equilibrium only if (see Fig. 8.7)

$$\psi_l = \bar{\psi}_l, \qquad \text{and}$$
$$\psi_h = \psi_h^c.$$

It remains to show that there is a separating equilibrium satisfying the intuitive criterion. We now construct one.

Let $\psi_l = \bar{\psi}_l$ and $\psi_h = \psi_h^c$. To define the insurance company's beliefs, $\beta(\cdot)$, in a manner that is compatible with the intuitive criterion, consider the following set of policies (see Fig. 8.13).

$$A = \left\{\psi \mid u_l(\psi) > u_l(\bar{\psi}_l) \text{ and } u_h(\psi) < u_h\left(\psi_h^c\right)\right\}.$$

This is the set of policies that only the low-risk type prefers to his equilibrium policy.

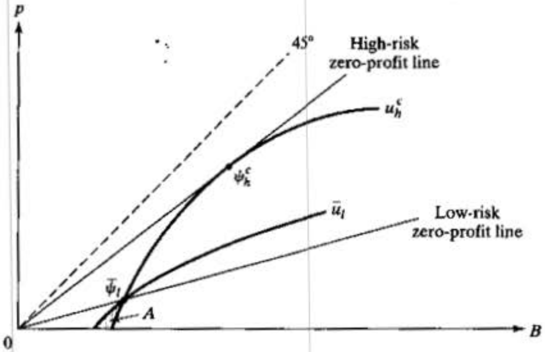


Figure 8.13. An equilibrium satisfying the intuitive criterion.

We now define $\sigma(\cdot)$ and $\beta(\cdot)$ as follows.

$$\beta(B,p)=\begin{cases}1, & \text{if } (B,p)\in A\cup\{\psi_l\}\\ 0, & \text{if } (B,p)\notin A\cup\{\psi_l\}.\end{cases}$$

$$\sigma(B,p)=\begin{cases}A, & \text{if } (B,p)=\psi_l, \quad \text{or } p\geq\bar{\pi}B,\\ R, & \text{otherwise.}\end{cases}$$

It is straightforward to check that by constriction, the beliefs satisfy the intuitive criterion. In addition, one can virtually mimic the relevant portion of the proof of Theorem 8.1 to conclude that the assessment $(\bar{\psi}_l, \psi_h^c, \sigma(\cdot), \beta(\cdot))$ constitutes a separating equilibrium. ∎

The inherent reasonableness of the additional restriction on the insurance company's beliefs embodied in the intuitive criterion suggests that the separating equilibrium that is best for the low-risk consumer is perhaps the most likely outcome in the signaling game. As we've discussed before, this particular outcome can outperform the competitive outcome under asymmetric information. Thus, signaling is indeed one way to improve the efficiency of this market.

There is another route toward improving the efficiency of competitive outcomes under asymmetric information. Indeed, in the insurance market of the real world, this alternative is the road more traveled.

8.1.3 SCREENING

When most consumers purchase auto insurance, they do not present the insurance company with a policy and await a reply, as in the model of the last section. Rather, the insurance company typically offers the consumer a menu of policies from which to choose, and the consumer simply makes a choice. By offering consumers a menu of policies, insurance companies are able to (implicitly) *screen* consumers by tailoring the offered policies so that

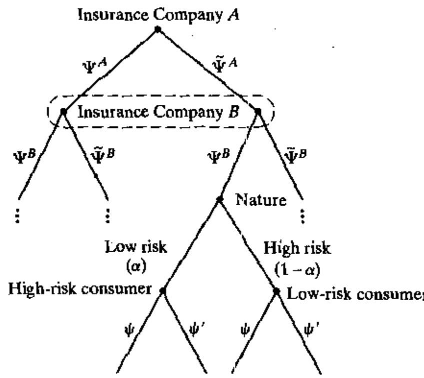


Figure 8.14. Insurance screening game. Note that, unlike the figure, the insurance companies actually have a continuum of actions. Thus, this game is not finite.

high-risk types are induced to choose one particular policy, and low-risk types are induced to choose another. We now analyze such a model.

Again, we shall formulate the situation as an extensive form game. Although it was possible to illustrate the essential features of signaling using just a single insurance company, there are nuances of screening that require two insurance companies to reveal. Thus, we shall add an additional insurance company to the model.[9]

As before, there will be two consumers, low- and high-risk, occurring with probability α and $1-\alpha$, respectively. And again, one can interpret this as there being many consumers, a fraction α of which is low-risk.

So consider the following "insurance screening game" involving two insurance companies and two consumers. Fig. 8.14 depicts its extensive form.

- The two insurance companies move first by simultaneously choosing a finite list (menu) of policies.
- Nature moves second and determines which consumer the insurance companies face. The low-risk consumer is chosen with probability α, and the high-risk consumer with probability $1-\alpha$.
- The chosen consumer moves last by choosing a single policy from one of the insurance companies' lists.

Now, because there are only two possible types of consumers, we may restrict the insurance companies to lists with at most two policies. Thus, a pure strategy for insurance company $j = A, B$ is a pair of policies $\Psi^j = (\psi_l^j, \psi_h^j)$. We interpret ψ_l^j (resp. ψ_h^j) as the policy that insurance company j includes in its list for the low- (resp. high-) risk consumer. However, keep in mind that the low- (resp. high-) risk consumer need not choose this policy because the insurance company cannot identify the consumer's risk type. The consumer will

[9] We could also have included two insurance companies in the signaling model. This would not have changed the results there in any significant way.

choose the policy yielding him the highest utility among those offered by the two insurance companies.

A pure strategy for consumer $i = l, h$ is a choice function $c_i(\cdot)$ specifying for each pair of policy pairs, (Ψ^A, Ψ^B), an insurance company and one of its policies or the null policy. Thus, we always give the consumers the option of choosing the null policy from either insurance company even if this policy is not formally on either company's list. This is simply a convenient way to allow consumers the ability not to purchase insurance. Thus, $c_i(\Psi^A, \Psi^B) = (j, \psi)$, where $j = A$ or B, and where $\psi = \psi_l^j, \psi_h^j$, or $(0, 0)$.

As is evident from Fig. 8.14, the only nonsingleton information set belongs to insurance company B. However, note that no matter what strategies the players employ, this information set must be reached. Consequently, it is enough to consider the subgame perfect equilibria of this game. You are asked to show in an exercise that were the game finite (so that the sequential equilibrium definition can be applied), its set of sequential equilibrium outcomes would be identical to its set of subgame perfect equilibrium outcomes.

Again, we can split the set of pure strategy subgame perfect equilibria into two kinds: separating and pooling. In a separating equilibrium, the two consumer types make different policy choices, whereas in a pooling equilibrium, they do not.

DEFINITION 8.4 ***Separating and Pooling Screening Equilibria***

The pure strategy subgame perfect equilibrium $(\Psi^A, \Psi^B, c_l(\cdot), c_h(\cdot))$ is separating if $\psi_l \neq \psi_h$, where $(j_l, \psi_l) = c_l(\Psi^A, \Psi^B)$, and $(j_h, \psi_h) = c_h(\Psi^A, \Psi^B)$. Otherwise, it is pooling.

Note then that in a pooling equilibrium, although the two types of consumers must choose to purchase the same policy, they needn't purchase it from the same insurance company.

Analyzing the Game

We wish to characterize the set of subgame perfect equilibria of the insurance screening game. An important driving force of the analysis is a phenomenon called *cream skimming*.

Cream skimming occurs when one insurance company takes strategic advantage of the set of policies offered by the other by offering a policy that would attract away *only the low-risk consumers* from the competing company. The "raiding" insurance company therefore gains only the very best consumers (the cream) while it leaves its competitor with the very worst consumers. In equilibrium, both companies must ensure that the other cannot skim its cream in this way. Note that at least two firms are required in order that cream skimming becomes a strategic concern. It is this that motivated us to introduce a second insurance company into the model.

We first provide a lemma that applies to all pure strategy subgame perfect equilibria.

LEMMA 8.2 *Both insurance companies earn zero expected profits in every pure strategy subgame perfect equilibrium.*

Proof: The proof of this result is analogous to that in the model of Bertrand competition from Chapter 4.

First, note that in equilibrium, each insurance company must earn nonnegative profits because each can guarantee zero profits by offering a pair of null policies in which $B = p = 0$. Thus, it suffices to show that neither insurance company earns strictly positive expected profits.

Suppose by way of contradiction that company A earns strictly positive expected profits and that company B's profits are no higher than A's. Let $\psi_l^* = (B_l^*, p_l^*)$ and $\psi_h^* = (B_h^*, p_h^*)$ denote the policies chosen by the low- and high-risk consumers, respectively, in equilibrium. We then can write the total expected profits of the two firms as

$$\Pi \equiv \alpha(p_l^* - \underline{\pi} B_l^*) + (1 - \alpha)(p_h^* - \bar{\pi} B_h^*) > 0.$$

Clearly, Π strictly exceeds company B's expected profits.

Now, we shall consider two cases.

Case 1: $\psi_l^* = \psi_h^* = (B^*, p^*)$. Consider the following deviation by company B. Company B offers the policy pair $\{(B^* + \varepsilon, p^*), (B^* + \varepsilon, p^*)\}$, where $\varepsilon > 0$. Clearly, each consumer type then will strictly prefer to choose the policy $(B^* + \varepsilon, p^*)$ from company B, and for ε small enough, company B's expected profits will be arbitrarily close to Π and so larger than they are in equilibrium. But this contradicts the equilibrium hypothesis.

Case 2: $\psi_l^* = (B_l^*, p_l^*) \neq \psi_h^* = (B_h^*, p_h^*)$. Equilibrium requires that neither consumer can improve his payoff by switching his policy choice to that of the other consumer. Together with this and the fact that the policy choices are distinct, the single-crossing property implies that at least one of the consumers strictly prefers his own choice to the other's; i.e., either

$$u_l(\psi_l^*) > u_l(\psi_h^*), \qquad \text{or} \tag{P.1}$$

$$u_h(\psi_h^*) > u_h(\psi_l^*). \tag{P.2}$$

Suppose then that (P.1) holds. Consider the deviation for company B in which it offers the pair of policies $\psi_l^\varepsilon = (B_l^* + \varepsilon, p_l^*)$ and $\psi_h^\varepsilon = (B_h^* + \beta, p_h^*)$, where $\varepsilon, \beta > 0$.

Clearly, each consumer $i = l, h$ strictly prefers policy ψ_i^ε to ψ_i^*. In addition, we claim that ε and $\beta > 0$ can be chosen arbitrarily small so that

$$u_l(\psi_l^\varepsilon) > u_l(\psi_h^\beta), \qquad \text{and} \tag{P.3}$$

$$u_h(\psi_h^\beta) > u_h(\psi_l^\varepsilon). \tag{P.4}$$

To see this, note that by (P.1), (P.3) will hold as long as ε and β are small enough. Inequality (P.4) then can be assured by fixing β and choosing ε small enough, because for $\beta > 0$ and fixed, we have

$$u_h(\psi_h^\beta) > u_h(\psi_h^*) \geq u_h(\psi_l^*) = \lim_{\varepsilon \to 0} u_h(\psi_l^\varepsilon),$$

where the weak inequality follows because, in equilibrium, the high-risk consumer cannot prefer any other policy choice to his own. See Fig. 8.15.

But (P.3) and (P.4) imply that subsequent to B's deviation, the low-risk consumer will choose the policy ψ_l^ε, and the high-risk consumer will choose the policy ψ_h^β. For ε

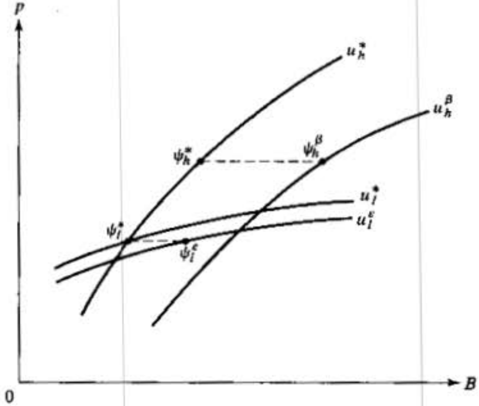


Figure 8.15. A difficult case: Depicted is the most troublesome case in which (P.1), $u_l^* \equiv u_l(\psi_l^*) > u_l(\psi_h^*)$ holds, but $u_h^* \equiv u_h(\psi_h^*) = u_h(\psi_l^*)$ so that (P.2) does not hold. For each $\beta > 0$, there is $\varepsilon > 0$ small enough so that $u_h^\beta \equiv u_h(\psi_h^\beta) > u_l^\varepsilon \equiv u_h(\psi_l^\varepsilon)$. When the policies ψ_l^ε and ψ_h^β are available, ψ_l^ε is strictly best for the low-risk consumer and ψ_h^β is strictly best for the high-risk consumer.

and β small enough, this will yield company B expected profits arbitrarily close to Π and therefore strictly above B's equilibrium expected profits. But this is again a contradiction.

A similar argument leads to a contradiction if instead (P.2) holds, so we conclude that both insurance companies must earn zero expected profits in every subgame perfect equilibrium. ∎

Pooling Equilibria

One might suspect that the set of pooling equilibria would be whittled down by the cream-skimming phenomenon. Indeed, the setting seems just right for cream skimming when both consumer types are treated the same way. This intuition turns out to be correct with a vengeance. Indeed, cream skimming eliminates the possibility of any pooling equilibrium at all.

THEOREM 8.4 ***Nonexistence of Pooling Equilibria***

There are no pure strategy pooling equilibria in the insurance screening game.

Proof: We shall proceed by way of contradiction.

Suppose the policy $\psi^* = (B^*, p^*)$ is chosen by both consumers in a subgame perfect equilibrium. By Lemma 8.2, the total expected profits of the two insurance companies must be zero, so

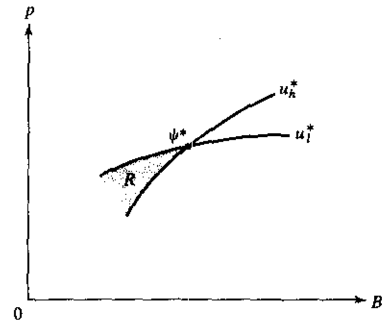
$$\alpha(p^* - \underline{\pi} B^*) + (1 - \alpha)(p^* - \bar{\pi} B^*) = 0. \tag{P.1}$$

Consider first the case in which $B^* > 0$. Then (P.1) implies that

$$p^* - \underline{\pi} B^* > 0. \tag{P.2}$$

Consequently, $p^* > 0$ as well, so that ψ^* does not lie on either axis as shown in Fig. 8.16.

Figure 8.16. ψ^* lies on neither axis.

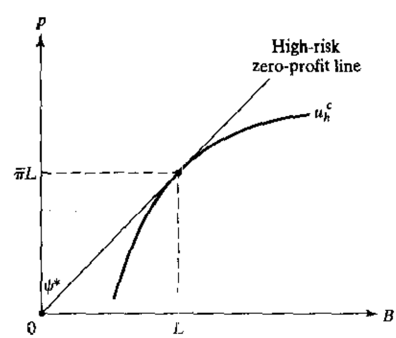


By the single-crossing property, there is a region, R (see Fig. 8.16), such that ψ^* is the limit of policies in R. Let ψ' be a policy in R very close to ψ^*.

Suppose now that insurance company A is offering policy ψ^* in equilibrium. If insurance company B offers policy ψ', and only ψ', then the high-risk consumer will choose policy ψ^* (or one he is indifferent to) from the first insurance company, whereas the low-risk consumer will purchase ψ' from insurance company B. If ψ' is close enough to ψ^*, then by (P.2), insurance company B will earn strictly positive profits from this cream-skimming deviation, and so must be earning strictly positive profits in equilibrium. But this contradicts Lemma 8.2.

Consider now the case in which $B^* = 0$. By (P.1), this implies that $p^* = 0$ as well. Thus, ψ^* is the null policy, as in Fig. 8.17. But either company now can earn positive profits by offering the single policy $(L, \bar{\pi} L + \varepsilon)$ where $\varepsilon > 0$ is sufficiently small. It earns strictly positive profits because it earns strictly positive profits on both consumer types (it is above both the high- and low-risk zero-profit lines), and the high-risk consumer certainly will choose this policy over the null policy. This final contradiction completes the proof. ∎

Figure 8.17. ψ^* is the null policy.



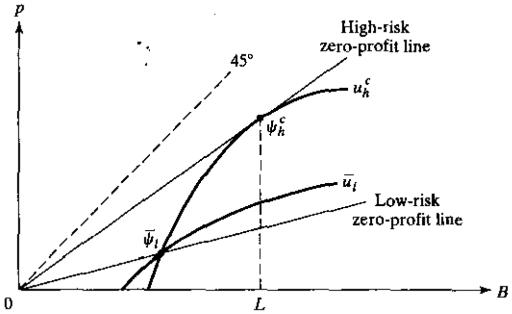


Figure 8.18. The only possible separating equilibrium. It coincides with the best separating equilibrium for consumers in the insurance signaling game from section 8.1.2.

Note the importance of cream skimming to the preceding result. This is a typical feature of competitive screening models wherein multiple agents on one side of a market compete to attract a common pool of agents on the other side of the market by simultaneously offering a menu of "contracts" from which the pool of agents may choose.

Separating Equilibria

The competitive nature of our screening model also has an important impact on the set of separating equilibria, as we now demonstrate.

THEOREM 8.5 ***Separating Equilibrium Characterization***

Suppose that ψ_l^* and ψ_h^* are the policies chosen by the low- and high-risk consumers, respectively, in a pure strategy separating equilibrium. Then $\psi_l^* = \bar{\psi}_l$ and $\psi_h^* = \psi_h^c$, as illustrated in Fig. 8.18.

Note then that the only possible separating equilibrium in the insurance screening model coincides with the *best* separating equilibrium for consumers in the insurance signaling game from section 8.1.1. By Theorem 8.4, this will be the only possible equilibrium in the game.

Proof: The proof proceeds in series of claims.

Claim 1. *The high-risk consumer must obtain at least utility u_h^c. (See Fig. 8.18.)*

By Lemma 8.2, both insurance companies must earn zero profits. Consequently, it cannot be the case that the high-risk consumer strictly prefers the policy $(L, \bar{\pi}L + \varepsilon)$ to ψ_h^*. Otherwise, one of the insurance companies could offer just this policy and earn positive profits. (Note that this policy earns positive profits on both consumers.) But this

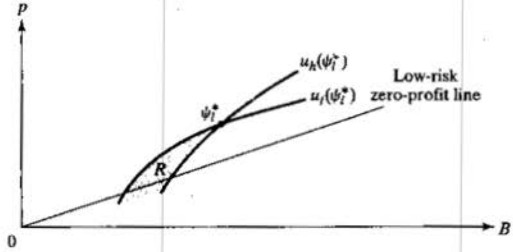


Figure 8.19. A cream-skimming region.

means that

$$u_h(\psi_h^*) \geq u_h(L, \bar{\pi} L + \varepsilon) \quad \text{for all} \quad \varepsilon > 0.$$

The result follows by taking the limit of the right-hand side as $\varepsilon \to 0$, because $u_h(\cdot)$ is continuous and $\psi_h^c = (L, \bar{\pi} L)$.

Claim 2. *ψ_l^* must lie on the low-risk zero-profit line.*

Note that by Claim 1, ψ_h^* must lie on or below the high-risk zero-profit line. Thus, nonpositive profits are earned on the high-risk consumer. Because by Lemma 8.2 the insurance companies' aggregate profits are zero, this implies that ψ_l^* lies on or above the low-risk zero-profit line.

So, suppose by way of contradiction that $\psi_l^* = (B_l^*, p_l^*)$ lies above the low-risk zero-profit line. Then $p_l^* > 0$. But this means that $B_l^* > 0$ as well because the low-risk consumer would otherwise choose the null policy (which is always available). Thus, ψ_l^* is strictly above the low-risk zero-profit line and not on the vertical axis as shown in Fig. 8.19.

Consequently, region R in Fig. 8.19 is present. Now if the insurance company who is *not* selling a policy to the high-risk consumer offers policies only strictly within region R, then only the low-risk consumer will purchase a policy from this insurance company. This is because such a policy is strictly preferred to ψ_l^* by the low-risk consumer and strictly worse than ψ_l^* (which itself is no better than ψ_h^*) for the high-risk consumer. This deviation would then result in strictly positive profits for this insurance company because all such policies are above the low-risk zero-profit line. The desired conclusion follows from this contradiction.

Claim 3. $\psi_h^* = \psi_h^c$.

By Claim 2, and Lemma 8.2, ψ_h^* must lie on the high-risk, zero-profit line. But by Claim 1, $u_h(\psi_h^*) \geq u_h(\psi_h^c)$. Together, these imply that $\psi_h^* = \psi_h^c$ (see Fig. 8.18).

Claim 4. $\psi_l^* = \bar{\psi}_l$.

Consult Fig. 8.20. By Claim 2, it suffices to show that ψ_l^* cannot lie on the low-risk zero-profit line strictly below $\bar{\psi}_l$ (such as ψ'') or strictly above $\bar{\psi}_l$ (such as ψ').

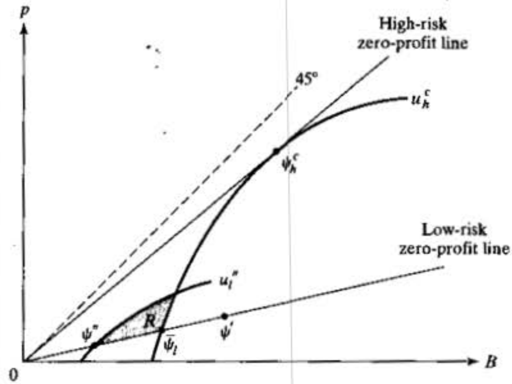


Figure 8.20. Another cream-skimming region.

So, suppose first that $\psi_l^* = \psi'$. The high-risk consumer would then strictly prefer ψ' to ψ_h^c and thus would not choose ψ_h^c contrary to Claim 3.

Next, suppose that $\psi_l^* = \psi''$. Then the low-risk consumer obtains utility u_l'' in equilibrium (see Fig. 8.20). Moreover, region R is then present. Consider the insurance company that does *not* sell ψ_h^c to the high-risk consumer. Let this insurance company offer any policy strictly within region R. This policy will be purchased only by the low-risk consumer and will earn strictly positive profits. This contradiction proves Claim 4 and completes the proof. ∎

Note that Theorem 8.5 does not claim that a separating screening equilibrium exists. Together with Theorem 8.4, it says only that *if* a pure strategy subgame perfect equilibrium exists, it must be separating and the policies chosen by the consumers are unique.

Cream skimming is a powerful device in this screening model for eliminating equilibria. But it can be too powerful. Indeed, there are cases in which no pure strategy subgame perfect equilibrium exists at all.

Consider Fig. 8.21. Depicted there is a case in which no pure strategy equilibrium exists. To see this, it is enough to show that it is not an equilibrium for the low- and high-risk consumers to obtain the policies $\bar{\psi}_l$ and ψ_h^c as described in Theorem 8.5. But this is indeed the case, because either insurance company can deviate by offering only the policy ψ', which will be purchased by both consumer types (because it is strictly preferred by them to their equilibrium policies). Consequently, this company will earn strictly positive expected profits because ψ' is strictly above the pooling zero-profit line (which is the appropriate zero-profit line to consider because *both* consumer types will purchase ψ'). But this contradicts Lemma 8.2.

Thus, when α is close enough to one, so that the pooling zero-profit line intersects the $\bar{u}_l$ indifference curve (see Fig. 8.21), the screening model admits no pure strategy

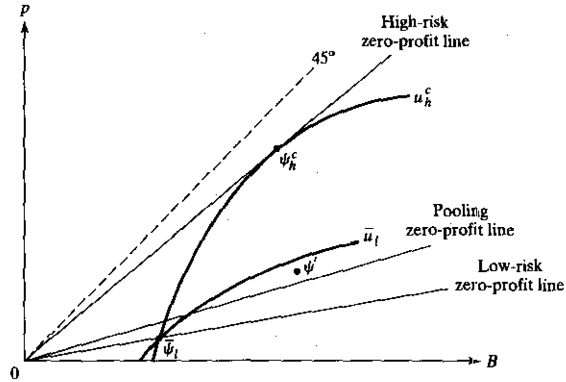


Figure 8.21. No equilibrium exists. If the best policies available for the low- and high-risk consumers are $\bar{\psi}_l$ and ψ_h^c, respectively, then offering the policy ψ' will attract both consumer types and earn positive profits because it lies above the pooling zero-profit line. No pure strategy subgame perfect equilibrium exists in this case.

subgame perfect equilibrium.[10] One can show that there always exists a subgame perfect equilibrium in behavioral strategies, but we shall not pursue this. We are content to note that nonexistence in this model arises only when the extent of the asymmetry of information is relatively minor, and in particular when the presence of high-risk consumers is small.

We next consider an issue that we have so far ignored. What is the effect of the availability of insurance on the driving behavior of the consumer?

8.2 Moral Hazard and the Principal Agent Problem

Insurance companies are not naive. They understand well that once a consumer has purchased auto insurance, he may not drive with as much care as he did before he had insurance. Moreover, a consumer's incentive to drive carefully is likely to diminish with the amount of coverage. Unfortunately for insurance companies, they cannot observe the effort consumers direct toward safe driving. Thus, they must structure their policies so that the policies themselves induce the consumers to take an appropriate level of care.

When a *principal* (like the insurance company) has a stake in the action taken by an *agent* (the consumer), but the agent's action cannot be observed by the principal, the situation is said to involve **moral hazard**. The **principal-agent problem** is for the principal

[10]Even when the pooling zero-profit line does not intersect the $\bar{u}_l$ indifference curve, an equilibrium is not guaranteed to exist. There may still be a *pair* of policies such that one attracts the low-risk consumers making positive profits, and the other attracts the high-risk consumers (keeping them away from the first policy) making negative profits, so that overall expected profits are strictly positive.

to design an incentive scheme so that the agent takes an appropriate action. We now explore these ideas in our insurance context.

To keep things simple, the model we shall consider involves a single insurance company and a single consumer. The consumer might incur an accident resulting in a varying amount of loss. There are L levels of losses, ranging from 1 dollar through L dollars, depending on the severity of the accident incurred. It is also possible that an accident is avoided altogether. It is convenient to refer to this latter possibility as an accident resulting in a loss of 0 dollars.

The probability of incurring an accident resulting in losses of $l \in \{0, 1, \ldots, L\}$ is given by $\pi_l(e) > 0$, where e is the amount of effort exerted toward safe driving. As discussed before, it is natural to think of these probabilities as being affected by such efforts. Note that $\sum_l \pi_l(e) = 1$ for each fixed effort level e.

To keep things simple, there are only two possible effort levels for the consumer. We let $e = 0$ denote low effort and $e = 1$ denote high effort. To capture the idea that higher effort results in a lower likelihood that the consumer will have a serious (i.e., expensive) accident, we make the following assumption.

ASSUMPTION 8.1 ***Monotone Likelihood Ratio***

$\pi_l(0)/\pi_l(1)$ *is strictly increasing in* $l \in \{0, 1, \ldots, L\}$.

The monotone likelihood ratio property says that conditional on observing the accident loss, l, the relative probability that low effort was expended versus high effort *increases* with l. Thus, one would be more willing to bet that the consumer exerted low effort when the observed accident loss is higher.

As in our previous models, the consumer has a strictly increasing, strictly concave, von Neumann-Morgenstern utility function, $u(\cdot)$, over wealth, and initial wealth equal to $w > L$. In addition, $d(e)$ denotes the consumer's disutility of effort, e. Thus, for a given effort level e, the consumer's von Neumann-Morgenstern utility over wealth is $u(\cdot) - d(e)$, where $d(1) > d(0)$.[11]

We assume that the insurance company can observe the amount of loss, l, due to an accident, but not the amount of accident avoidance effort, e. Consequently, the insurance company can only tie the benefit amount to the amount of loss. Let B_l denote the benefit paid by the insurance company to the consumer when the accident loss is l. Thus, a *policy* is a tuple $(p, B_0, B_1, \ldots, B_L)$, where p denotes the price paid to the insurance company in return for guaranteeing the consumer B_l dollars if an accident loss of l dollars occurs.

The question of interest is this: What kind of policy will the insurance company offer the consumer, and what are its efficiency properties?

8.2.1 SYMMETRIC INFORMATION

To understand the impact of the unobservability of the consumer's accident avoidance effort, we first shall consider the case in which the insurance company can observe the consumer's effort level.

[11] All of the analysis to follow generalizes to the case in which utility takes the form $u(w, e)$, where $u(w, 0) > u(w, 1)$ for all wealth levels w.

Consequently, the insurance company can offer a policy that pays benefits *only if* a particular effort level was exerted. In effect, the insurance company can choose the consumer's effort level.

Thus, the insurance company wishes to solve the following problem:

$$\max_{e,p,B_0,\dots,B_L} \quad p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \text{subject to} \tag{8.7}$$

$$\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u},$$

where $\bar{u}$ denotes the consumer's reservation utility.[12]

According to the maximization problem (8.7), the insurance company chooses a policy and an effort level to maximize its expected profits subject to the constraint that the policy yields the consumer at least his reservation utility—hence, the consumer will be willing to accept the terms of the policy and exert the required effort level.

The easiest way to solve (8.7) is to assume that $e \in \{0, 1\}$ is fixed and to then form the Lagrangian considered as a function of $p, B_0, \dots, B_L$ only. This gives

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(e) B_l + \lambda \left[\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \right].$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda \left[\sum_{l=0}^{L} \pi_l(e) u'(w - p - l + B_l) \right] = 0, \tag{8.8}$$

$$\frac{\partial \mathcal{L}}{\partial B_l} = -\pi_l(e) + \lambda \pi_l(e) u'(w - p - l + B_l) = 0, \qquad \forall l \geq 0, \tag{8.9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \geq 0, \tag{8.10}$$

where (8.10) holds with equality if $\lambda \neq 0$.

Note that the first condition, (8.8), is redundant because it is implied by the $(L+1)$ equations in (8.9). Thus, the preceding is a system of at most $(L+2)$ independent equations in $(L+3)$ unknowns.

The equalities in (8.9) imply that $\lambda > 0$, and that

$$u'(w - p - l + B_l) = 1/\lambda, \qquad \forall l \geq 0.$$

Hence, $B_l - l$ must be constant for all $l = 0, 1, \dots, L$.

[12] Because the consumer always can choose not to purchase insurance, $\bar{u}$ must be at least as large as $\max_{e \in \{0,1\}} \sum_{l=0}^{L} \pi_l(e) u(w - l) - d(e)$. However, $\bar{u}$ may be strictly larger than this if, for example, there are other insurance companies offering policies to the consumer as well.

Because $\lambda > 0$, the first-order condition associated with the constraint must hold with equality, which implies that

$$u(w - p - l + B_l) = d(e) + \bar{u}, \qquad \forall l \geq 0. \tag{8.11}$$

Because there are only $(L+2)$ independent equations and $(L+3)$ unknowns, we may set $B_0 = 0$ without any loss.[13] Thus, setting $l = 0$ in (8.11) gives us an equation in p alone and so determines p. Moreover, because $B_l - l$ is constant for all $l = 0, 1, \ldots, L$, and because $B_0 - 0 = 0$, we therefore must have

$$B_l = l, \qquad \text{for all} \quad l = 0, 1, \ldots, L.$$

Therefore, for either fixed effort level $e \in \{0, 1\}$, the symmetric information solution provides full insurance to the consumer at every loss level. This is no surprise because the consumer is strictly risk-averse and the insurance company is risk-neutral. It is simply an example of efficient risk sharing. In addition, the price charged by the insurance company equates the consumer's utility from the policy at the required effort level with his reservation utility.

Now that we have determined for each effort level the optimal policy, it is straightforward to optimize over the effort level as well. Given $e \in \{0, 1\}$, the optimal benefit levels are $B_l = l$ for each l, so using (8.11) the optimal price $p(e)$ is given implicitly by

$$u(w - p(e)) = d(e) + \bar{u}. \tag{8.12}$$

Therefore, the insurance company chooses $e \in \{0, 1\}$ to maximize

$$p(e) - \sum_{l=0}^{L} \pi_l(e) l.$$

Note the trade-off between requiring high versus low effort. Because $d(0) < d(1)$, (8.12) implies that requiring *lower* effort allows the insurance company to charge a higher price, increasing profits. On the other hand, requiring *higher* effort reduces the expected loss due to an accident (by the monotone likelihood ratio property; see the exercises), and so also increases expected profits. One must simply check which effort level is best for the insurance company in any specific case.

What is important here is that regardless of which effort level is best for the firm, the profit-maximizing policy always involves full insurance. This is significant and it implies that the outcome here is Pareto efficient. We have seen this sort of result before, so we shall not give another proof of it.

[13] Indeed, it was clear from the start that setting $B_0 = 0$ was harmless because changes in B_0 always can be offset by corresponding changes in the price p and in the benefit levels $B_1, \ldots, B_L$ without changing the consumer's utility or the insurance company's profits.

8.2.2 ASYMMETRIC INFORMATION

We now turn our attention to the more interesting case in which the consumer's choice of effort cannot be observed by the insurance company. The insurance company continues to seek the policy that will maximize expected profits. However, if it now cannot observe the effort level chosen by the consumer, how should it go about choosing the optimal policy?

Think of the problem this way. The insurance company must design a policy with a desired accident avoidance effort level in mind. However, because the consumer's effort level cannot be observed, the insurance company must ensure that *the nature of the policy renders it optimal for the consumer to voluntarily choose the desired effort level.*

This effectively adds an additional constraint to the insurance company's maximization problem. The policy and effort level must be chosen not only to provide the consumer with at least his reservation utility; it must also induce the consumer to voluntarily choose the desired effort level. Thus, the insurance company's problem is

$$\max_{e, p, B_0, \ldots, B_L} p - \sum_{l=0}^{L} \pi_l(e) B_l \qquad \text{subject to} \tag{8.13}$$

$$\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u}, \qquad \text{and} \tag{8.14}$$

$$\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \sum_{l=0}^{L} \pi_l(e') u(w - p - l + B_l) - d(e'), \tag{8.15}$$

where $e, e' \in \{0, 1\}$ and $e \neq e'$.

The new constraint is (8.15). It ensures that e, the accident avoidance effort level that the insurance company has in mind when calculating its profits, is the same as that actually chosen by the consumer, for it guarantees that this effort level maximizes the consumer's expected utility given the proposed policy.

We shall follow the same procedure as before in solving this problem. That is, we will first fix the effort level, e, and then determine for this particular effort level the form of the optimal policy. Once this is done for both effort levels, it is simply a matter of checking which effort level together with its associated optimal policy maximizes the insurance company's profits.

The Optimal Policy for $e = 0$

Suppose we wish to induce the consumer to exert low effort. Among policies that have this effect, which is best for the insurance company? Although we could form the Lagrangian associated with this problem, it is simpler to take a different route.

Recall that if the incentive constraint (8.15) were absent, then the optimal policy when $e = 0$ is given by choosing $p, B_0, \ldots, B_L$ to satisfy

$$\begin{aligned} u(w - p) &= d(0) + \bar{u}, \\ B_l = l, &\qquad l = 0, 1, \ldots, L. \end{aligned} \tag{8.16}$$

Now, adding the incentive constraint to the problem cannot increase the insurance company's maximized profits. Therefore, if the solution to (8.16) satisfies the incentive constraint, then it must be the desired optimal policy. But, clearly, the solution does indeed satisfy (8.15). Given the policy in (8.16), the incentive constraint when $e = 0$ reduces to

$$d(0) \geq d(1),$$

which holds (strictly) by assumption.

Consequently, inducing the consumer to exert low effort in a manner that maximizes profits requires the insurance company to offer the same policy as it would were effort observable.

The Optimal Policy for $e = 1$

Suppose now that we wish to induce the consumer to exert high effort. To find the optimal policy for the insurance company, we shall consider the effort level as fixed at $e = 1$ in the maximization problem (8.13). Thus, the maximization is over the choice variables $p, B_0, \ldots, B_L$. Also, because $e = 1$, we have $e' = 0$ in the incentive constraint (8.15).

The Lagrangian for this problem is then

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(1)B_l + \lambda\left[\sum_{l=0}^{L} \pi_l(1)u(w - p - l + B_l) - d(1) - \bar{u}\right] \tag{8.17}$$
$$+ \beta\left[\sum_{l=0}^{L} \pi_l(1)\, u\,(w - p - l + B_l) - d(1) - \left(\sum_{l=0}^{L} \pi_l(0)u(w - p - l + B_l) - d(0)\right)\right],$$

where λ and β are the multipliers corresponding to constraints (8.14) and (8.15), respectively.

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda\left[\sum_{l=0}^{L} (\pi_l(1) + \beta(\pi_l(1) - \pi_l(0)))u'(w - p - l + B_l)\right] = 0, \tag{8.18}$$

$$\frac{\partial \mathcal{L}}{\partial B_l} = -\pi_l(1) + [\lambda\pi_l(1) + \beta(\pi_l(1) - \pi_l(0))]u'(w - p - l + B_l) = 0, \quad \forall l, \tag{8.19}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{l=0}^{L} \pi_l(1)u(w - p - l + B_l) - d(1) - \bar{u} \geq 0, \tag{8.20}$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \sum_{l=0}^{L} (\pi_l(1) - \pi_l(0))u(w - p - l + B_l) + d(0) - d(1) \geq 0, \tag{8.21}$$

where (8.20) and (8.21) hold with equality if $\lambda \neq 0$ and $\beta \neq 0$, respectively.

As in the previous problem, the first of these conditions (8.18) is implied by the next $L + 1$ given in (8.19). As before, this redundancy will allow us to set $B_0 = 0$ without loss of generality.

Now, (8.19) can be rewritten as

$$\frac{1}{u'(w-p+B_l-l)}=\lambda+\beta\left[1-\frac{\pi_l(0)}{\pi_l(1)}\right]. \tag{8.22}$$

We now argue that both λ and β are nonzero.

Suppose that $\beta=0$. Then (8.22) would imply that the left-hand side is constant in l, which implies that $w-p+B_l-l$ is constant in l. But this cannot hold because then condition (8.21) fails, as its left-hand side reduces to $d(0)-d(1)$, which is strictly negative. We conclude that $\beta\neq 0$.

To see that $\lambda\neq 0$, first note that the monotone likelihood ratio property implies that there is an l such that $\pi_l(0)\neq\pi_l(1)$. Because $\sum_l \pi_l(0)=\sum_l \pi_l(1)=1$, there must exist l and l' such that $\pi_l(0)>\pi_l(1)$, and $\pi_{l'}(0)<\pi_{l'}(1)$. Consequently, the term in square brackets in (8.22) takes on both positive and negative values.

Now, if $\lambda=0$, then because $\beta\neq 0$, the right-hand side of (8.22) takes on both positive and negative values. However, the left-hand side is always strictly positive. Therefore, $\lambda\neq 0$. Indeed, this argument shows that $\lambda>0$.

The fact that both λ and β are nonzero implies that both constraints, (8.20) and (8.21), are binding in the optimal solution. Thus, the consumer is held down to his reservation utility, and he is just indifferent between choosing high and low effort.

To gain more insight into the optimal policy for $e=1$, it is helpful to show that $\beta>0$. So suppose that $\beta<0$. The monotone likelihood ratio property then implies that the right-hand side of (8.22) is strictly increasing in l. Consequently $u'(w-p+B_l-l)$ is strictly decreasing in l, so that B_l-l, and therefore $u(w-p+B_l-l)$ are strictly increasing in l. But the latter together with the monotone likelihood ratio property imply that $\sum_l(\pi_l(1)-\pi_l(0))u(w-p+B_l-l)<0$ (see Exercise 8.11). This contradicts (8.21), because $d(0)<d(1)$. We conclude that $\beta>0$.

Now because $\beta>0$, the monotone likelihood ratio property implies that the right-hand side of (8.22) is strictly decreasing, so that $u'(w-p+B_l-l)$ is strictly increasing. Consequently, the optimal policy must display the following feature:

$$l-B_l \text{ is strictly increasing in } l=0,1,\ldots,L. \tag{8.23}$$

Recall that we may set $B_0=0$ without any loss of generality. Consequently, condition (8.23) indicates that the optimal high-effort policy does not provide full insurance—rather, it specifies a deductible payment that increases with the size of the loss.

This is, of course, very intuitive. To give the consumer an incentive to choose high effort, there must be something in it for the consumer. When $l-B_l$ is strictly increasing, there is a positive utility benefit to exerting high effort, namely,

$$\sum_{l=0}^{L}(\pi_l(1)-\pi_l(0))u(w-p-l+B_l)>0.$$

That this sum is strictly positive follows from (8.23) and the monotone likelihood ratio property (again, see Exercise 8.11). Of course, there is also a utility cost associated with high effort, namely, $d(1) - d(0) > 0$. The optimal policy is crafted so that the utility benefit of high effort just equals the utility cost.

The Optimal Policy and Efficiency

As we have seen, the policy that is best for the insurance company differs depending on whether it wishes to induce the consumer to choose high or low accident avoidance effort. The overall optimal policy—the one that solves the maximization problem (8.13)—is simply the one of these two that yields the larger expected profits.

Now, suppose that in the symmetric information case, the optimal effort level required of the consumer by the insurance company is *low*. Then precisely the same (full insurance) policy will be optimal in the asymmetric information case. This follows because this policy yields the same expected profits as in the symmetric information case, and the maximum expected profits when $e = 1$ is no higher in the asymmetric information case versus the symmetric information case because there is an additional constraint present under asymmetric information. Consequently, because the symmetric information outcome is Pareto efficient, so, too, will be the asymmetric information outcome in this case.

On the other hand, suppose that the optimal effort level required by the insurance company of the consumer is *high* in the symmetric information case. It may well be that the insurance company's maximized expected profits are substantially lower when it attempts to induce high effort in the asymmetric information case. Because expected profits conditional on low effort are identical in both the symmetric and asymmetric information cases, it may then be optimal for the insurance company in the asymmetric information setting to induce low effort by offering the full insurance policy. Although this would be optimal for the insurance company, it would not be Pareto efficient. For compared to the symmetric information solution, the consumer's utility is unchanged (and equal to $\bar{u}$), but the insurance company's profits are strictly lower.

Thus, once again, the effects of asymmetric information can reveal themselves in Pareto-inefficient outcomes.

8.3 INFORMATION AND MARKET PERFORMANCE

The distribution of information across market participants can have a profound and sometimes startling impact on market equilibrium. Indeed, as we've seen in this chapter, asymmetric equilibrium may cause markets to fail in that mutually beneficial trades go unexploited. This failure of market outcomes to be Pareto efficient is a most troubling aspect from a normative point of view.

We've devoted this chapter to a careful study of just one market—the market for insurance.[14] But the problems we've identified here are present in many other markets, too. Adverse selection arises in the market for used cars and in the market for labor.[15] Moral hazard arises in the employer–employee relationship, in the doctor–patient relationship, and even in marriages.[16]

[14] Much of our analysis was drawn from Rothschild and Stiglitz (1976).

[15] See Akerlof (1970) and Spence (1973).

[16] See Grossman and Hart (1983) and Holmstrom (1979, 1982).

For the most part in this chapter, we've concentrated on the disease and its symptoms, only occasionally hinting at a potential cure. We end this chapter by noting that very often these information problems can be mitigated if not surmounted. If adverse selection is the problem, signaling or screening can help. If moral hazard is the problem, contracts can be designed so that the agents' incentives lead them nearer to Pareto-efficient outcomes.

The analysis of markets with asymmetric information raises new questions and offers important challenges to economists. It is an area that offers few simple and broadly applicable answers, but it is an area where all the analyst's creativity, insight, and logical rigor can pay handsome dividends.

8.4 EXERCISES

8.1 Consider the insurance model of section 8.1, but treat each insurance company as if it were a risk-neutral consumer with wealth endowment $\bar{w} \geq L$ in every state, where L is the size of the loss should one of the m risk-averse consumers have an accident. Also assume that the number of risk-neutral consumers exceeds the number of risk-averse ones. Show that the competitive equilibrium derived in section 8.1 is a competitive equilibrium in this exchange economy.

8.2 Suppose that in the insurance model with asymmetric information, a consumer's accident probability is a function of his wealth. That is, $\pi = f(w)$. Also suppose that different consumers have different wealth levels, and that $f' > 0$. Does adverse selection necessarily occur here?

8.3 In our insurance model of section 8.1, many consumers may have the same accident probability. We allowed policy prices to be person specific. Show that, with symmetric information, equilibrium policy prices depend only on probabilities, not on the particular individuals purchasing them.

8.4 Answer the following questions related to the insurance model with adverse selection.

(a) When there are finitely many consumers, F, the distribution of consumer accident probabilities is a step function. Show that $g : [0, \bar{\pi} L] \rightarrow [0, \bar{\pi} L]$ then is also a step function and that it is nondecreasing.

(b) Show that g must therefore possess a fixed point.

(c) More generally, show that a nondecreasing function mapping the unit interval into itself must have a fixed point. (Note that the function need not be continuous! This is a special case of a fixed-point theorem due to Tarski (1955)).

8.5 Suppose there are two states, 1 and 2. State 1 occurs with probability π, and w_i denotes a consumer's wealth in state i.

(a) If the consumer is strictly risk-averse and $w_1 \neq w_2$, show that an insurance company can provide her with insurance rendering her wealth constant across the two states so that she is better off and so that the insurance company earns positive expected profits.

(b) Suppose there are many consumers and many insurance companies and that a feasible allocation is such that each consumer's wealth is constant across states. Suppose also that in this allocation, some consumers are insuring others. Show that the same wealth levels for consumers and expected profits for insurance companies can be achieved by a feasible allocation in which no consumer insures any other.

8.6 (Akerlof) Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller of type θ sells his car (of quality θ) for a price of p, his utility is $u_s(p, \theta)$. If he does not sell his car, then his utility is 0. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price p and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential k buyers.

(a) Argue that in a competitive equilibrium under asymmetric information, we must have $E(\theta | p) = p$.

(b) Show that if $u_s(p, \theta) = p - \theta/2$, then every $p \in (0, 1/2]$ is an equilibrium price.

(c) Find the equilibrium price when $u_s(p, \theta) = p - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?

(d) Find an equilibrium price when $u_s(p, \theta) = p - \theta^3$. How many equilibria are there in this case?

(e) Are any of the preceding outcomes Pareto efficient? Describe Pareto improvements whenever possible.

8.7 Show that in the insurance signaling game, if the consumers have finitely many policies from which to choose, then an assessment is consistent if and only if it satisfies Bayes' rule. Conclude that a sequential equilibrium is then simply an assessment that satisfies Bayes' rule and is sequentially rational.

8.8 Analyze the insurance signaling game when benefit B is restricted to being equal to L.

(a) Show that there is a unique sequential equilibrium when attention is restricted to those in which the insurance company earns zero profits.

(b) Show that among all sequential equilibria, there are no separating equilibria. Is this intuitive?

(c) Show that there are pooling equilibria in which the insurance company earns positive profits.

8.9 Consider the insurance signaling game.

(a) Show that there are separating equilibria in which the low-risk consumer's policy proposal is rejected in equilibrium if and only if $MRS_l(0, 0) \leq \bar{\pi}$.

(b) Given a separating equilibrium in which the low-risk consumer's policy proposal is rejected, construct a separating equilibrium in which it is accepted without changing any player's equilibrium payoff.

(c) Continue to consider this setting with one insurance company and two types of consumers. Also, assume low-risk consumers strictly prefer no insurance to full insurance at the high-risk competitive price. Show that when α (the probability that the consumer is low-risk) is low enough, the only competitive equilibrium under asymmetric information gives the low-risk consumer no insurance and the high-risk consumer full insurance.

(d) Returning to the general insurance signaling game, show that every separating equilibrium Pareto dominates the competitive equilibrium described in part (c).

8.10 Consider the insurance screening game. Suppose that the insurance companies had only finitely many policies from which to construct their lists of policies. Show that a joint strategy is a subgame perfect equilibrium if and only if there are beliefs that would render the resulting assessment a sequential equilibrium.

8.11 Consider the moral hazard insurance model where the consumer has the option of exerting either high or low accident avoidance effort (i.e., $e = 0$ or 1). Recall that $\pi_l(e) > 0$ denotes the probability that a loss of l dollars is incurred due to an accident. Show that if the monotone likelihood ratio property holds so that $\pi_l(0)/\pi_l(1)$ is strictly increasing in l, then $\sum_{l=0}^{L} \pi_l(0)x_l > \sum_{l=0}^{L} \pi_l(1)x_l$ for every increasing sequence of real numbers $x_1 < x_2 < \cdots < x_L$.

8.12 Consider the moral hazard insurance model.

(a) Show that when information is symmetric, the profit-maximizing policy price is higher when low effort is induced compared to high effort.

(b) Let the consumer's reservation utility, $\bar{u}$, be the highest she can achieve by exerting the utility-maximizing effort level when no insurance is available. Suppose that when information is asymmetric, it is impossible for the insurance company to earn nonnegative profits by inducing the consumer to exert high effort. Show then that were no insurance available at all, the consumer would exert low effort.

8.13 Consider once again the moral hazard insurance model. Let the consumer's von Neumann-Morgenstern utility of wealth be $u(w) = \sqrt{w}$, let her initial wealth be $w_0 = \$100$, and suppose that there are but two loss levels, $l = 0$ and $l = \$51$. As usual, there are two effort levels, $e = 0$ and $e = 1$. The consumer's disutility of effort is given by the function $d(e)$, where $d(0) = 0$ and $d(1) = 1/3$. Finally, suppose that the loss probabilities are given by the following entries, where the rows correspond to effort and the columns to loss levels.

e \ l	$l = 0$	$l = 51$
$e = 0$	1/3	2/3
$e = 1$	2/3	1/3

So, for example, the probability that a loss of \$51 occurs when the consumer exerts high effort is 1/3.

(a) Verify that the probabilities given in the table satisfy the monotone likelihood ratio property.

(b) Find the consumer's reservation utility assuming that there is only one insurance company and that the consumer's only other option is to self-insure.

(c) What effort level will the consumer exert if no insurance is available?

(d) Show that if information is symmetric, then it is optimal for the insurance company to offer a policy that induces high effort.

(e) Show that the policy in part (d) will not induce high effort if information is asymmetric.

(f) Find the optimal policy when information is asymmetric.

(g) Compare the insurance company's profits in the symmetric and asymmetric information cases. Also, compare the consumer's utility in the two cases. Argue that the symmetric information solution Pareto dominates that with asymmetric information.