9 DESIGNING SUPPLY CONTRACTS: CONTRACT TYPE AND INFORMATION ASYMMETRY

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9.1 INTRODUCTION

In designing supply contracts, a supplier has to consider the type of contract he can offer and the information he has about the buyer's cost structure. In this paper we provide a framework for fleshing out these two effects in the context of a simple single-supplier single-buyer supply chain facing price-sensitive deterministic demand. There are two well-known reasons for the resulting suboptimality (from both the supplier's and the joint perspectives):

- *Double marginalization:* because the buyer and the supplier only receive a portion of the total contribution margin, their decisions do not reflect the supply-chain wide incentive structure. As a result of receiving less than the full margin at any given quantity, they will produce less than a vertically integrated monopolist.
- *Asymmetric information:* the supplier rarely has complete information about the buyer's cost structure. However, the quantity the buyer will purchase (and therefore the supplier's profits) depend on that cost structure. Somehow, the supplier will have to take this information asymmetry into account.

In this paper we provide a simple but effective framework that allows us to study the following questions:

- If the supplier is faced with decreased buyer demand (due to a buyer cost increase), should the supplier sacrifice unit margin so as to maintain volume or should he do the opposite?
- What is the value to the supplier of obtaining additional information about the buyer's cost structure?
- What is the value to the supplier of being able to offer progressively more sophisticated supply contracts, eg. contracts with side payments or nonlinear contracts as opposed to merely specifying a constant unit wholesale price?
- Combining the previous two questions, when should a supplier focus on obtaining additional information and when should he focus on offering more sophisticated contracts?
- Under what circumstances can the double marginalization problem be overcome?

To study these questions, we examine the interaction between two key issues in designing supply contracts. The first deals with the type of contract that the supplier can offer while the second deals with the supplier's knowledge about the buyer's cost structure. We consider three types of contracts: one-part linear contracts, two-part linear contracts, and two-part nonlinear contracts. Under the one-part linear contract, the supplier charges a constant unit wholesale price. Under the two-part linear contract, the supplier charges a constant unit wholesale price but offers a fixed lump sum side payment to the buyer. Under a two-part nonlinear contract, the supplier offers a "menu" of contracts, where each item on that menu consists of a pair of unit wholesale price and lump sum side payment, leaving it to the buyer to select the pair of his choice. In all cases, the buyer chooses the order quantity based on the wholesale price and side payment specified in the contract selected. Clearly, the design of an optimal supply contract requires a good understanding of how the buyer will choose an order quantity under different types of contract. However, the buyer's order quantity will depend on the buyer's internal cost structure, which may not be known to the supplier. This leads us to consider the two situations in which the supplier has complete or incomplete information about the buyer's cost structure. Therefore, there are six possible scenarios (see Table 9.1 at the end of this chapter) to be examined.

In this paper, we determine the optimal supply contract, the buyer's optimal order quantity, and the corresponding profits for supplier and buyer under each of the six scenarios depicted in Table 9.1. By comparing the profits for different scenarios, we aim to examine the value of offering successively more complex contracts (from one-part linear to two-part linear to two-part nonlinear) and the value to the supplier of getting better information about the buyer's cost structure. These comparisons enable us to gain a better understanding about the impact of different types of contracts and information asymmetry on supplier's and buyer's profits.

We assume that demand is deterministic and decreases linearly in price. This case captures the situation in which the product market is relatively mature and demand for any given price level is known. In this case, the supplier specifies the supply contract while the buyer determines the order quantity and the price at which he resells the product. For a more general demand function, the reader is referred to the fundamental work of Ha (1997) for details. Ha (1997) analyzes two-part nonlinear contracts under complete and incomplete information about the buyer's cost structure, in the case of stochastic price-sensitive demand. His results are therefore more general; however, his results do not lend themselves to simple interpretations. By focusing on this simpler case, we are able to obtain more qualitative insights into the value of information and the value of contracting flexibility.

Let us briefly summarize our findings. First, when the buyer's cost increases, we find it is optimal for the supplier to sacrifice part of his margin to maintain volume when he can observe the buyer's costs. However, when the supplier cannot observe the buyer's costs, the opposite may be true: the supplier will sometimes end up *increasing* his margin and thus sacrifice volume. Second, under the one-part linear contract, we find that the value to the supplier of information about the buyer's cost structure increases with the price-sensitivity of demand and with the uncertainty about the buyer's cost structure (measured by the variance of the supplier's prior distribution). Under the two-part linear contract, the value of information to the supplier also increases with the difference between the expected and worst-case value for the buyer's retail cost. Third, we find that the value of offering two-part contracts instead of one-part decreases with price-sensitivity; ie., when demand is more price sensitive, the supplier loses less when forced to contract on wholesale price alone.

The organization of this paper is as follows. Below, we first discuss the relevant literature in supply-chain management and in economics. In Section 9.3, we present the model and some of the underlying assumptions. In Section 9.4, we analyze the optimal contracts for each of the six scenarios for the case in which the demand is decreasing linearly in the selling price. In Section 9.5, we compare the supplier's and the buyer's profits and profit margins for different scenarios and comment on the value of more sophisticated contracts and the value to the supplier of obtaining information about the buyer's cost structure. Section 9.6 contains numerical examples, and Section 9.7 concludes the paper and provides some suggestions for future research.

9.2 LITERATURE

Although the learnings from this paper are targeted at researchers in the operations management community, the paper draws on both the supply-chain literature in operations and on the economics literature. We briefly review some of the key papers leading to this work below. (Any omission is our oversight.)

Most of the supply-chain literature on contracting has focused either on deriving optimal ordering policies in the context of a given contract, or on deriving optimal contract parameters given the functional form of that contract. Some exceptions have started studying questions related to coordination within supply chains, the value of information and various alternative contracting schemes. Lee, So and Tang (1998) quantify the value of sharing demand information to retailers and manufacturers in a two-level supply chain. Their work examines the case in which the demand follows an AR(1) process and is not directly observed by the manufacturer. Other recent work that examines the benefits of information sharing when demand is i.i.d. include Bourland, Powell and Pyke (1996), Cachon and Fisher (1997), and Gavirneri, Kapuscinski and Tayur (1996). Lee and Whang (1996) derive an incentive scheme for a multi-echelon supply chain that can be implemented by a central planner where each echelon uses local information only and which leaves each party with at least the same expected profit as the classic Clark and Scarf (1960) decomposition scheme. Other recent work that examines various incentive schemes includes Cachon and Zipkin (1997) and Chen, Federgruen and Zheng (1997). In contrast, Corbett (1996,1998) shows how asymmetric information (about upstream setup costs or downstream backorder penalty) in the absence of a central planner generally does lead to suboptimal outcomes. Weng (1995) quantifies the value of channel coordination (eg. using quantity discounts) in a two-level supply chain where both retailer and manufacturer face setup costs, and finds that quantity discounts (ie. one-part nonlinear contracts) alone are not sufficient to achieve coordination. Corbett and de Groote (1997) compare various coordination schemes for a two-level deterministic-demand supply chain with setup costs at both levels, and show how the equivalence of these schemes under full information breaks down when one party holds private information; that paper also derives preference orderings of these schemes for the supplier, buyer, and vertically-integrated firm. The current paper adds to this literature by explicitly quantifying the value of information and the value of more complex contracts.

The economics literature has a rich history of studying vertical contracting relationships, a brief introduction to which can be found in eg. Tirole (1988) and the references therein. The basic problem is that of two successive monopolists, where the downstream one faces a price-sensitive demand curve (often taken to be linear). Demand is generally deterministic or, if it is stochastic, the uncertainty is resolved before the buyer places his order, so that safety stock is not an issue. Left to their own devices, both parties add a markup to their costs, leading to the classic "double marginalization" phenomenon of higher prices and lower output, and lower hence profits, than an integrated monopolist would offer. Most work in this area focuses on comparing total surplus under various schemes and on finding what type of contract a manufacturer should offer to mitigate the double marginalization issue. A comprehensive and critical discussion of this issue, including an overview of the three schools of thought on bilateral monopoly in economics up to then, is given by Machlup and Taber (1960). The work by Gal-Or (1991a) is probably the most closely related to the current paper. Her paper focuses on a single-manufacturer single-retailer situation, where the retailer has private information about demand and about retail costs. She finds that, in general, neither franchise fees (wholesale price plus fixed side payment) nor retail price maintenance (supplier forces a particular retail price as part of the contract) can achieve the vertically-integrated solution under asymmetric information. She does not go beyond showing suboptimality to actually quantifying that suboptimality as we do here. Gal-Or (1991b) studies the situation with two suppliers but complete information, and finds that equilibrium is sometimes achieved with linear pricing and sometimes with a franchise fee contract. Bresnahan and Reiss (1985) look at the ratio of the profit margins of manufacturer and retailer under simple wholesale pricing with full information, and show how this ratio (which can also be seen as a measure of relative power) depends on the convexity of the demand function.

The current paper attempts to combine these two strands of theory, by building on the basic bilateral monopoly framework offered in economics and asking the normative and more micro-level questions more typical of the supply-chain literature. For instance, rather than focusing on achieving first-best outcomes and profits or on showing suboptimality of certain contract types, we explicitly measure the cost of that suboptimality. The contribution of this paper lies not in the analysis of the individual cases (which is either well-known or trivial), but in the comparisons between the cases, the quantification of the differences between them, and the insights that result.

9.3 THE MODEL

Consider a supply chain that consists of a single supplier and a single buyer. The supplier is a manufacturer who provides an important product to the buyer, who in turn resells it to final consumers. Alternatively, the supplier can provide a critical component to the buyer who in turn integrates this component with other components to form the finished products. For mathematical convenience, we assume that each unit of finished product requires one unit of the component. Demand per period for the finished product is denoted by q, and the selling price is denoted by p. We assume demand decreases linearly in price; i.e., $q = a - bp$, where $a \geq 0$ and $b \geq 0$ are known parameters. In addition, since q is a linear function of price p, the buyer's order quantity q and profit Π_b can be uniquely determined by the selling price p. Thus, it is sufficient for the buyer to select either the quantity q or the selling price p that maximizes his profit.

The supplier's marginal costs (including production costs) are given by s, the buyer's internal marginal costs (i.e. excluding the part cost) by c. When there is complete information, the supplier knows the actual value of the buyer's internal marginal cost c. However, when there is information asymmetry, the supplier does not know the actual value of the buyer's internal marginal cost c but holds a prior distribution $F(c)$ (with continuous density function $f(c)$), where the distribution is defined on a finite interval $[\underline{c}, \overline{c}]$. To ensure nonnegative demand, we assume that $a - b(s + \overline{c}) \geq 0$, and to establish tractable results we also assume that $\frac{F(c)}{f(c)}$ is increasing in c. When this assumption does not hold, tractable results are essentially impossible to obtain. The reader is referred to the work of Ha (1997) that examines various general cases. (To justify that this assumption is not too restrictive, let us examine the conditions under which it does hold. Consider a random variable $k = \overline{c} - c$, where k has a probability distribution $G(k)$ and density function $g(k)$. In this case, it can be easily shown that $\frac{F(c)}{f(c)}$ is increasing in c if and only if $\frac{g(k)}{1-G(k)}$ is increasing in k. Notice that $\frac{g(k)}{1-G(k)}$ is increasing in k so long as G is an IFR (increasing failure rate) distribution, which includes the exponential, normal and truncated normal.)

We consider three types of contract: a basic one-part linear contract, in which wholesale price w does not depend on the order quantity q, a two-part linear contract (w, L) consisting of wholesale price w and a per-period lump-sum side payment L from the supplier to the buyer, where both w and L are independent of order quantity q. When $L > 0$, the side payment can be interpreted as a slotting fee, which is common in dealing with large retailers. When $L < 0$ this lump sum can be seen as a franchise fee, more common when the supplier's product has a strong brand. The most complex type of contract we consider is that of two-part nonlinear contracts $\{(w(q), L(q)\}$, where actual wholesale price and side payment both depend on the order quantity selected by the buyer. Clearly, the two-part nonlinear contracts generalize the one-part linear contract and the two-part linear contract. As such, it is sufficient to formulate the supplier's problem for the two-part nonlinear contract first, and analyze different types of contracts subsequently as special cases. To develop an optimal supply contract that maximizes his expected profit for the two-part nonlinear contract, the supplier solves the problem below; if the supplier is restricted to offering simpler contracts, the problem becomes correspondingly more constrained.

$$
\begin{array}{llcll}
\mathcal{S} & \max\limits_{\{w(q),L(q)\}} \Pi_s(w,L) & := & E_c[(w(q(c)) - s)q(c) - L(q(c))] & (9.1) \\
& \text{s.t.} \quad \Pi_b(c,q) & := & (p(q) - w(q) - c)q + L(q) \ \geq \Pi_b^- & \forall\ c\ (9.2) \\
& D_q\Pi_b(c,q) & = & 0 & \forall\ c\ (9.3)
\end{array}
$$

The supplier's expected net profits Π_s in (9.1) depend on the quantity $q(c)$ ordered by the buyer. This in turn depends on the buyer's internal cost structure c, which may be unknown to the supplier, hence the expectation $E_c[\cdot]$ using the prior distribution $F(c)$. Depending on the type of contract, the supplier may also offer a side payment $L(q)$. With $p(q)$ as the selling price, inequality (9.2) represents the buyer's individual rationality constraint: the buyer's net profits Π_b (which include the side payment $L(q)$), the term to the left of the inequality in (9.2), must exceed his reservation profit level Π_b^-. (For a retailer, Π_b^- could be the profit that the buyer could obtain from allocating the shelf space to another supplier's product.) Finally, condition (9.3) is the buyer's incentive-compatibility constraint: presented with a menu $\{w(q), L(q)\}$, a buyer with cost c will choose $q(c)$ so as to maximize his net profit, denoted by $\Pi_b(c,q)$. Throughout this paper, we reserve the notation $D_x\, H$ to denote the derivative of any given function H with respect to x.

We shall consider the following six cases:

1. Case F1: the supplier offers a one-part linear contract on wholesale price w only, but has full information about c.

2. Case F2: the supplier offers a two-part contract (w, L), and has full information about c.

3. Case F3: the supplier offers a two-part nonlinear menu of contracts $\{w(q), L(q)\}$, and has full information about c.

4. Case A1: the supplier offers a one-part linear contract on w only, and does not know c.

5. Case A2: the supplier offers a two-part contract (w, L), but does not know c.

6. Case A3: the supplier offers a two-part menu of contracts $\{w(q), L(q)\}$, but does not know c.

Table 9.1 depicts the cases to be analyzed in Section 9.4. The sequence of events is always:

1. The supplier offers one type of contract (one-part linear contract, two-part linear contract, or two-part nonlinear contract).

2. The buyer (who has internal cost c) chooses a specific contract by selecting the order quantity q (when a one-part linear or two-part linear contract is offered) or by selecting $(w(q(c)), L(q(c)))$, and the corresponding order quantity q (when a two-part nonlinear contract is offered).

3. All sales and financial transactions take place simultaneously.

When the buyer's marginal cost is unknown to the supplier, we consider the case in which the supplier treats the buyer's marginal cost as a random variable C and imposes a prior probability distribution $F(c)$ over the buyer's marginal cost C. During the analysis of case A3 (asymmetric information, two-part nonlinear contract), we transform our model into an equivalent but mathematically more convenient form. This transformation utilizes the revelation principle from economics (see Fudenberg and Tirole 1991 for more detail or Corbett 1998 for a simple proof in a similar context). The revelation principle can be explained as follows. Intuitively, for any given order quantity q selected by the buyer, the supplier can deduce the buyer's corresponding cost c. Hence, the buyer's selecting q is essentially equivalent to his announcing a cost parameter c. This implies that the supplier can reformulate the contracts in terms of c, i.e. optimizing over $\{w(c), L(c)\}$ rather than $\{w(q), L(q)\}$. Although the buyer could announce a cost parameter $\hat{c}$ where $\hat{c} \neq c$, the revelation principle assures us that there is an optimal contract under which the buyer will reveal truthfully, i.e. $\hat{c} = c$. Throughout this paper, our analyses will be based on $\{w(c), L(c)\}$ and $\{w(q), L(q)\}$ interchangeably. Note, however, that the contribution of this paper lie in the framework and the insights derived from comparing the six scenarios, not in the application of the revelation principle which has been done previously.

For any function $H(\cdot)$, we reserve $\dot{H}$ to denote the derivative of the function H with respect to the variable $\hat{c}$ and evaluated at $c = \hat{c}$, i.e. $\dot{H} = D_{\hat{c}}H$. The notation is summarized in Table 9.2.

9.4 THE SUPPLIER'S OPTIMAL SUPPLY CONTRACTS

The basic steps for determining the optimal supply contract for each of the six cases are as follows. We first solve the buyer's optimization problem and determine the buyer's optimal order quantity when either the one-part contract or the two-part linear or nonlinear contract are being offered by the supplier. Then we solve the supplier's optimzation problem that accounts for the buyer's optimal order quantity and determine the optimal supply contract. After we complete the analysis for those six possible scenarios, we examine the impact of the contract type and of information asymmetry on the supplier's and the buyer's profits. Though the first parts below (the buyer's optimization under a given contract, and the supplier's problem under full information) are already well-established, we briefly review them below for the sake of completeness.

9.4.1 The buyer's optimization problem

9.4.1.1 The buyer's optimization problem under a one-part or two-part linear contract. Under the one-part linear contract, the supplier selects the wholesale price w. For any given w, the buyer solves:

$$\mathcal{B}_{1,2} \qquad \max_{q} \qquad \Pi_b \;:=\; (p(q) - w - c)q$$

Since $q = a - bp$, it is well-known that the buyer will select

$$p^* = \frac{a + b(w + c)}{2b} \tag{9.4}$$

$$q^* = \frac{1}{2}[a - b(w + c)] \tag{9.5}$$

Under the two-part linear contract, the supplier also selects a lump-sum side payment L, but as L is independent of q it will clearly not affect the buyer's order quantity selection. Hence, when the supplier offers a two-part linear contract, it is optimal for the buyer to order q^* as given in (9.5).

9.4.1.2 The buyer's optimization problem under a two-part nonlinear contract. Under the two-part nonlinear contract, the supplier offers a menu of contracts $\{w(q), L(q)\}$, and the buyer's decision is to choose the order quantity q. Or, equivalently (as explained above), the supplier offers a menu of contracts $\{w(\hat{c}), L(\hat{c})\}$, and the buyer announces a cost parameter $\hat{c}$. For a given menu of contracts $\{w(\hat{c}), L(\hat{c})\}$, the buyer would choose to announce whatever level of cost $\hat{c}$ will maximize his profit. Associated with the cost $\hat{c}$ announced by the buyer to the supplier, the buyer would pay wholesale price $w(\hat{c})$, and would order $q^*(w(\hat{c}))$ and set the selling price $p^*(w(\hat{c}))$, where p^* and q^* are as given in (9.4) and (9.5), respectively. Thus, the buyer solves:

$$\begin{aligned} \mathcal{B}_3 \qquad \max_{\hat{c}} \qquad \Pi_b(c, \hat{c}) \;&:=\; L(\hat{c}) + [p^*(w(\hat{c})) - w(\hat{c}) - c)q^*(w(\hat{c})] = \\ &=\; L(\hat{c}) + b[\tfrac{a}{2b} - \tfrac{1}{2}(w(\hat{c}) + c)]^2 \end{aligned} \tag{9.6}$$

We know, from the revelation principle, that there is an optimal menu of contracts under which it is optimal for the buyer to reveal his true costs so that $\hat{c}^* = c$. This implies that at the optimum, the first-order condition is solved at $\hat{c} = c$. The second-order condition is verified below. By considering the first-order condition, the buyer's incentive-compatibility constraint is given as:

$$\dot{L}(c) = \frac{1}{2}[a - b(w + c)]\dot{w}(c) \tag{9.7}$$

In addition, the supplier will need to set the side payment L so as to meet the buyer's individual-rationality constraint (9.2); i.e., $\Pi_b(c) \geq \Pi_b^-$ for all $c \in [\underline{c}, \overline{c}]$.

To verify sufficiency of the first-order condition for the buyer's optimization problem, first note that for given w, the buyer's price selection or order quantity problem is concave in p or q respectively. Now, to verify sufficiency, we examine the second-order condition. Specifically, for any given menu $\{L(\cdot), w(\cdot)\}$, the second-order condition is given as:

$$D_{\hat{c}}^2\Pi_b = -b[\frac{a}{2b} - \frac{1}{2}(w+c)]\ddot{w} + \frac{1}{2}b\dot{w} + \ddot{L} \quad (9.8)$$

$$= \frac{1}{2}a\ddot{w} + \frac{1}{2}b(w+c)\ddot{w} + \frac{1}{2}b\dot{w} + \ddot{L} \quad (9.9)$$

Substituting

$$D_{\hat{c}}^2 L = \frac{1}{2}a\dot{w} - \frac{1}{2}b(w+\hat{c}))\dot{w} \quad (9.10)$$

gives

$$D_{\hat{c}}^2\Pi_b = -\frac{1}{2}b\dot{w} \leq 0 \quad (9.11)$$

In the analysis below, we will see that the optimal wholesale price in case A3 w_{A3}^* is increasing in c, so that Π_b is concave in c, which verifies sufficiency.

9.4.2 *Optimal supply contracts under complete information*

By using the information about the buyer's optimal order quantity (given in (9.5)) when a one-part or two-part linear contract is offered and the information about the buyer's cost announced to the supplier, we can derive the optimal supply contract for each of the six cases below. The characterization of the optimal supply contract, the buyer's optimal order quantity, and the optimal supplier's and buyer's profit margins for each of these six cases are summarized in Table 9.3.

9.4.2.1 Case F1: one-part linear contract with complete information. When the supplier has complete information about the buyer's marginal cost c, the supplier knows the buyer's optimal order quantity q^* as given in (9.5) for any wholesale price w. Thus, the supplier needs to determine the wholesale price w so that his profits $\Pi_{s,F1}$ are maximized. In this case, the supplier solves:

$$\mathcal{S}_{F1} \qquad \max_{w} \quad \Pi_{s,F1}(w) := (w-s)q^* = (w-s)\frac{1}{2}[a - b(w+c)] \quad (9.12)$$

It is easy to check from the first and second order conditions that the problem is concave and that the optimal wholesale price w^* can be written as:

$$w_{F1}^* = \frac{a}{2b} + \frac{1}{2}(s-c) \quad (9.13)$$

Substituting the optimal wholesale price into the objective function of problem $\mathcal{S}_{F1}$, one can show that the supplier's profits satisfy

$$\Pi_{s,F1}(w_{F1}^*) = \frac{b}{2}(\frac{a}{2b} - \frac{(s+c)}{2})^2 \quad (9.14)$$

and that the supplier's profit margin is equal to $w^* - s = \frac{1}{2}[\frac{a}{b} - (s+c)]$.

Similarly, we can substitute w^* into the expressions for p^* and q^* given in (9.4) and (9.5) to determine the buyer's profit. In this case, it is easy to show that the buyer's profit for case F1, denoted by $\Pi^*_{b,F1}$, is $\Pi^*_{b,F1} = (p^* - w^* - c)q* = \frac{b}{4}(\frac{a}{2b} - \frac{(s+c)}{2})^2$. Also, notice that the buyer's profit margin is equal to $p - w^* - c = \frac{1}{4}[\frac{a}{b} - (s+c)]$.

By comparing the supplier's and the buyer's profit margins and profits, it is clear that the supplier's profit and profit margin are double those of the buyer. This well-known result corresponds to the special linear demand case in Bresnahan and Reiss (1985); in general, this ratio is equal to $\frac{1}{2+\eta}$, where $\eta = q\frac{D^2_{qP}}{D_{qP}}$, a local measure of the curvature of the demand curve. Recall that, throughout this paper, the supplier is the party with the initiative to propose contract terms, so it is not surprising to find the supplier capturing a larger proportion of total profits.

9.4.2.2 Case F2: two-part linear contract with complete information.

Two-part contracts are often referred to as franchise fee (FF) contracts in the literature. Using the same argument as presented for case F1, it is easy to see that the supplier can determine the contract of the form $\{w, L\}$ that optimizes his profits by solving the following problem:

$$\begin{array}{llcl} \mathcal{S}_{F2} & \max_{\{w,L\}} \Pi_{s,F2}(w,L) & := & (w-s)q^* - L = (w-s)\frac{1}{2}(a - b(w+c)) - L \\ & \text{s.t.} \quad \Pi_{b,F2}(c,c) & \geq & \Pi_b^- \end{array} \tag{9.15}$$

The problem can be solved by using two observations. First, because the supplier has complete information, he can set inequality (9.15) to be binding. Second, the supplier's profits $\Pi_{s,F2} = \Pi_{j,F2} - \Pi_{b,F2} = \Pi_{j,F2} - \Pi_b^-$ where Π_j denotes joint profits. Hence, problem $\mathcal{S}_{F2}$ is equivalent to maximizing joint profits $\Pi_{j,F2}$. It is easy to verify that

$$L^*_{F2} = \Pi_b^- - \frac{1}{4b}(a - b(w+c))^2 \tag{9.16}$$

and

$$w^*_{F2} = s \tag{9.17}$$

Hence, when the supplier has complete information about the buyer's marginal cost information, it is optimal for the supplier to set the wholesale price equal to the supplier's marginal cost and use the lump sum side payment to extract all profits from the buyer in excess of his reservation profit level Π_b^-. Note that this means that L^*_{F2} will be negative whenever the buyer's reservation profit level is not too high. This corresponds to a franchise fee paid by the buyer to the supplier. In case F2, the supplier's profits satisfy

$$\Pi_{s,F2}(w^*_{F2}, L^*_{F2}) = -L^*_{F2} = \Pi_b^- - \frac{1}{4b}(a - b(w+c))^2 \tag{9.18}$$

and the buyer's profit $\Pi^*_{b,F2} = \Pi_b^-$. In this case, the sum of the supplier's and the buyer's profits is equal to $\Pi_{j,F2} = \frac{1}{4b}(a - b(s+c))^2$.

9.4.2.3 Case F3: two-part nonlinear contract with complete information. Although the supplier now has the added flexibility of being able to offer nonlinear contracts, it is clear that in case F2 he is already extracting all profits beyond the minimum level Π_b^- from the buyer, so this additional flexibility has no value to the supplier in the complete information case. All results for case F2 carry over directly to case F3. Below, though, we will find that in the case of asymmetric information this equivalence no longer holds.

Later, in Section 9.5, we shall compare these three contracts, to examine the value of two-part linear and nonlinear contracts vs. one-part contracts under full information. First, though, let us analyze the asymmetric information cases.

9.4.3 Optimal supply contracts under asymmetric information

Most models that explicitly include asymmetric information assume the supplier can offer two-part nonlinear contracts (or even more sophisticated contracts, including resell price maintenance). We do not make this assumption, and in doing so we can precisely quantify the value of information and the value of contracting sophistication to the supplier.

9.4.3.1 Case A1: one-part linear contract with asymmetric information. In this case, the supplier holds a prior probability distribution $F(c)$ over the buyer's marginal cost c, and the supplier needs to specify a wholesale price so as to maximize his expected profit. We can utilize the buyer's optimal quantity q^* in (9.5) to formulate the supplier's optimization problem as follows:

$$\begin{aligned} \mathcal{S}_{A1} \quad \max_w \; E_c[\Pi_{s,A1}(w)] &:= E_c[(w-s)q^*] = \\ &= \int_{\underline{c}}^{\overline{c}} \frac{1}{2}(w-s)(a-b(w+c))dF(c) \end{aligned} \tag{9.19}$$

The first-order condition $D_w E_c[\Pi_{s,A1}(w)] = 0$ yields:

$$D_w E_c[\Pi_{s,A1}(w)] = \frac{1}{2}a - \frac{1}{2}b(2w-s) - \frac{1}{2}bE[c] = 0$$

which is solved at

$$w^*_{A1} = \frac{a}{2b} + \frac{1}{2}(s - E[c]) \geq 0 \tag{9.20}$$

The inequality follows from our assumption that $a - b(s+\overline{c}) \geq 0$. Substituting the optimal wholesale price in to the objective function of problem $\mathcal{S}_{A1}$, one can show that the supplier's expected profits satisfy

$$E_c[\Pi_{s,A1}(w^*_{A1})] = \frac{1}{8b}(a - b(s+E[c]))^2 \tag{9.21}$$

and that the supplier's profit margin is equal to $w^*_{A1} - s = \frac{a}{2b} - \frac{(s+E[c])}{2} \geq 0$.

Similarly, we can substitute w^*_{A1} into the expressions for p^* and q^* given in (9.4) and (9.5) to determine the buyer's profit. In this case, it is easy to show that the buyer's profit $\Pi^*_{b,A1} = (p^* - w^* - c)q* = \frac{b}{4}(\frac{a}{2b} - \frac{(s+c)}{2} + \frac{E[c]-c}{2})^2$, and that the buyer's profit margin is equal to $p - w^* - c = \frac{a}{4b} - \frac{(s+c)}{4} + \frac{E[c]-c}{4}$. Observe that the supplier's and the buyer's profits and profit margins depend on the accuracy of the estimated buyer's marginal cost $E[c]$. Therefore the supplier has the incentive to induce the buyer to reveal his true cost c so as to gain a higher profit; however, within the limited flexibility allowed by the one-part linear contract, the supplier cannot achieve this. We shall show how the supplier can induce the buyer to reveal his true cost in case A3, i.e., the two-part nonlinear contract case.

9.4.3.2 Case A2: two-part linear contract with asymmetric information. By following the same approach as in case A1, we can formulate the supplier's optimization problem for case A2 as:

$$\mathcal{S}_{A2} \qquad \max_{w,L} E_c[\Pi_{s,A2}(w,L)] := E_c[(w-s)q^* - L] =$$

$$= \int_{\underline{c}}^{\overline{c}} [\frac{1}{2}(w-s)(a-b(w+c)) - L]dF(c) \quad (9.22)$$

For any given w, the supplier will always choose the lowest L that still satisfies the buyer's individual rationality constraint $\Pi_b(c) \geq \Pi_b^-$ for all c. Since $\Pi_b(c)$ is decreasing in c (because, by assumption, $a-b(w+c) = q \geq 0$), this constraint holds for all c if it holds at $c = \overline{c}$. thus, the buyer's profits can be written as

$$\Pi_{b,A2}(c,q) \quad = \quad \frac{1}{4b}(a-b(w+c))^2 + L \quad (9.23)$$

Setting $\Pi_b(\overline{c}, q) = \Pi_b^-$, we can determine the optimal side payment L^*_{A2}, where

$$L^*_{A2} \quad = \quad \Pi_b^- - \frac{1}{4b}(a-b(w+\overline{c}))^2 \quad (9.24)$$

Substituting this expression for L into $\Pi_{s,A2}$ as given in (9.22) and noting that the resulting expression is concave in w, we can solve the first-order condition for w to find that:

$$w^*_{A2} \quad = \quad s + \overline{c} - E[c] \quad (9.25)$$

Substitute (9.25) into (9.24) and (9.22), to get:

$$L^*_{A2} \quad = \quad \Pi_b^- - \frac{1}{4b}(a-b(s+2\overline{c}-E[c]))^2 \quad (9.26)$$

$$E_c[\Pi_{s,A2}(w^*_{A2}, L^*_{A2})] \quad = \quad -\Pi_b^- + \frac{1}{2}(\overline{c}-E[c])(a-b(s+\overline{c}))+$$

$$+ \quad \frac{1}{4b}(a-b(s+2\overline{c}-E[c]))^2 \quad (9.27)$$

Clearly, under complete information, $\bar{c} = E[c] = \underline{c}$ and case A2 reduces to case F2. The information asymmetry means the supplier must now offer a larger side payment than before, i.e. $L^*_{A2} \geq L^*_{F2}$, to meet the "worst-case" buyer's minimum profit requirements. To compensate, the supplier adds a markup $\bar{c} - E[c]$, based on how far removed the "worst-case" buyer is from the mean, to his marginal cost s. We will discuss the qualitative differences in more depth below, after analyzing the final case A3.

9.4.3.3 Case A3: two-part nonlinear contract with asymmetric information. In this case the supplier has the flexibility to offer a two-part nonlinear menu of contracts $\{w(\hat{c}), L(\hat{c})\}$. By selecting any specific pair $(w(\hat{c}), L(\hat{c}))$ the buyer is essentially revealing a marginal cost $\hat{c}$ which, by the revelation principle explained earlier, will be his true marginal cost c. In this case, we can utilize the buyer's optimal quantity q^* in (9.5) and requirement (9.7) that the optimal lump sum payment satisfies the first order condition for problem $\mathcal{B}_3$ to formulate the supplier's optimization problem as follows:

$$\begin{aligned} \mathcal{S}_{A3} \quad \max_{\{w(\cdot),L(\cdot)\}} E_c[\Pi_{s,A3}] &:= E_c[(w-s)q^* - L] = \\ &= E[(w-s)\frac{1}{2}(a - b(w+c)) - L] && (9.28) \\ \text{s.t.} \qquad \dot{L} &= \frac{1}{2}[a - b(w+c)]\dot{w} && \forall\ c \quad (9.29) \\ \Pi_b(c,c) &\geq \Pi_b^- && \forall\ c \quad (9.30) \end{aligned}$$

The solution procedure is given in the Appendix; the optimal wholesale price for case A3, w^*_{A3}, has the following form that is based on Laffont and Tirole (1993) and other related work:

$$w^*_{A3}(c) = s + \frac{F(c)}{f(c)} \qquad \forall\ c \quad (9.31)$$

Recall that the function $\frac{F}{f}$ is assumed to be increasing in c, so one immediately sees from (9.31) that the optimal wholesale price is increasing in c. This is in contrast to the earlier cases, where wholesale price was decreasing in c (case F1), constant in c (case F2), or decreasing in $E[c]$ (cases A1 and A2). We further discuss this contrast in Section 9.5.1. As we had also assumed that $q^* = \frac{1}{2}a - \frac{1}{2}b(w + \hat{c})) \geq 0$, we can verify from (9.7) that the lump sum payment is also increasing in c. In this case, the buyer faces a tradeoff: accepting a higher lump sum payment and a higher unit wholesale price versus accepting a lower lump sum payment and a lower unit wholesale price.

9.4.3.4 Special case A3: uniform prior distribution. To generate some managerial insights for case A3, let us consider the case when the prior distribution is uniformly distributed on $[\underline{c}, \bar{c}]$. In this case, it follows from (9.31) and (9.7) that:

$$w^*_{A3}(c) = s + c - \underline{c} \qquad (9.32)$$

$$L^*_{A3}(c) \quad = \quad \frac{1}{2}c[a - b(s + c - \underline{c})] + k \tag{9.33}$$

where k depends on Π_b^- and other parameters but not on c.

In this case, one can write w and L explicitly as function of q by inverting c out of $q^* = \frac{1}{2}[a - b(s + 2c - \underline{c})]$ from (9.5). This gives

$$w^*_{A3}(q) \quad = \quad \frac{1}{2}(s + \frac{a - 2q}{b} - \underline{c}) = \frac{1}{2}(s - \underline{c} + \frac{a}{b}) - \frac{1}{b}q \tag{9.34}$$

$$L^*_{A3}(q) \quad = \quad \Pi_b^- + \frac{1}{8b}[(a - b(s - \underline{c}))^2 - 4q^2] + k \tag{9.35}$$

One can easily verify that when $c = \underline{c}$, $q^* = \frac{1}{2}[a - b(s + \underline{c})]$ so that (9.34) reduces to $w^*_{A3}(q^*) = s$ (as indeed it must from (9.31)). The structure of w^*_{A3} in (9.34) is interesting: the unit wholesale price can be interpreted as the average of a constant part and a part decreasing in q, illustrating how unit wholesale price decreases with quantity.

9.5 COMPARISON OF THE SIX CASES

Using the analysis above and the results summarized in Table 9.3, we now compare the six cases (three contract types, full or asymmetric information) along various dimensions:

- Wholesale prices w_j and (if applicable) lump-sum side payments L_j for case j.
- Supplier's and buyer's profit margin for case j, denoted by $m_{s,j}$ and $m_{b,j}$ respectively, where $m_{s,j} := w_j - s$ and $m_{b,j} := p^*_j - w_j - c$.
- "Effective" wholesale prices $w^e_j := w_j - L_j/q^*_j$ for case j, the average unit wholesale price taking the lump-sum rebate L_j into account. Although ex post, w^e_j is the averge wholesale price paid by the buyer, we prefer not to call it that because ex ante the buyer's ordering behavior is based on w_j, not on w^e_j.
- Supplier's and buyer's "effective" unit profit margin for case j, denoted by $m^e_{s,j} := w^e - s$ and $m^e_{b,j} := p - w^e - c$ respectively.
- Profits of supplier, buyer, and joint profits, for case j.

9.5.1 *The impact of the buyer's cost on the supplier's profit margin*

One of the questions we are now able to answer is the following. As the buyer's cost c increases, the buyer's unit profit margin m_b decreases, leading the buyer to order less, which in turn decreases the supplier's profit. How should the supplier respond to this? The answer depends on the information structure and contract type allowed. In case F1, the supplier's margin $m_{s,F1} = \frac{1}{2}\frac{a}{b} - \frac{1}{2}(c + s)$ decreases with c. This means that a supplier faced with a buyer cost increase should accept a smaller profit margin in order to maintain volume. Similarly, in case A1, the supplier's margin $m_{s,F1} = \frac{1}{2}\frac{a}{b} - \frac{1}{2}(E[c]+s)$ decreases with $E[c]$, leading to the analogous insight that a supplier should respond to an increase in buyer's expected cost by decreasing his own margin. In cases F2 and F3, the supplier always sets wholesale price equal to marginal cost s, leading to $m_{s,F2} = m_{s,F3} = 0$, regardless of buyer cost c. In case A2, however, the supplier's margin $m_{s,A2} = \bar{c} - E[c]$; this is because the supplier needs to offer a side payment L that will satisfy even a buyer with the highest possible cost $\bar{c}$, so the supplier wishes to recoup part of that relatively high side payment by charging a higher unit profit margin than in the complete information case. The unit margin increases with $\bar{c}$, and decreases in $E[c]$ as in case A1. In all these cases, the supplier sacrifices margin for volume.

Interestingly, though, in case A3, the supplier should do the reverse: his margin is $m_{s,A3} = \frac{F(c)}{f(c)}$, which is increasing in c. This seems to suggest that the supplier can use the additional flexibility offered by nonlinear contracts to maintain a high unit profit margin in a way that he cannot do when restricted to offering linear contracts (with or without side payment).

To examine this phenomenon more precisely, one should take the side payment into account and evaluate the "effective" unit wholesale price $w^e = w - \frac{L}{q}$ and profit margins. Assume the buyer incurs a cost increase but the supplier cannot observe it, so his prior $F(c)$ remains unchanged. What will this cost increase do to the supplier? Although we do not yet have analytical results for this case, the numerical example in Section 9.6 and Figure 9.2 provides an instance where under full information, the supplier will lower his effective margin, but will increase it under asymmetric information (except in case A1 where average margins depends only on $E[c]$).

9.5.2 *The value of information to the supplier*

How much can the supplier gain from obtaining better information about the buyer's cost structure, without changing the type of contract he can offer? We answer this question for one-part and two-part linear contracts. First, introduce the difference operator $\Delta_{ij}\Pi_s := \Pi_{s,i} - \Pi_{s,j}$ where i and j denote the cases being compared.

Starting with the one-part linear contracts, one can easily verify from the expressions for supplier's (expected) profits in case F1 (9.14) and A1 (9.21) respectively, that the expected difference is equal to:

$$\Delta_{F1,A1}\Pi_s = E_c[\Pi_{s,F1}(w^*_{F1}) - \Pi_{s,A1}(w^*_{A1})] = \frac{b}{8}\mathrm{Var}(c) \tag{9.36}$$

This implies that the supplier would increase his expected profit by $\frac{b}{8}\mathrm{Var}(c)$ if he had complete information about the buyer's marginal cost. In addition, the more price-sensitive the demand (i.e. the greater b), the more valuable is the information to the supplier; this confirms what one would expect intuitively.

Now turning to the two-part linear contract cases, similar analysis based on the equations for case F2 (9.18) and A2 (9.27) respectively, one finds

$$\begin{aligned}\Delta_{F2,A2}\Pi_s &= E_c[\Pi_{s,F2}(w^*_{F2}, L^*_{F2}) - \Pi_{s,A2}(w^*_{A2}, L^*_{F2})] = \\ &= \frac{b}{4}\mathrm{Var}(c) + \frac{1}{2}(\bar{c} - E[c])(a - b(s + \bar{c}))\end{aligned} \tag{9.37}$$

So now the value of information still depends on price-sensitivity b and on $\mathrm{Var}(c)$, but also on the worst-case deviation from the supplier's expected value $E[c]$. This is because in cases F2 and A2 the supplier offers a side payment which the supplier can use to make the buyer's individual rationality constraint binding at $c = \bar{c}$; he can not do so in cases F1 and A1. We also see that $\Delta_{F2,A2}\Pi_s \geq 2\Delta_{F1,A1}\Pi_s$ because $\bar{c} \geq E[c]$ and $a - b(s + \bar{c}) \geq 0$ by assumption (to ensure nonnegative quantities). This leads to the important and intuitive finding that the value of information to the supplier is (significantly) greater when the supplier has the flexibility to offer two-part contracts.

9.5.3 The value to the supplier of offering side payments

Here we ask ourselves, how much can the supplier gain from offering more sophisticated contracts, without changing the information structure? In the full information case, we use the expressions for case F2 (9.18) and F1 (9.14) respectively, to find

$$\begin{aligned}\Delta_{F2,F1}\Pi_s &= \Pi_{s,F2}(w^*_{F2}, L^*_{F2}) - \Pi_{s,F1}(w^*_{F1}) = \\ &= -\Pi_b^- + \frac{1}{8}\frac{(a - b(s + c))^2}{b}\end{aligned} \tag{9.38}$$

This expression is exactly as one would expect: the second term is the difference between the supplier's profits in case F1 and an integrated firm's profits, the first term reflects the fact that the supplier cannot extract an arbitrary level of profit from the buyer. We see that the value of offering two-part contracts instead of one-part contracts decreases with the buyer's reservation profit level Π_b^- and with price-sensitivity b. The latter is perhaps surprising: as demand becomes more price-sensitive, the absolute penalty from using only wholesale price without side payments decreases. Measuring the penalty in relative terms by looking at $\frac{\Pi_{s,F2}(w^*_{F2}, L^*_{F2})}{\Pi_{s,F1}(w^*_{F1})}$ gives the same result: the relative penalty decreases in b (for $\Pi_b^- \geq 0$).

Moving to the asymmetric information case, we use the expressions for case A2 (9.27) and A1 (9.21) respectively, to find

$$\begin{aligned}\Delta_{A2,A1}\Pi_s &= E_c[\Pi_{s,A2}(w^*_{A2}, L^*_{A2}) - \Pi_{s,A1}(w^*_{A1})] = \\ &= -\Pi_b^- + \frac{1}{8}\frac{(a - b(s + 2\bar{c} - E[c]))^2}{b}\end{aligned} \tag{9.39}$$

The comparative statics are as above: the value of contracting flexibility decreases with the buyer's reservation profit level Π_b^- and with price-sensitivity b. Moreover, because $\Delta_{A2,A1}\Pi_s \leq \Delta_{F2,F1}\Pi_s$, the value of contracting flexibility is greater under full information.

9.5.4 *The value of information versus the value of contracting flexibility*

Many suppliers in practice find themselves in case A1, with the simplest possible type of contract and incomplete information about the buyer's cost structure. Should such a supplier focus on offering more sophisticated contracts or on obtaining better information about the buyer's costs? Two observations are of interest here:

- The value of information increases with b, while the value of contracting flexibility decreases with b. This suggests that in more price-sensitive environments, the supplier should focus more on obtaining information about the buyer's costs.

- Whichever step the supplier takes first (A1 to F1 or A1 to A2), the value of that step would have been greater had he taken the other step first. In other words, a supplier investing in efforts to reduce his uncertainty about the buyer's costs should realize that, without increases contracting flexibility, he will not realize the full value of those efforts.

9.6 NUMERICAL EXAMPLES

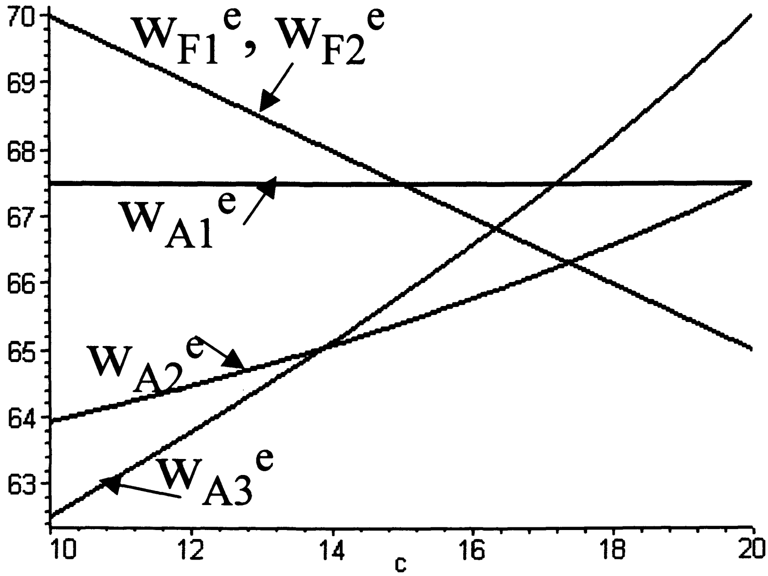
To illustrate the behaviour of the various types of contract we provide a simple numerical example. Assume the supplier's prior distribution over c is uniformly distributed on $[\underline{c}, \overline{c}] = [10, 20]$, that the demand function is $q = a - bp = 200 - 2p$, that the manufacturing cost is $s = 50$, and that the buyer's reservation profits $\Pi_b^- = 0$. The figures below show how "effective" unit wholesale prices, "effective" margins for supplier and buyer, and both parties' profits, depend on c. Clearly, when the supplier cannot observe c, unit wholesale price w cannot depend directly on c (other than through a revelation mechanism), but w^e and the other variables displayed will depend on c through their dependence on q which in turn depends on c.

Figure 9.1 shows how the effective unit wholesale price, which we defined as $w^e = w - L/q$, depends on c. The figure is in accordance with our earlier suggestion that that in all full-information cases, the effective wholesale price decreases with c, which means the supplier is sacrificing margin to maintain volume; in the asymmetric information cases A2 and A3, the opposite occurs. (In case A1, w_{A1}^* cannot depend on c and there is no side payment which could introduce dependence on c.) Figure 9.2 shows the effective unit margins obtained by the supplier, and is similar to Figure 9.1. In Figure 9.3 we see that the buyer's unit margin always decreases in c, which is as one would expect. Figures 9.4 and 9.5 show which cases the supplier and buyer respectively will prefer.

9.7 CONCLUSIONS

In this paper we have used the simple bilateral monopoly framework with price-sensitive demand to study the interactions between information structure and contracting sophistication. We observed that, under full information, a supplier

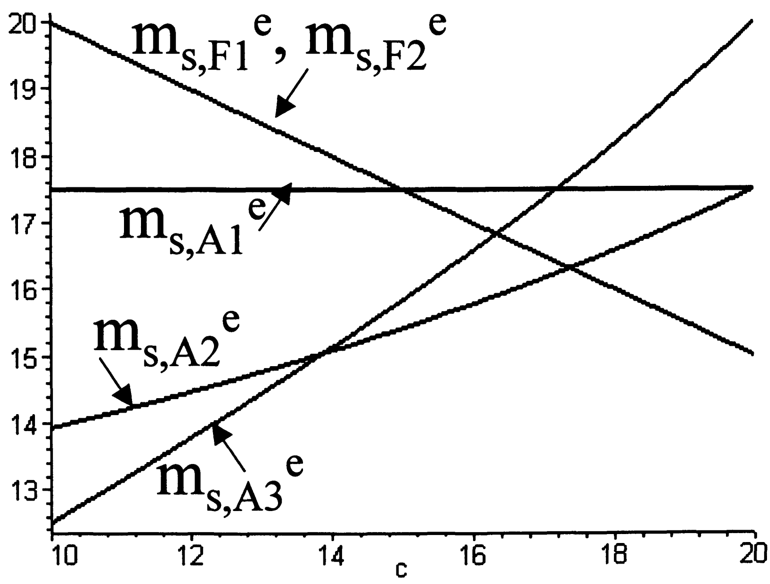
Figure 9.1 "Effective" (average) unit wholesale prices



will decrease his wholesale price in reaction to a buyer cost increase, maintaining volume while sacrificing margin. Under asymmetric information, however, the supplier may do the opposite: increase average wholesale price, thus maintaining margin but sacrificing volume. We found that the value to the supplier of obtaining better information about the buyer's cost structure increases with the variance of the supplier's prior distribution about that cost parameter and with price-sensitivity of demand. We also found that the value of better information is greater when the supplier can offer two-part contracts rather than only one-part contracts. We saw that the value of being able to offer two-part contracts rather than one-part contracts is decreasing in price-sensitivity b.

Clearly, even in such a simple contracting framework as bilateral monopoly with asymmetric information, non-intuitive behaviour can occur. Clearly, many questions remain to be addressed, both within the framework presented here and by expanding the framework. In many contracting situations, the supplier starts in case A1: offering a simple linear wholesale price with no side payment, without knowing the buyer's cost structure. When should the supplier focus

Figure 9.2 Supplier's "effective" (average) unit margins



on obtaining better information about the buyer's cost structure, and when should he offer more sophisticated contracts? How would our results change if we introduce stochastic price-sensitive demand, as in Ha (1997) and Gal-Or (1991a)? What changes if the supplier cannot observe the price-sensitivity parameter b? We plan to investigate these questions in our future research.

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Figure 9.3 Buyer's "effective" (average) unit margins

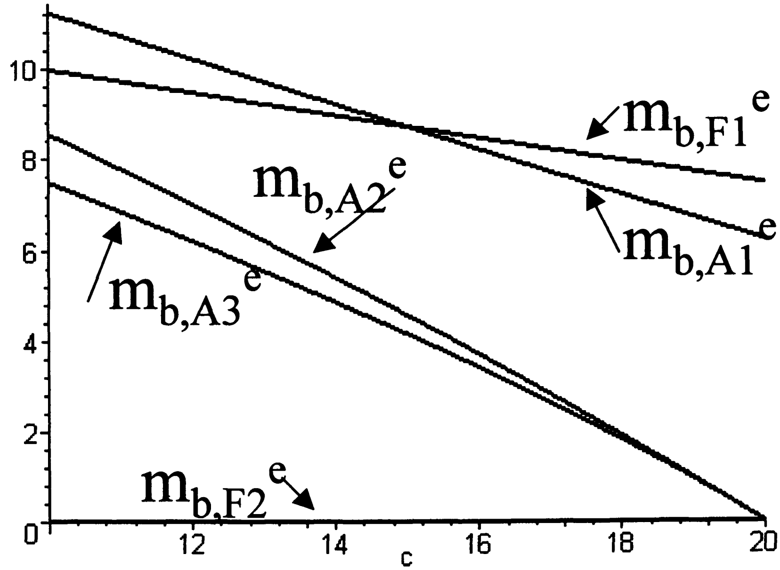

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Figure 9.4 Supplier's profits

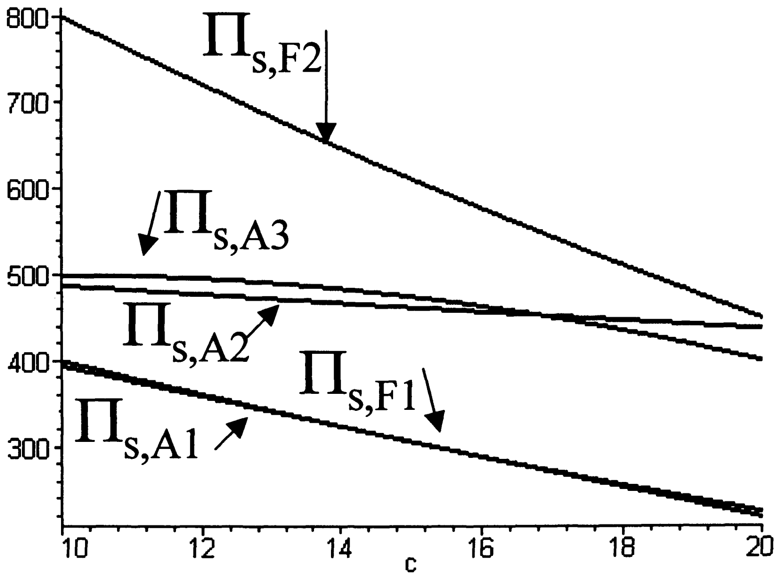

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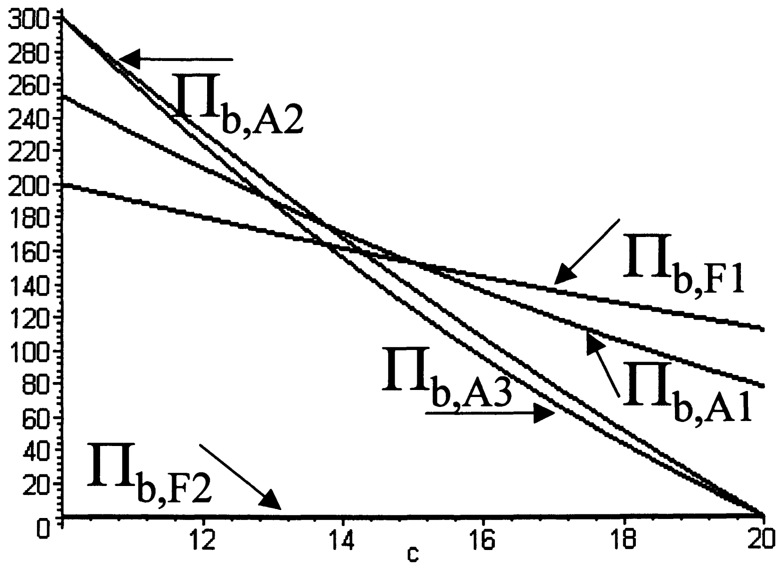
Figure 9.5 Buyer's profits



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Proofs for case A3

Writing $u := \dot{w}$, formulate the Hamiltonian:

$$H = -fL + \frac{1}{2}f(w-s)(a-b(w+c)) + \lambda\frac{1}{2}b[\frac{a}{b}(w+c)]u + \mu u$$

Necessary optimality conditions:

$$\begin{aligned}\dot{\lambda} &= -D_L H\\ \dot{\mu} &= -D_w H\\ D_u H &= 0\end{aligned}$$

The first gives $\dot{\lambda} = -D_L H = f$, so that (combined with the transversality condition) $\lambda = F$. The second gives

$$\begin{aligned}\dot{\mu} &= -D_w H = -\frac{1}{2}f(a-b(w+c)) + \frac{1}{2}f(w-s)b + \frac{1}{2}\lambda ub =\\ &= -\frac{1}{2}f[a-b(w+c)-b(w-s)] + \frac{1}{2}Fub\end{aligned}$$

The third gives

$$\begin{aligned}\frac{1}{2}\lambda b[\frac{a}{b} - (w+c)] + \mu = 0&\\ \mu &= -\frac{1}{2}Fb[\frac{a}{b} - (w+c)]\\ \dot{\mu} &= -\frac{1}{2}fb[\frac{a}{b} - (w+c)] + \frac{1}{2}Fb(u+1)\end{aligned}$$

Equating the two expressions for $\dot{\mu}$ gives:

$$\tfrac{1}{2}Fb = \frac{1}{2}fb(w-c)$$

from which the desired expression for w^*_{A3} follows immediately. We still need to verify several assumptions that we had temporarily set aside:

1. the buyer's individual rationality constraint;

2. sufficiency for supplier's optimization problem $\mathcal{S}_{A3}$.

Individual rationality: set the individual rationality constraint to equality for $c = \bar{c}$. A sufficient condition for it to hold for all c is that $D_c\Pi_b \leq 0$.

$$
\begin{aligned}
D_c\Pi_b &= -b[\frac{a}{2b} - \frac{1}{2}(w+c)](\dot{w}+1) + \dot{L} \\
&= -\frac{1}{2}[a - b(w+c)]
\end{aligned}
$$

As we had assumed $a - b(w + c) \geq 0$ (i.e. demand q is always nonnegative), we need only check the buyer's individual rationality constraint for $c = \bar{c}$ to ensure it is satisfied for all possible c.

Sufficiency, supplier: unfortunately with this type of analysis, this condition is generally impossible to verify, and we have indeed been unable to prove sufficiency in this case. However, the results are fully in line with what one would expect based on e.g. Laffont and Tirole (1993).

Table 9.1 The six combinations of contract type and information structure

Type of contract offered by supplier:	*Does the supplier know the buyer's cost structure?* *yes (Full information)*	*no (Asymmetric information)*
1: one part, linear contract: w	Case F1: ■ supplier offers a one-part contract w, where w is the unit wholesale price, *independent* of quantity purchased q ■ buyer selects order quantity q ■ buyer pays supplier wq	Case A1: ■ supplier offers a one-part contract w, where w is the unit wholesale price, *independent* of quantity purchased q ■ buyer selects order quantity q ■ buyer pays supplier wq
2: two-part, linear contract: (w, L)	Case F2: ■ supplier offers a two-part contract (w, L), where w is the unit wholesale price and L the lump-sum side payment, both *independent* of q ■ buyer selects order quantity q ■ buyer pays supplier $wq - L$	Case A2: ■ supplier offers a two-part contract (w, L), where w is the unit wholesale price and L the lump-sum side payment, both *independent* of q ■ buyer selects order quantity q ■ buyer pays supplier $wq - L$
3: two-part, nonlinear contract: $\{w(q), L(q)\}$	Case F3: ■ supplier offers a two-part menu of contracts $\{w(q), L(q)\}$, where $w(q)$ is the unit wholesale price and $L(q)$ the lump-sum side payment, both *dependent* on q ■ buyer selects order quantity q ■ buyer pays supplier $w(q)q - L(q)$	Case A3: ■ supplier offers a two-part menu of contracts $\{w(q), L(q)\}$, where $w(q)$ is the unit wholesale price and $L(q)$ the lump-sum side payment, both *dependent* on q ■ buyer selects order quantity q ■ buyer pays supplier $w(q)q - L(q)$

Table 9.2 Notation

p	final selling price, selected by the buyer
w	wholesale price
L	lump sum payment per period from supplier to buyer
c	buyer's unit manufacturing costs
s	supplier's unit manufacturing costs
m_s	supplier's unit profit margin, $m_s := w - s$
m_b	buyer's unit profit margin, $m_b := p - w - c$
w^e	"effective" (or average) unit wholesale price, $w^e := w - L/q$
m_s^e	supplier's "effective" unit profit margin, $m_s^e := w^e - s$
m_b^e	buyer's "effective" unit profit margin, $m_b^e := p - w^e - c$
Π_b	buyer's profits
Π_b^-	buyer's reservation profit level
Π_s	supplier's profits
a, b	parameters of demand function $q = a - bp$
q	buyer's order quantity
$\hat{c}$	buyer's (announced) marginal unit manufacturing cost
$F(c)$, $f(c)$	supplier's prior distribution over the buyer's marginal costs C, where F and f are defined on the finite interval $[\underline{c}, \overline{c}]$
$D_x H$	derivative of function H with respect to x
$\dot{H}$	derivative of a function H with respect to $\hat{c}$ evaluated at $c = \hat{c}$, ie. $\dot{H} = D_{\hat{c}} H$
$\Delta_{ij}\Pi_k$	increase in profits of player k going from case j to case i, ie. $\Delta_{ij}\Pi_k := \Pi_{k,i} - \Pi_{k,j}$, where $i, j \in \{F1, F2, F3, A1, A2, A3\}$

Table 9.3 Summary of optimal supply contracts

Type of contract offered by supplier:	*Does the supplier know the buyer's cost structure?* *yes (full information, F)*	*no (asymmetric information, A)*
1: one part, ***linear*** *contract:* w	$w^*_{F1} = \frac{a}{2b} + \frac{1}{2}(s - c)$ $q^*_{F1} = \frac{1}{4}a - \frac{1}{4}(s + c)$ $p^*_{F1} = \frac{3}{4}a + \frac{1}{4}(s + c)$ $m^*_{s,F1} = \frac{a}{2b} - \frac{1}{2}(s + c)$ $m^*_{b,F1} = \frac{a}{4b} - \frac{1}{4}(s + c)$ $\Pi_{s,F1}(w^*_{F1}) = \frac{1}{8b}[a - b(s + c)]^2$ $\Pi^*_{b,F1} = \frac{1}{16b}[a - b(s + c)]^2$	$w^*_{A1} = \frac{a}{2b} + \frac{1}{2}(s - E[c])$ $q^*_{A1} = \frac{1}{4}a - \frac{1}{4}(s + c) - \frac{1}{4}b(c - E[c])$ $p^*_{A1} = \frac{3}{4}a + \frac{1}{4}(s + c) + \frac{1}{4}(c - E[c])$ $m^*_{s,A1} = \frac{a}{2b} - \frac{1}{2}(s + E[c])$ $m^*_{b,A1} = \frac{a}{4b} - \frac{1}{4}(s + c) - \frac{1}{4}(c - E[c])$ $E_c[\Pi_{s,A1}(w^*_{F1})] = \frac{1}{8b}[a - b(s + E[c])]^2$ $\Pi^*_{b,A1} = \frac{1}{16b}[a - b(s + 2c - E[c])]^2$
2: two-part, ***linear*** *contract:* (w, L)	$w^*_{F2} = s$ $L^*_{F2} = \Pi^-_b - \frac{1}{4b}[a - b(s + c)]^2$ $q^*_{F2} = \frac{1}{2}[a - b(s + c)]$ $p^*_{F2} = \frac{1}{2b}[a + b(s + c)]$ $m^*_{s,F2} = 0$ $m^*_{b,F2} = \frac{a}{2b} - \frac{1}{2}(s + c)$ $\Pi_{s,F2}(w^*_{F2}, L^*_{F2}) = -\Pi^-_b +$ $+\frac{1}{4b}[a - b(s + c)]^2$ $\Pi^*_{b,F2} = -\Pi^-_b$	$w^*_{A2} = s + \overline{c} - E[c]$ $L^*_{A2} = \Pi^-_b - \frac{1}{4b}[a - b(s + \overline{c}) - b(\overline{c} - E[c])]^2$ $q^*_{A2} = \frac{1}{2}[a - b(s + \overline{c} + c - E[c])]$ $p^*_{A2} = \frac{1}{2b}[a + b(s + \overline{c} + c - E[c])]$ $m^*_{s,A2} = \overline{c} - E[c]$ $m^*_{b,A2} = \frac{a}{2b} - \frac{1}{2}(s + \overline{c} + c - E[c])$ $E_c[\Pi_{s,A2}] = -\Pi^-_b + \frac{1}{4b}[a - b(s + 2\overline{c} - E[c])]^2 +$ $+\frac{1}{2}(\overline{c} - E[c])[a - b(s + \overline{c})]$ $\Pi^*_{b,A2} = \Pi^-_b + \frac{1}{4b}[a - b(s + \overline{c} + c - E[c])]^2 +$ $-\frac{1}{4b}[a - b(s + 2\overline{c} - E[c])]^2$
3: two-part, ***nonlinear*** *contract:* $\{w(q), L(q)\}$	Case F3: results are the same as case F2	*Note:* most results assume uniform prior $w^*_{A3} = s + \frac{F(c)}{f(c)} = s + c - \underline{c}$ $L^*_{A3} = \frac{1}{2}c[a - b(s + c - \underline{c})] + k$ $q^*_{A3} = \frac{1}{2}[a - b(s + c) - b(c - \underline{c})]$ $p^*_{A3} = \frac{1}{2b}[a + b(s + c) + b(c - \underline{c})]$ $m^*_{s,A3} = \frac{F(c)}{f(c)} = c - \underline{c}$ $m^*_{b,A3} = \frac{a}{2b} - \frac{1}{2}(s + c) - \frac{1}{2}(c - \underline{c})$