



Constrained optimization of component reliabilities in complex systems

Kazuyoshi Nishijima^{a,*}, Marc A. Maes^b, Jean Goyet^c, Michael Havbro Faber^d

^a Institute of Structural Engineering, ETH Zurich, ETH Hönggerberg, HIL E 22.3, Zürich 8093, Switzerland

^b Civil Engineering Department, Schulich School of Engineering, University of Calgary, 2500 University Avenue N.W., Calgary, Canada T2N1N4

^c Bureau Veritas, Marine Division, Research Department, 17 bis Place des Reflets, La Defense 2, 92400 Courbevoie, France

^d Institute of Structural Engineering, ETH Zurich, ETH Hönggerberg, HIL E 23.2, Zürich 8093, Switzerland

ARTICLE INFO

Article history:

Available online 20 August 2008

Keywords:

Constrained optimization
Complex system
Acceptance criteria
Bayesian probabilistic network
Influence diagram

ABSTRACT

The present paper proposes an approach for identifying target reliabilities for components of complex engineered systems with given acceptance criteria for system performance. The target reliabilities for components must be consistent in the sense that the system performance resulting from the choice of the components' reliabilities satisfy the given acceptance criteria, and should be optimal in the sense that the expected utility associated with the system is maximized. To this end, the present paper first describes how complex engineered systems may be modelled hierarchically by use of Bayesian probabilistic networks and influence diagrams. They serve as functions relating the reliabilities of the individual components of the system to the overall system performance. Thereafter, a constrained optimization problem is formulated for the optimization of the component reliabilities. In this optimization problem the acceptance criteria for the system performance define the constraints, and the expected utility from the system is considered as the objective function. Two examples are shown: (1) optimization of design of bridges in a transportation network subjected to an earthquake, and (2) optimization of target reliabilities of welded joints in a ship hull structure subjected to fatigue deterioration in the context of maintenance planning.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

Typically engineered systems are complex systems comprised of geographically distributed and/or functionally interrelated components, which through their connections with other components provide the desired functionality of the system expressed in terms of one or more attributes. This perspective may indeed be useful for interpreting and modelling a broad range of engineered systems ranging from construction processes over water and electricity distribution systems to structural systems. One of the characteristics of engineered systems is that, while the individual components may be standardized in regard to quality and reliability, the systems themselves often cannot be standardized due to their uniqueness. The performance of the systems will depend on the way their components are interconnected to provide the functionalities of the systems as well as on the choice of reliabilities of their components. Thus, the design and maintenance of such systems effectively concern the requirements to the reliability of their components, which can be translated from given requirements to the attributes of the performance of systems in accordance with the way the components are connected.

Due to the complex nature of the problem, modelling and optimization of such systems generally require that different levels of analyses provided by different experts and supported by data are integrated interdisciplinary. Taking basis in engineered structures, at component level physical failure mechanisms may be analyzed, such as yielding, fracture and corrosion. The component failure modes now constitute the building stones for the development of systems failure modes including the formation of failure modes for sequences of sub-systems, for which the corresponding consequences may be assessed. An optimization of the target reliability for components of a given system, i.e., a system with a given interrelation between its components, must take basis in such analyses. Seen in this light, it is useful to hierarchically establish models for complex engineered systems which accommodate for the integration of the different levels of analyses. Such a hierarchical approach may also prove to be beneficial as a mean of communication between professionals representing the expertise required for the modelling of the performance of the different types of components, sub-systems and systems.

The present paper addresses the problem outlined in the foregoing in the context of a hierarchical system modelling developed for risk assessment of engineered systems by the Joint Committee on Structural Safety [1], where taking basis in structural systems a framework is formalized in regard to how the hierarchical system

* Corresponding author. Tel.: +41 44 633 43 16; fax: +41 44 633 10 64.
E-mail address: Nishijima@ibk.baug.ethz.ch (K. Nishijima).

model can be established and then applied to optimize the reliability for components of structures based on specified requirements to the acceptable risks for the considered structural system.

The present paper first provides a short summary of available techniques on the modelling of complex systems. Following this, a general approach for the optimization of the reliability of system components with given criteria to the acceptable system risk is proposed. The proposed approach is composed of three steps; (1) adaptation of Bayesian probabilistic network and influence diagram representation for hierarchical system modelling, (2) linking of acceptance criteria for system level to component level through the Bayesian probabilistic networks and the influence diagrams, and (3) optimizing the target reliabilities of individual components. The original contribution of the presented approach is the effective use of the commonly available techniques, i.e. Bayesian probabilistic networks, influence diagrams and generic algorithms for constrained optimization problems. The approach suggested allows for the assessment of optimal target reliabilities for the individual components of systems for which the risk acceptance criteria are specified in regard to the system performance. The proposed approach is most useful in cases where (1) the components that constitute the system or the sub-system can be categorized into groups with identical probabilistic characteristics and/or (2) the components are hierarchically related. Finally, two illustrative examples are provided. The first example addresses the design of bridges in a transportation network subject to earthquake hazards. Through this example the individual steps of the proposed approach are explained. The second example considers a floating production storage and offloading unit (FPSO), which constitutes a typical complex engineered system. In this example, the target reliabilities of welded joints subject to fatigue deterioration in the framework of inspection and maintenance planning are optimized with given acceptance criteria for the performance of the ship hull structure as a whole.

2. Problem setting

2.1. Modelling of complex systems

The requirements to the probabilistic modelling of complex engineered systems in the context of risk based decision making concern the consistent and tractable representation of the physical characteristics of the considered system and the appropriate detailing to facilitate the assessment of the benefit associated with different decision alternatives. In addition, of course the modelling should facilitate an efficient analysis of the probabilities and consequences required for the ranking of decision alternatives. Fault tree analyses comprise classical techniques for the representation and analysis of systems failure modes, see e.g. [2]. Assuming that components in a system have only two states (failure and success) and that the component failures are statistically independent, the probability that a predefined state of the system (top-event) occurs may be quantitatively assessed [3]. Fault tree analyses have been applied to a variety of fields, e.g., among others, risk assessments of nuclear power plants [4,5] and the reliability analysis of control systems for gas turbine plants [3]. Fault tree analysis is from a technical perspective relatively simple, and hence in many ways attractive, however, for the same reason subject to important limitations. Among these limitations, the difficulty in representing dependencies between basic events as well as the problems associated with updating based on new information should be mentioned. Bayesian probabilistic networks (BPNs) and influence diagrams (IDs) seem to provide an interesting and promising alternative to the classical techniques for system analysis. Any fault trees can be mapped into BPNs as is shown in [6]. The BPN ap-

proach for systems modelling has been utilized for the analysis of structural systems, see e.g. [7]. The applications of BPNs in the context of hierarchical modelling are briefly reviewed in the subsequent section.

When modelling the performance of systems it is important to consider temporal aspects. Petri Nets provide a powerful platform based on which temporal dependencies associated with e.g. repair or replacement actions which may provoke cyclic references to states of the components in the model can be accounted for, see [8]. However, the evaluation of the reliability of a given system through a Petri Net often takes basis in Monte Carlo simulation, which in general requires a considerable amount of computational effort, and the generic algorithms applicable to a broader range of problems are not yet available. BPNs are not immediately appropriate for the representation of cyclic effects, however, by introducing time slices in a BPN (so-called dynamic BPN), BPNs may also be applied for such analysis. Several efficient time slice BPN algorithms have been developed for calculating probabilistic characteristics of state variables of BPNs, e.g. expected values and conditional probabilities, see e.g. [9]. It should be noted that a dynamic BPN representation is equivalent to a Markov chain representation [10].

Another approach for the probabilistic modelling and analysis of complex systems is proposed by Der Kiureghian and Song [11]. In this approach, the probability of an event of interest (related to the system performance) is formulated as a sum of the probabilities of the mutually exclusive combinations of the component states that govern this event. Upper and lower probability bounds on the system performance are calculated based on an out-crossing formulation and using linear programming techniques. Moreover, it is shown in [11] that by aggregating several components as “super-components” and applying the linear programming method in a hierarchical way, the approach provides reasonable probability bounds on the system performance with a manageable computational effort. However, the applied scheme for component aggregation affects the efficiency of the computation and the width of the obtained probability bounds. An optimization of the aggregation scheme in principle requires trial and error, although general guidelines are provided in [11].

2.2. Bayesian hierarchical modelling

The applications of the Bayesian hierarchical models range from, for instance, sociology, biology, environmental studies to engineering. In experiments in sociology, e.g., experiments for studying school effect in educational research, it is difficult to control all the experimental conditions. Ignoring dependences between the uncontrolled experimental conditions at different levels – for the example of school effect, student level, classroom level and school level – and applying simple statistical analysis are proven to produce misleading results as is summarized in [12]. Raudenbush and Bryk [12] propose a hierarchical approach for studying school effect taking basis in the Bayesian multi-level linear model proposed by Lindley and Smith [13]. It provides a flexible statistical tool for estimating how variations in school policies and practices influence educational processes, whereby the different levels of interrelations are taken into account. Environmental sciences face similar situations where due to the complex nature of processes and interactions between systems, observing all the relevant variables that may influence the process of interest is not realistic. Furthermore, it is difficult to realize the identical conditions in different experiments. Thus, the comprehensive use of data obtained for different conditions is necessary for efficiently estimating the parameters of the models, see [14]. In these contexts the Bayesian hierarchical models are employed in such ways that the causal relation or interrelation of variables at different lev-

els in whole systems are first established based on scientific knowledge without specifying the probabilistic characteristics of the variables or assuming weak prior distributions. The parameters of the variables are then estimated or updated using observed data. Other applications of Bayesian hierarchical models can be found in the area of pattern categorization/recognition, see e.g. [15,16]. Due to the characteristics of the applications of the models for the pattern categorizations or recognitions, it is important that these models allow for promptly updating the parameters in the models for a broader range of objects. To this end, flexible representations and systematic learning algorithms which the BPN approach provides are extensively utilized. The Bayesian hierarchical approach has been applied also for engineered complex systems. Among others, Johnson et al. [17] applies the hierarchical model for estimating the reliability of missile systems, where the fault tree analysis is extended using the Bayesian approach to accommodate the integration of available expert knowledge and data.

Emphasizing the difference of the use of the Bayesian hierarchical models, the present paper appreciates the fact that input–output relations of phenomena in engineering at different levels are often quantitatively available in probabilistic terms. For instance, given the geometry and material properties of an engineered component, it is possible to calculate the probability of failure of the component using data and by physical modeling and analysis techniques, e.g. finite element methods. Fatigue deterioration can be probabilistically modelled for given environments, with using physical models and data, see [18]. As the events of interest such as component failure and fatigue degradation are subject to given circumstances, which themselves might be associated with uncertainty, the probabilities of the events are appropriately represented in terms of conditional probabilities. Therefore, in the context of modelling of complex engineered systems, the main focus is how the system can be hierarchically modelled using these conditional probabilities of components at different levels.

As observed in the above the applications of Bayesian hierarchical models are rather diverse. However, all Bayesian hierarchical models utilize generic algorithms developed for estimating parameters and/or obtaining conditional or posterior distributions. The algorithms themselves are indifferent to the contexts where the Bayesian hierarchical models are employed.

2.3. Optimization of engineering decisions under constraints

It is often the case that the optimization of decisions for engineering systems must be performed under constraints. These constraints are typically given a priori to the decision problems in terms of acceptance criteria regarding risks and/or practical operational limitations. Acceptance criteria are generally defined for the attributes of the performance of systems considering the consequences due to possible failures. Recent design codes e.g. [19] provide acceptance criteria in terms of minimum requirements to structural performance. The Joint Committee on Structural Safety [20] recommends different target reliabilities for engineered structures in accordance with the different magnitude of the consequence of failure as well as the relative cost of safety measures. Also, safety to personnel must be considered. Recently, a general principle for evaluating the acceptability of a life saving measure has been proposed using the concept of life quality index (LQI), e.g. [21,22]. Based on the LQI principle it is possible to optimize and specify requirements for the reliability of engineered systems based on the costs of improving their reliability. Additionally, several practical constraints, e.g., available budget, cost-benefit ratios and allowable environmental impacts, may be given for projects involving design and maintenance of engineered systems. Together with acceptance criteria given from normative perspectives, these exogenously given constraints constitute important boundary con-

ditions for the optimization of the performance of engineered systems.

A number of approaches have been proposed for optimizing decisions under constraints in engineering [23–25]. Thereby, one of the central issues is how the optimization process can be transformed in such ways that it allows for the utilization of commonly available techniques for the probability calculations as well as for numerical optimization. Royset et al. [23] propose algorithms for reliability-based optimal design problems with which the required calculations of reliability and optimizations are completely decoupled, hence, allowing for a flexible choice of the optimization algorithm and the reliability calculation method. Guikema and Paté-Cornell [24] propose a method for the optimization whereby the performances of engineered systems are related discontinuously to decision variables. These approaches are in fact highly sophisticated and also efficient in the treatment of some optimization problems. However, for the same reason they may be cumbersome to apply in practical situations where complex engineered systems are of interest, since different levels of models established by different experts must be reformulated to fit the format which these approaches require. To overcome this difficulty Bayesian probabilistic network and influence diagram representations are employed in the present paper as is described in the following sections.

2.4. Objective of proposed approach

The acceptance criteria mentioned in the foregoing may be seen to constitute the boundary conditions, which any engineered system must satisfy during its service life. The present paper takes the standpoint that the acceptance criteria for systems are a priori given. This situation is often the situation encountered in practice. The goal of the present paper is to establish an approach for the optimization of the target reliability for components of systems for given system performance requirements in terms of acceptance criteria, by minimizing life cycle costs for the design and operation of the system, or more generally by maximizing the service life expected utility.

3. Proposed approach

3.1. Hierarchical system modelling with Bayesian probabilistic networks

A hierarchical system modelling for complex systems facilitates the representation of complex systems at an early stage of risk analysis, e.g. at the concept evaluation, but may also serve to optimize the final design as well as the management of the risk during operation. Hierarchical BPN models appear suitable as a platform for modelling complex systems, since they provide a causal and mind mapping representation of the system characteristics and functionalities. In Fig. 1 it is illustrated how the system functions are represented in terms of a hierarchical aggregation of components and their interrelations. At the same time the requirements to the system performance may be disaggregated into reliability performance requirements for the components. In what follows, the proposed approach is explained in accordance with Fig. 1.

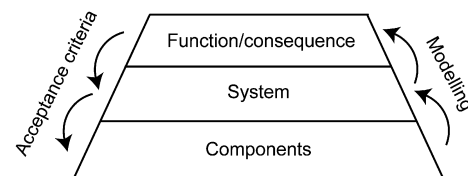


Fig. 1. Hierarchical modelling and translation of acceptance criteria.

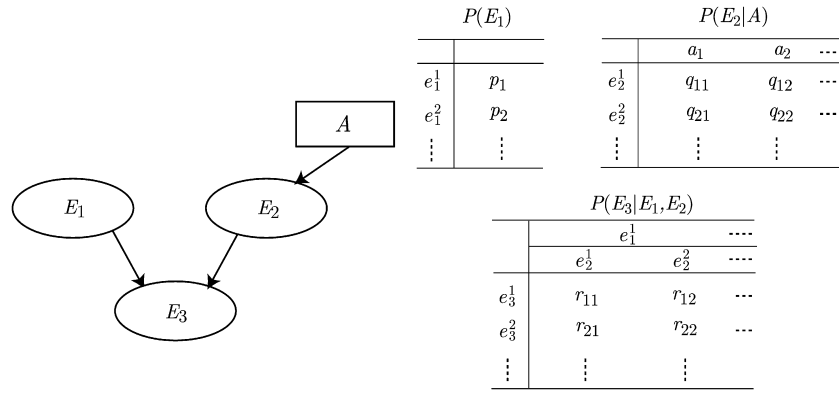


Fig. 2. Example of a BPN and conditional probability tables.

Let A and $E = (E_1, E_2, \dots, E_n)$ denote the sets of possible actions and possible states of a system respectively. The combination of $a \in A$ and $\mathbf{e} \in E$ specifies the joint probability conditional on the action $P(\mathbf{e}|a)$ and the consequences $\mathbf{C}(a, \mathbf{e}) = (C_1(a, \mathbf{e}), C_2(a, \mathbf{e}), \dots, C_m(a, \mathbf{e}))$. In general these quantities are the functions describing how the components and the sub-systems in the system are interconnected. However, in the following it is assumed that the interconnectivity is fixed. Note that the consequences $\mathbf{C}(a, \mathbf{e})$ may be a vector when two or more attributes of the system performance are considered, e.g. financial cost, fatalities and damages to the qualities of the environment. It is assumed that the consequences $\mathbf{C}(a, \mathbf{e})$ can be represented as an attribute-wise sum of the consequences $\mathbf{C}_A(a)$ associated with action a and the consequences $\mathbf{C}_E(\mathbf{e})$ associated with event \mathbf{e} , namely

$$\mathbf{C}(a, \mathbf{e}) = \mathbf{C}_A(a) + \mathbf{C}_E(\mathbf{e}). \tag{1}$$

A Bayesian probabilistic network is a probabilistic model representation in terms of a directed acyclic graph that consists of nodes representing uncertain state variables, so-called chance nodes and edges that logically link the nodes, and conditional probability assignments, see Fig. 2 for example, and see e.g. [26] for general introduction. An influence diagram is an extension of Bayesian probabilistic network that includes so-called decision nodes and utility nodes in a graph in addition to chance nodes. Using the chain rule for Bayesian probabilistic networks [26], the joint probability $P(E|a)$ can be decomposed as

$$P(E|a) = \prod_i P(E_i|pa(E_i), a), \tag{2}$$

where $pa(E_i)$ is the parent set of E_i . From Eq. (2) it can be seen that the joint probability $P(E|a)$ can be built up by conditional probabilities. Any marginal probabilities of the states of the subset of E can be efficiently calculated from the joint probability $P(E|a)$ with generic algorithms and software tools commonly available, see the appendix of [27]. For the BPN shown in Fig. 2, the parents of E_3 are the nodes E_1 and E_2 , and the node E_2 is a function of action A . The joint probability is then written as

$$P(E|a) = P(E_3|E_1, E_2)P(E_1)P(E_2|a). \tag{3}$$

Each term in Eq. (3) thus the joint probability is fully characterized by the conditional probability tables shown in Fig. 2.

Let $\mathbf{F}(\mathbf{C}, P) = (F_1(\mathbf{C}, P), F_2(\mathbf{C}, P), \dots, F_i(\mathbf{C}, P))$ denote a vector function of $\mathbf{C}(a, \mathbf{e})$ and $P(E|a)$. For instance, the expected total cost, may be one of the attribute of a system performance to be considered, and is written as one element of $\mathbf{F}(\mathbf{C}, P)$ as

$$F_i(\mathbf{C}, P) = \sum_{\mathbf{e} \in E} C_i(\mathbf{e}, a)P(\mathbf{e}|a), \tag{4}$$

where $C_i(\cdot, \cdot)$ represent the cost. The probability that the damage to environmental quality exceeds a given threshold c_{acc} may be another element of $\mathbf{F}(\mathbf{C}, P)$ and is written as

$$F_j(\mathbf{C}, P) = \sum_{\mathbf{e} \in E} I[C_j(\mathbf{e}, a) > c_{acc}]P(\mathbf{e}|a), \tag{5}$$

where $C_j(\cdot, \cdot)$ represents the environmental damage and $I[\cdot]$ is the indicator function, which returns unity if the condition in the bracket is satisfied and zero otherwise. Such environmental damages may be represented e.g. in terms of release volumes, the geographical release extent and/or temporal release extent of agents. The conditional expected value of the number of fatalities given the state $E_m = e_m$ may be other element of $\mathbf{F}(\mathbf{C}, P)$ and is written as

$$F_k(\mathbf{C}, P) = \frac{\sum_{\mathbf{e}' \in E \setminus E_m} C_k(a, (e_m, \mathbf{e}'))P((e_m, \mathbf{e}')|a)}{\sum_{\mathbf{e}' \in E \setminus E_m} P((e_m, \mathbf{e}')|a)}, \tag{6}$$

where $C_k(\cdot, \cdot)$ represents the number of fatalities and

$$\mathbf{e}' \in E \setminus E_m = \{E_1, E_2, \dots, E_{m-1}, E_{m+1}, \dots, E_n\}.$$

Note that any functions represented in terms the elements of $\mathbf{F}(\mathbf{C}, P)$ can be systematically calculated by the algorithms developed for the analyses of BPNs and IDs when the state variables $E = (E_1, E_2, \dots, E_n)$ and their interrelations and the (conditional) probabilities corresponding to the interrelations of the variables are defined in an ID, see e.g. [26]. Thus, the remaining task for developing models for engineered complex systems is to represent the physical understanding, the relevant experience and the data available at different hierarchical levels in terms of (conditional) probabilities of states of variables or in terms of decision nodes or utility nodes, and then link them together. Thereby, the general characteristic that engineered systems are comprised and built up by components, which are standardized by codes and industrial standards in regard to quality and reliability may add value to the use of object-oriented BPN representations. This special type of BPN models allows for creating classes of BPNs, which are representative for sub-systems that have identical characteristics, see e.g. [28,29].

3.2. Objective function and constraints

Having established the hierarchical system model in terms of IDs, the objective function such as service life utility or expected total cost may be assessed from the ID as a function of the chosen action utilizing the functional representation of $\mathbf{F}(\mathbf{C}, P)$ as shown in the previous section, i.e.:

$$u(a) = F_1(\mathbf{C}(a, \cdot), P(\cdot|a)). \tag{7}$$

Acceptance criteria are typically defined in regard to the functional-ity or performance of the considered system measured in terms of

risks and/or probability of failure. Since the design and maintenance of a system usually specifically addresses the components of the system, it is of interest how the acceptance criteria for the components may be derived from the acceptance criteria specified for the system performance. Thus, the optimization of reliabilities for components in a system constitutes an inverse problem, see Fig. 1. The acceptance criteria for the system performance can be related to the target reliabilities for the components using the function type of $F(\mathbf{C}, P)$ as is shown in the previous section as

$$F_i(\mathbf{C}(a, \cdot), P(\cdot|a)) \leq c_i \quad (i = 2, 3, \dots, m), \tag{8}$$

where $F_i (i = 2, 3, \dots, m)$ represent the functions on the ID calculating the quantities for which the acceptance criteria for the system are defined, and c_i are acceptance levels for the corresponding quantities.

3.3. Optimization of actions for components of complex system

Since several combinations of target reliabilities for different components in a system may satisfy the prescribed acceptance criteria for the system, the optimal combination of target reliabilities for components may be identified as the combination of the target component reliabilities associated with action a which maximizes the expected utility u using Eqs. (7) and (8) formulated in accordance with the previous sections as

$$\begin{aligned} \text{Maximize } & u(a) = F_1(\mathbf{C}(a, \cdot), P(\cdot|a)) \text{ s.t.} \\ & F_i(\mathbf{C}(a, \cdot), P(\cdot|a)) \leq c_i \quad (i = 2, 3, \dots, m). \end{aligned} \tag{9}$$

Since the functions $F_i (i = 1, 2, \dots, m)$ are readily calculated, the problem is reduced to a standard non-linear constrained optimization problem for which efficient algorithms are available, see e.g. [30].

4. Example 1

This example considers the simple optimization of the design of bridges subject to earthquake hazards. The aim of this example is to explain in detail how the proposed approach may be applied in practical situations. The bridges b_1, b_2 and b_3 geographically connect the location a with c , and thus constitute the system components in a transportation network system, see Fig. 3. It is assumed that the state of the system is fully described through the combinations of the states of the three bridges, and hence, the failures of e.g. the road sections besides the bridges in the network are not

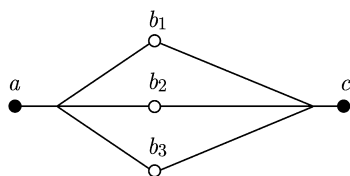


Fig. 3. Transportation network system.

considered. The system failure is assumed to be defined as the joint failures of all three bridges. The objective function to be minimized is the expected discounted total cost, which consists of the initial cost and the expected cost associated with the failures of bridges. The acceptance criteria are assumed to be given for (1) the expected number of fatalities in the system given that an earthquake occurs as 10, and (2) the conditional probability that the system fails given that an earthquake occurs as 1%. The life time considered in the design of the bridges is 100 years, and it is assumed that an earthquake occurs at most once in the system's life time. The discounting rate applied for evaluating the future costs is assumed equal to 3% per annum.

4.1. Model description

The earthquake hazard is modelled in the earthquake class BPN as is shown in Fig. 4 (left). It consists of five nodes, namely, "Scenario", "Time", "V1", "V2" and "V3". The node "Scenario" contains different possible earthquake scenarios with corresponding probabilities. The term *scenario* may refer to an earthquake occurring at different seismic zones and different faults, or more specifically, different combinations of the values of ground motion intensities at different locations. The latter corresponds to the cases where the joint probability density of ground motion intensities at different sites is identified by seismic hazard analyses and thereafter the joint probability density is discretized into a finite number of probabilities corresponding to the intervals of the ground motion intensities at different sites. When the different combinations of the values of ground motion intensities are taken as the identifiers of the scenarios, the spatial correlations between the intensities at different locations can be suitably considered in the earthquake hazard model. In this example, however, for illustrative purposes only one scenario "eq1" is considered.

The node "Time" specifies the probability of the yearly discretized time T when the scenario eq1 occurs. T is assumed to follow a geometric distribution with an occurrence probability for each year given as $v\Delta t = 0.01$. The nodes "V1", "V2" and "V3" represent the logarithms of the peak ground accelerations (cm/s^2) at the locations where the bridges are to be built, and are assumed to follow normal distributions given the scenario eq1 with the parameters shown in Table 1. When the probabilistic characteristics are implemented into the conditional probability table in BPNs they have to be discretized. The intervals and the upper and lower bounds must be chosen carefully assuring the efficiency and accuracy of the discretizations. They are chosen in this example as shown in Table 1. Note that the BPN in Fig. 4 (left) assumes that "V1", "V2" and "V3" are conditionally independent given the scenario. The nodes "Time", "V1", "V2" and "V3" (surrounded by the bold line) are output nodes, and are connected to other nodes in the BPN for the transportation network system, Fig. 5.

The bridges are modelled in the Bridge class BPN as shown in Fig. 4 (right). The bridges b_1, b_2 and b_3 are assumed to be identically modelled through the Bridge class BPN. However, the different probabilities in the input nodes "V", "X" and "Theta2" (highlighted

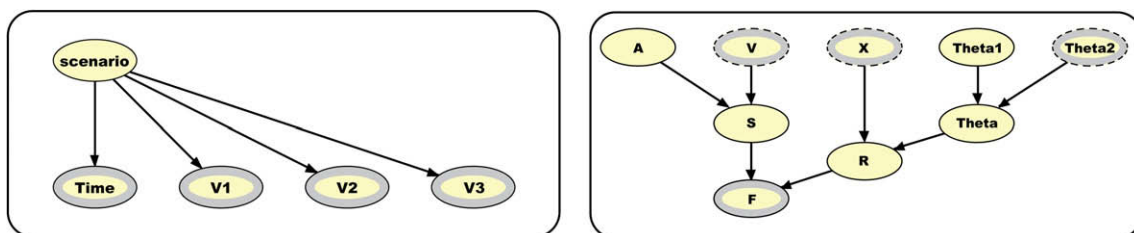


Fig. 4. Classes of BPNs for Earthquake hazard (left) and for Bridge (right).

Table 1
Assumed distributions of nodes in BPNs and ID

| Variables | Distributions | Bounds |
|--|---------------------------------------|-------------|
| Earthquake class BPN | | |
| Scenario | $P[\text{Scenario} = \text{eq1}] = 1$ | |
| $V1 \text{eq1}$ | Normal ($\ln 200, 0.5$) | [0, 9] |
| $V2 \text{eq1}$ | Normal ($\ln 300, 0.5$) | [0, 9] |
| $V3 \text{eq1}$ | Normal ($\ln 400, 0.5$) | [0, 9] |
| $\text{Time} \text{eq1}$ | Geometric (0.01) | [1, 100] |
| Bridge class BPN | | |
| A | Normal ($\ln 2, 0.1$) | [0, 2] |
| Theta1 | Normal ($\ln 1, 0.1$) | [-0.5, 0.5] |
| ID for transportation network system | | |
| Theta2 | Normal ($\ln 1, 0.1$) | [-0.5, 0.5] |
| X1, X2 and X3 given design alternative a_1 | Normal ($\ln 600, 0.1$) | [0, 9] |
| X1, X2 and X3 given design alternative a_2 | Normal ($\ln 800, 0.1$) | [0, 9] |
| X1, X2 and X3 given design alternative a_3 | Normal ($\ln 1000, 0.1$) | [0, 9] |

Normal (μ, σ) abbreviates the normal distribution with the mean μ and the standard deviation σ , and Geometric (p) abbreviates the geometric distribution with occurrence probability p . The geometric distribution is discretized by the interval of 1 and the Normal distributions are discretized by the interval of 0.1 when implemented into the conditional probability tables in the BPNs. The last column shows the upper and lower bounds in the corresponding conditional probability tables.

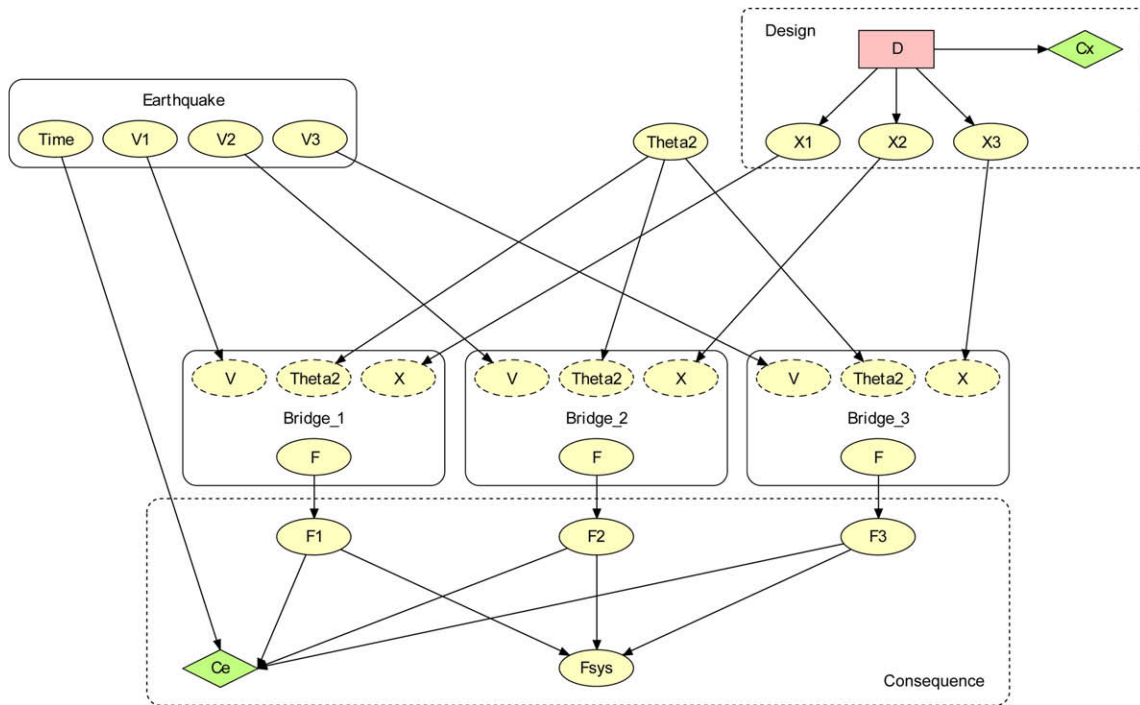


Fig. 5. ID for transportation network system (cost).

with bold dashed line) facilitate the differentiation between the resistances of the bridges and the corresponding probabilities of failure. In the Bridge class BPN, S denotes the load effect, which is represented by

$$S = V + A, \tag{10}$$

where A represents the logarithm of the soil amplification factor. A is assumed to follow a normal distribution with the parameters given in Table 1. The resistance R of the bridge is modelled as

$$R = X + \theta = X + (\theta_1 + \theta_2), \tag{11}$$

where X specifies the design of the bridges and θ represents the uncertainties associated with the resistance of the bridge. θ can be decomposed into two types of uncertainties, θ_1 and θ_2 . θ_1 is the uncertainty associated with individual realizations of bridges, and can be assumed independent between the different bridges,

whereas θ_2 denotes the common uncertainty that affects all realizations of bridges thus introduces the statistical dependence. For example, uncertainty on material geometry or uncertainties associated with construction work may belong to the former type of uncertainty. Modelling and statistical uncertainties belong to the latter type of uncertainty. The assumed probabilistic characteristics of θ_1 and θ_2 are shown in Table 1. The failure of a bridge, which is defined as the event $R < S$, is denoted by the Boolean node “ F ”, and the probability of failure is expressed as

$$P[F = \text{‘true’}] = P[R < S]. \tag{12}$$

The node “ F ” is the output node from the Bridge class BPN and is utilized for the assessment of consequences in the ID, see Fig. 5.

Fig. 5 shows the ID for the whole transportation network system. “Earthquake” is an instance of the Earthquake class BPN, and “Bridge_1”, “Bridge_2” and “Bridge_3” corresponding to b_1 ,

b_2 and b_3 , respectively, are instances of the Bridge class BPN, for which only input and output nodes are shown. The node “Fsys” represents system failure, which is connected with the nodes “F1”, “F2” and “F3” representing the individual failures of the bridges b_1 , b_2 and b_3 , respectively. These are required for checking if the acceptance criterion is satisfied for the conditional probability of system failure given that an earthquake occurs. The node “Theta2” specifies the probability distribution of the common uncertainty Θ_2 , see Table 1. Finally, the decision node “D” represents the set of design alternatives for the three bridges. Three design alternatives a_1 , a_2 and a_3 are considered for each bridge, hence, there are $3^3 = 27$ possible actions in the decision node. The nodes “X1”, “X2” and “X3” represent the probability distribution of state of the bridges b_1 , b_2 and b_3 respectively, corresponding to the choice of the design alternatives, see in Table 1. For each action, the corresponding initial cost is defined in the utility node “Cx” whose values are shown in Table 2. The utility node “Ce” defines the discounted failure costs for all combinations of the states of the three bridges for each year up to 100 years. The failure costs assumed in the example are shown in Table 3. From the utility nodes “Ce” and “Cx” the expected discounted total cost is calculated. Similarly, the expected number of fatalities in the system given that an earthquake occurs can be calculated with a similar ID as the one shown in Fig. 6. In the figure input and output nodes of the instances of the class BPNs (earthquake class, bridge class, design class and consequence class) are abbreviated. The summary of

the magnitudes of the consequences are given in Table 3. Failure costs and fatalities shown in the tables should be considered as the expected values over possible consequences given the states of the bridges when an earthquake occurs. In practice the development of the table requires that the consequences must be analyzed for all possible combinations of the states of all bridges in the network. While it requires considerable efforts, it allows for flexibility considering the significance of each bridge in the network, e.g. consideration of the topology of network.

4.2. Results

The expected discounted total costs, the expected number of fatalities and the probabilities of system failure given that an earthquake occurs for the 27 possible actions are calculated using the established IDs. The result is shown in Fig. 7. At the bottom of the figure the correspondence between the actions and the combinations of the design alternatives for the three bridges is also shown. The optimal action consistent with the two acceptance criteria regarding the expected number of fatalities and conditional probability of system failure given the occurrence of an earthquake is identified as action 25 (design alternative a_3 for the bridges b_1 and b_2 , and design alternative a_1 for the bridge b_3); action 17 results in the minimum expected discounted total cost, but it does not satisfy the acceptance criteria. The strategy behind action 25 may be interpreted as follows; considering the non-linear relation between the number of failed bridges and the failure costs, a sound strategy may be to avoid, by all means, the simultaneous failures of the three bridges in an economically efficient way, which may be realized with higher reliabilities for one or two of the three bridges and comparatively low reliability for the other bridge(s). Since the earthquake hazard is smallest for bridge b_1 , the highest reliability of the system can be realized most efficiently through bridge b_1 and be realized relatively efficiently for the bridge b_2 , by adopting the design alternative a_3 for the bridges b_1 and b_2 ; corresponding to the highest design resistance in the three design alternatives. At the same time, by accepting a relatively higher failure probability for bridge b_3 , the expected discounted total cost can be reduced. This becomes clearer by comparing action 25 with action 9, which is composed of the same set of design alternatives but applied for different bridges, i.e., a_1 for the bridge b_1 and a_3 for the bridges b_2 and b_3 . Action 9 requires the same initial cost as action 25, and results in almost the same amount of the expected discounted total cost, but significantly high conditional probability of system failure given an earthquake. This strategy seems tricky, and may not be considered in practical situations where typically the resistances of structures may be designed in a proportional way to the magnitudes of hazards. However, from a system optimization point of view, this is the best strategy that satisfies the acceptance criteria given for the system. It should be noted that in practical situations decision makers might accept slightly higher costs to further reduce the risk of fatalities (e.g. Action 27 instead of Action 25 in this example). However, if the objective function and the constraints are established to fully represent the decision maker’s preference, such a subjective decision may lead to sub-optimal decisions.

4.3. Discussion

The hierarchical Bayesian approach provides a clear perspective of how the whole system should be built up using the modules representative of different levels of analyses. In this example, the transportation network system can be built up with four modules, i.e., earthquake module represented by the earthquake class BPN, a bridge module represented by the bridge class BPN, a design module and a consequence module, see Fig. 5. These modules can be

Table 2
Initial costs

| Design alternative | Initial cost (Monetary unit) |
|--------------------------|------------------------------|
| Design alternative a_1 | 10 |
| Design alternative a_2 | 11 |
| Design alternative a_3 | 12 |

Table 3
Failure costs and fatalities

| | State of Bridge | | | | | | | |
|------------------------------|-----------------|----|----------|----|----------|----|----------|-----|
| | Bridge 1 | | Bridge 2 | | Bridge 3 | | Bridge 3 | |
| | F | NF | F | NF | F | NF | F | NF |
| Failure cost (Monetary unit) | 0 | 10 | 10 | 50 | 10 | 50 | 50 | 200 |
| Fatality | 0 | 10 | 10 | 20 | 10 | 20 | 20 | 30 |

Failure costs are not discounted. F and NF are abbreviations for failure and no failure, respectively.

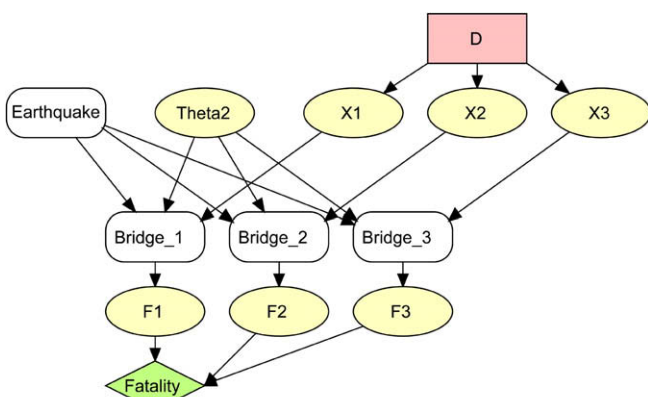


Fig. 6. ID for transportation network system (fatality).

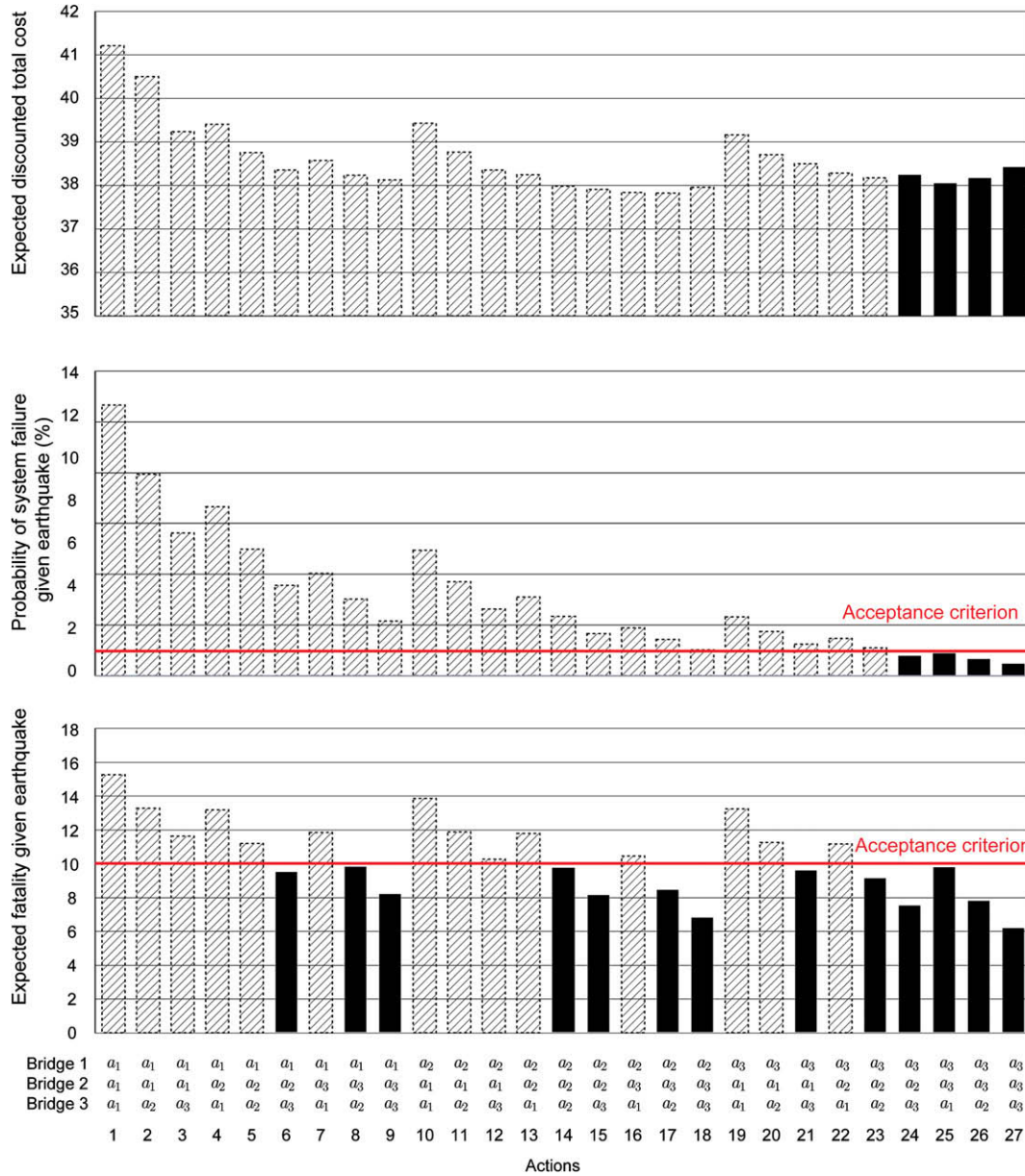


Fig. 7. Expected discounted total cost, and expected fatality and probability of system failure given that an earthquake occurs.

built up separately, whereas the interfaces between the modules must be specified. Such a module oriented modelling in the hierarchical Bayesian approach not only enhances the integration of the knowledge of different experts, experience and data available at different levels, but also increases the productivity of risk assessments, since the modules are re-useable.

Updating of the probabilistic characteristics in BPNs is of practical use, although this aspect is not emphasized in the example. For instance, when the data on the damage states of the bridges and the load effects are obtained after the occurrence of an earthquake, the uncertainties associated with the resistance of the bridges can be updated by conditioning the corresponding nodes. Hence, the updated probability can be used for future risk assessment.

While only a small number of discrete action alternatives are considered in this example, there are other cases where a large number of discrete action alternatives or continuous action alternatives are to be considered. In such cases it is not feasible to per-

form the ID analysis for every action, thus adaptation of efficient algorithms for solving optimization problems under constraints are needed. In this context, IDs serve as the function in the process of calculating the value of the expected utility and the values of the quantities for which acceptance criteria are defined which then in turn can be implemented into optimization algorithms. In the next example, it is shown how this may be realized using commonly available software tools.

5. Example 2

Optimal reliability for components in Floating Production Storage and Offloading Units (FPSOs) subject to fatigue deteriorations is considered in this example. The main function of FPSOs is to produce and store oil at offshore oil fields with given requirements to reliability in production and safety to persons and environment. Typically considered events of system failure for FPSOs are:

- Loss or damage of ship due to loss of buoyancy or explosions/fires.
- Loss of production due to reduced functionality.
- Loss of lives due to foundering or explosion/fires.
- Leaks and other damages to the quality of the environment.

Considering the hull as an assembly of components, the hull may be considered to comprise an assembly of tanks tied together with deck plates, tank partitions, and bottom and side plates. The individual components are furthermore stiffened by girders and web frames to ensure a sufficient structural integrity of the hull, see Fig. 8. The corresponding hierarchical model representation is shown in Fig. 9.

The hull components as described above have basically two functions, namely, to ensure that the overall ship has a sufficient structural integrity and provide the means for containing cargo and ballast. Failure of the components of the hull at this level can be assumed as the events of:

- Loss of or reduced structural integrity.
- Loss of containment due to explosion.
- Leaks of the individual tanks.

Considering now the individual components as outlined in the above these may be viewed upon as assembly of plates connected by welded joints. Failure of these components may lead to:

- Crack or pit through plate thickness.
- Reduced overall plate thickness.
- Joint stiffness reduction or failure.

Thus, the losses or damages at component level may lead to the hull failure or undesired economic and environmental losses as well as loss of lives given the way how the components are interconnected. The problem in this example is to optimize the target reliabilities for the welded joints in plate and tank partition components given the requirements to the functionality/consequence of the ship hull, e.g. the probability of hull failure. It is emphasized in this example how commonly available software tools can be used in accordance with the proposed approach. For this purpose a software tool is developed using Hugin® for BPN/ID representation and Microsoft Excel® (hereafter Excel) for the optimization algorithm as well as the user interface. In the subsequent section, the overview of the software tool development is illustrated.

5.1. Optimization of target reliability for welded joints in components

The developed software tool provides an easy interface to obtain the optimal target reliabilities for welded joints subjected to fatigue deterioration. Excel is used as a platform for integrating the various computational modules and storing information required for calculations. The Excel platform is linked dynamically to the Hugin ActiveX server (hereafter Hugin). In order to use the

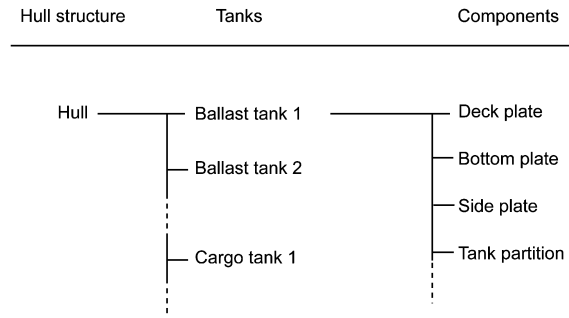


Fig. 9. Hierarchical modelling of hull structure.

software tool the user has to define, through Hugin files, the BPNs corresponding to the hierarchical model of the hull structure as described above. The outputs, i.e. optimized target reliabilities for all welded joints, are written into the Excel file.

In Fig. 10, the illustration of the hierarchical Bayesian representation of the ship hull structure is given. Two BPNs in the top of the figure represent the performances of tanks. The tank performances are characterized by the states of the plates that constitute the tanks. As is described above, at this level the possible consequences due to component failures are capacity reduction, explosion and environmental damage due to leaks. The ID in the bottom of the figure concerns how the component failures may propagate and lead to further consequences at system level. Here, three attributes of the consequences are identified, i.e. economic loss, loss of lives and environmental damage measured in terms of leak intensities. These BPNs and ID are interconnected as shown in the figure. In the entire ID the conditional probability tables are assumed established with the help of experts, see e.g. Fig. 11 (which is the conditional probability table for node “Explosion_1” as implemented into a Hugin file), whereas the nodes that represent the components serve as root nodes whose probabilities are represented in terms of unconditional probabilities, which are derived from the target reliabilities for welded joints in each component. Therefore, by changing the target reliabilities for the welded joints which are set in the Excel file, the unconditional probabilities for the components are changed accordingly. In turn, the corresponding probabilistic characteristics, e.g. expected total cost or probability of ship hull failure are changed and stored in the Excel file, see Fig. 12. This process is made automatically through ActiveX. The design and service life maintenance cost for the different welded joints is in general a function of the target reliability in regard to fatigue failure, and this is implemented as a VBA code in the Excel file. For the assessment of the relationship between the reliability of the welded joints subjected to fatigue failure and the service life cost, the iPlan software described in [31] may be utilized. Finally, the optimal target reliabilities for welded joints are obtained using the Solver add-in provided in Excel – target reliabilities correspond to “changing cells”, and acceptance criteria for the ship hull correspond to “constraints” in the Solver add-in.

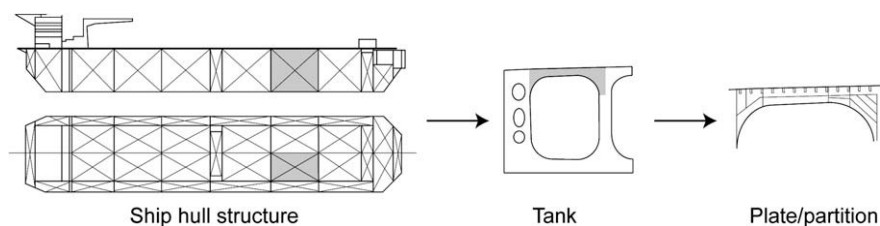


Fig. 8. Hierarchy of ship hull structure considered.

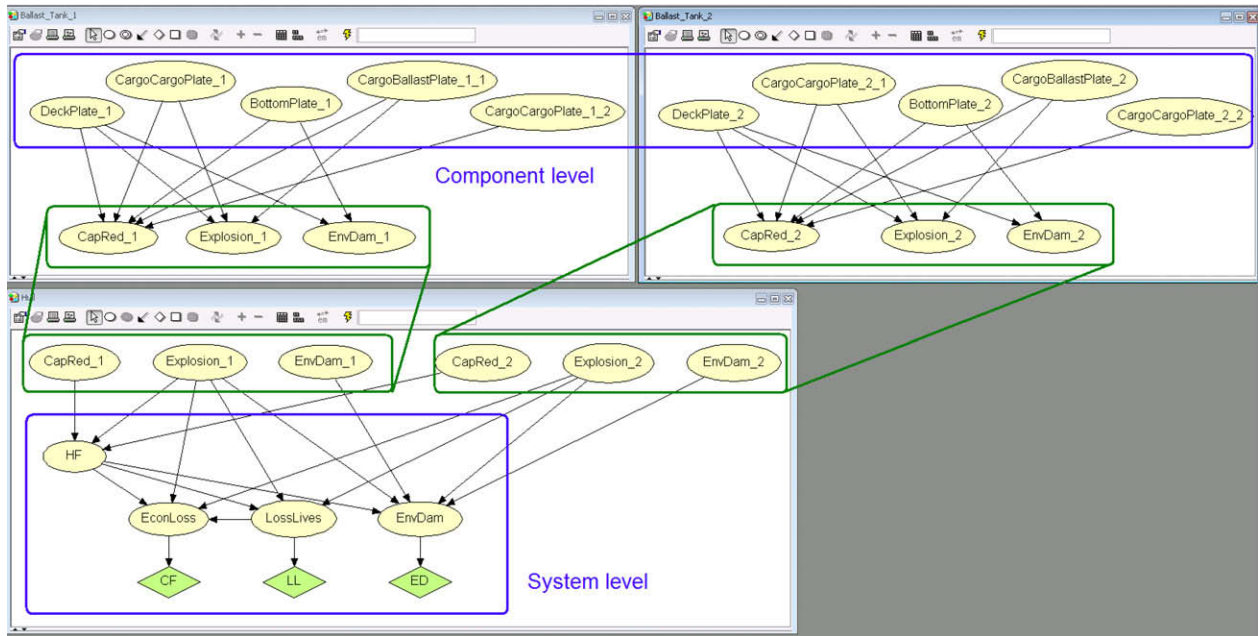


Fig. 10. ID for the tanks and the hull structure.

Explosion_1

| CaraoBallast | Fail | | | | Survive | | | |
|--------------|------|---------|------|---------|---------|---------|------|---------|
| | Fail | Survive | Fail | Survive | Fail | Survive | Fail | Survive |
| CaraoCaraoF | | | | | | | | |
| DeckPlate 1 | Fail | Survive | Fail | Survive | Fail | Survive | Fail | Survive |
| No | 0.4 | 0.5 | 0.8 | 0.8 | 0.4 | 0.5 | 0.6 | 1 |
| Minor | 0.5 | 0.45 | 0.1 | 0.19 | 0.55 | 0.49 | 0.35 | 0 |
| Maioir | 0.1 | 0.05 | 0.1 | 0.01 | 0.05 | 0.01 | 0.05 | 0 |

Fig. 11. Illustration of conditional probability table.

| Tank name | Engineering component | Correlation coefficient | Target reliability of each hot spot | Number of hot spots to be inspected |
|----------------|-----------------------|-------------------------|-------------------------------------|-------------------------------------|
| Ballast_Tank_2 | CargoCargoPlate_2_1 | 0.30 | 2.30E-03 | 10 |
| | BottomPlate_2 | 0.30 | 1.68E-02 | 10 |
| | DeckPlate_2 | 0.30 | 1.43E-03 | 10 |
| | CargoBallastPlate_2 | 0.30 | 2.89E-03 | 10 |
| Ballast_Tank_1 | CargoCargoPlate_2_2 | 0.30 | 1.45E-02 | 10 |
| | CargoCargoPlate_1 | 0.30 | 1.36E-02 | 10 |
| | BottomPlate_1 | 0.30 | 1.44E-02 | 10 |
| | DeckPlate_1 | 0.30 | 1.17E-02 | 10 |
| | CargoBallastPlate_1_1 | 0.30 | 1.64E-02 | 10 |
| | CargoCargoPlate_1_2 | 0.30 | 1.62E-02 | 10 |

| Summary | |
|--|----------|
| Total risk | 2148.4 |
| Hypothetical output from iPlan | 1947.00 |
| Expected failure cost | 181.41 |
| Expected Loss Lives | 0.07 |
| Expected Environmental Damage | 0.04 |
| Probability of failure of hull structure | 1.00E-03 |

Fig. 12. User interface of developed software tool.

5.2. Results and discussion

In this illustrative example, the acceptable probability of system failure is set as 10^{-3} per annum which constitutes the boundary condition in the optimization problem. The objective function is the expected total cost including the inspection cost, the repair cost and the failure cost due to ship hull failure. As is shown in Fig. 12, different optimal target reliabilities are obtained for the components in different tanks, reflecting the different contribution to the system failure. The set of these optimal target reliabilities correspond to the set of the target reliabilities that satisfy the acceptance for the probability of system failure and that minimizes the expected total cost.

Although in this example, the exposure to the ship hull structure, e.g. wave load, is not directly considered and thus the failures of the individual tanks are assumed to be independent, it is possible to take into account the exposures which may introduce the correlation between the failures of the components and/or sub-systems by adding the node for the exposure scenario in the ID as is found in the previous example.

6. Conclusions

The present paper proposes a framework for the modeling and the optimization of reliabilities for components in complex engineered systems subject to requirements specified in terms of system performance. It is shown how the identification of the target component reliabilities that are optimal and consistent with given acceptance criteria for system performance can be treated as an optimization problem with constraints. Appreciating the perspective that engineered systems are built up by standardized components which through their connections with other components provide the desired functionality and that the system performance will depend on the way the components are interconnected, the proposed framework takes basis in a hierarchical system modelling facilitated by use of (object-oriented) BPNs and IDs. Using the established BPNs and IDs it is possible to calculate the objective function such as service life utility, and the quantities for which the acceptance criteria are given, both of which are required for solving the optimization problems with constraints. Two examples are shown: (1) optimization of the design of bridges in a transportation network subject to earthquake hazards, and (2) optimization of target reliabilities of welded joints in a ship hull structure subject to fatigue deterioration in the context of maintenance planning. The first example serves as the introduction how the proposed approach is implemented step by step. The second example illustrates how complex engineered system may be modelled and how the target component reliabilities may be optimized using commonly available software tools.

References

- [1] Faber MH, Maes MA, Baker JW, Vrouwenvelder T, Takada T. Principles of risk assessment of engineered systems. In: Tenth international conference on applications of statistics and probability in civil engineering. The University of Tokyo, Kashiwa Campus, Japan; 2007.

- [2] Vesely WE, Goldberg FF, Roberts NH, Haasl DF. Fault tree handbook (NUREG-0492); 1981.
- [3] Bobbio A, Ciancamerla E, Franceschinis G, Gaeta R, Minichino M, Portinale L. Sequential application of heterogeneous models for the safety analysis of a control system: a case study. *Reliab Eng Syst Safety* 2003;81(3):269–80.
- [4] USNRC. Reactor safety study – an assessment of accident risks in US commercial nuclear power plants, WASH-1400 (NUREG-75/014); 1975.
- [5] USNRC. Severe accident risks: an assessment for five US nuclear power plants (NUREG-1150); 1990.
- [6] Bobbio A, Portinale L, Minichino M, Ciancamerla E. Improving the analysis of dependable systems by mapping fault trees into Bayesian networks. *Reliab Eng Syst Safety* 2001;71(3):249–60.
- [7] Baker JW, Schubert M, Faber MH. On the assessment of robustness. *Struct Safety* 30(3):253–67.
- [8] Volovoi V. Modeling of system reliability Petri nets with aging tokens. *Reliab Eng Syst Safety* 2004;84(2):149–61.
- [9] Kjaerulf U. dHugin: a computational system for dynamic time-sliced Bayesian networks. *Int J Forecast* 1995.
- [10] Smyth P. Belief networks, hidden Markov models, and Markov random fields: a unifying view. *Pattern Recognit Lett* 1997;18(11–13):1261–8.
- [11] DerKiureghian A, Song J. Multi-scale reliability analysis and updating of complex systems by use of linear programming. *Reliab Eng Syst Safety* 2008;93(2):288–97.
- [12] Raudenbush S, Bryk AS. A hierarchical model for studying school effects. *Sociol Educ* 1986;59(1):1–17.
- [13] Lindley DV, Smith AFM. Bayes estimates for the linear model. *J Royal Stat Soc Ser B* 1972;34(1):1–41.
- [14] Clark JS, Gelfand AE. Hierarchical modelling for the environmental sciences. New York: Oxford University Press; 2006.
- [15] Li F-F, Pietro P. A Bayesian hierarchical model for learning natural scene categories. In: IEEE conference on computer vision and pattern recognition, San Diego, USA; 2005.
- [16] George D, Hawkins J. A hierarchical Bayesian model of invariant pattern recognition in the visual cortex. *Neural Networks* 2005;3:1812–7.
- [17] Johnson VE, Graves TL, Hamada M, Reese CS. A hierarchical model for estimating the reliability of complex systems. *Bayesian Stat* 2002;7:199–214.
- [18] Straub D. Generic approaches to risk based inspection planning for steel structures. Department of Civil, Environmental and Geomatic Engineering; 2004.
- [19] ASCE7-98. Minimum design loads for buildings and other structures. Revision of ANSI/ASCE; 2000. p. 7–95.
- [20] JCSS. Probabilistic model code; 2001.
- [21] Nathwani JS, Lind NC, Pandey MD. Affordable safety by choice: the life quality method. Waterloo: University of Waterloo; 1997.
- [22] Rackwitz R. Optimization and risk acceptability based on the life quality index. *Struct Safety* 2002;24(2–4):297–332.
- [23] Royset JO, Kiureghian AD, Polak E. Reliability-based optimal design: problem formulations, algorithms and application. In: Proceedings of the 11th IFIP WG7.5 working conference on reliability and optimization of structural systems, Banff, Canada; 2003.
- [24] Guikema SD, Pate-Cornell ME. Component choice for managing risk in engineered systems with generalized risk/cost functions. *Reliab Eng Syst Safety* 2002;78(3):227–38.
- [25] Salazar D, Rocco CM, Galvan BJ. Optimization of constrained multiple-objective reliability problems using evolutionary algorithms. *Reliab Eng Syst Safety* 2006;91(9):1057–70.
- [26] Jensen FV. Bayesian networks and decision graphs. New York: Springer; 2001.
- [27] Korb KB, Nicholson AE. Bayesian artificial intelligence. Chapman & Hall/CRC; 2004.
- [28] Bangso O, Flores MJ, Jensen FV. Plug & play OOBns. Lecture notes in artificial intelligence 2003;3040:457–67.
- [29] Bangso O, Olesen KG. Applying object oriented Bayesian networks to large (medical) decision support systems. In: Proceedings of 8th scandinavian conference on artificial intelligence, SCAI'03, Bergen, Norway; 2003.
- [30] Press WH, Flannery BP, Teukolsky SA, Vetterling WT. Numerical recipes in C. Cambridge University Press; 1988.
- [31] Straub D, Faber MH. Computational aspects of risk-based inspection planning. *Comput Aided Civil Infrastruct Eng* 2006;21(3):179–92.