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FURTHER APPLICATIONS OF FUZZY LOGIC TO RELIABILITY ASSESSMENT AND SAFETY ANALYSIS

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ABSTRACT

A methodology is presented for situations which are by nature imprecise and where quantitative data is either not available or appropriate. The concept of possibility theory and fuzzy logic are presented and the methodology is illustrated by applications to reliability and risk assessment. It is shown that fuzzy logic is a potential tool for creating expert bases.

INTRODUCTION

In many scientific areas; such as operations research, management and decision analysis, a system is modelled using "soft" data which is often inaccurate, inexact and sometimes incomplete. Scientists have, until now, tended to formulate mathematical models using the rigid conceptual mould of Boolean logic. However, limitations and inadequacies of Classical methods are increasingly becoming obvious. In this paper, concepts of possibility theory and fuzzy logic are explored and applied to problems of reliability and safety engineering.

FUZZY SET THEORY

In Classical set theory, a set is defined by an indicator function, which for any element of the set is either 0 or 1. A fuzzy subset[2] A of a universe of discourse $U=\{x\}$ is defined as a mapping by which x is assigned a number in [0,1] indicating the extent to which x belongs to A. Given fuzzy sets A and B of U, the basic operations are:

a) $\bar{A}$: Complement of A defined by:

$$u_{\bar{A}}(x)=1-u_A(x)$$

b) $C=A\cap B$: Conjunction of A and B defined by:

$$u_C(x)=\min(u_A(x),u_B(x))$$

c) $D=A\cup B$: Disjunction of A and B defined by:

$$u_D(x)=\max(u_A(x),u_B(x))$$

d) R : A fuzzy relation from $U=\{x\}$ and $V=\{y\}$ is a fuzzy set of the Cartesian product U.V characterised by $u_R(x,y)$, by which each pair (x,y) is assigned a number in [0,1].

e) min-max : Composition. Given a fuzzy relation from U to V and a fuzzy subset of A of U, a fuzzy subset B of V is obtained by the compositional rule of inference:

$$B=AoR \text{ and } u_B(y)=\underset{x}{\mathrm{Max}}\{\min(u_A(x),u_R(x,y))\}$$

This operation is very similar to matrix multiplication.

These basic operations in fuzzy set theory together with practical applications to risk and reliability are illustrated in reference 3.

APPLICATION OF POSSIBILITY THEORY TO RELIABILITY ASSESSMENT

It is essential to note that the concepts of probability and possibility are not equivalent. The fundamental distinction between the two concepts is that events with a high possibility do not necessarily have a high probability. As stated by Zadeh[4]: "What is possible may not be probable and what is improbable need not be impossible".

Probability is the degree of likelihood and belief based on frequency and proportion. On the other hand, possibility is the degree of feasibility or ease of attainment[4]. In practice and in every day semantics, possibility is used rather than probability particularly when the system is still at design stage and very little is known about its components. Kara-Zaitri and Keller[5] have shown the use of possibility concepts in fuzzy logic reliability applications.

In many cases, it is convenient to express the membership function of the possibility distribution in terms of a standard function in an approximate fashion. A large number of different types of functions[1,6] can be found in the literature for this purpose. In this paper, π[7] functions are utilised for describing the possibility of failure of components and systems. The parameters of the π function are:

s : Scale
ls : Left shape
rs : Right shape

$$\pi((x,s,ls,rs) = 0 \qquad x\varepsilon[0,s-2.ls]$$
$$= 2.\frac{(x-s+2.ls)^2}{(2.ls)^2} \qquad x\varepsilon[s-2.ls,s-ls]$$
$$= 1-2.\frac{(x-s)^2}{(2.ls)^2} \qquad x\varepsilon[s-ls,s]$$
$$= 1-2.\frac{(x-s)^2}{(2.rs)^2} \qquad x\varepsilon[s,s+rs]$$
$$= 2.\frac{(x-s-2.rs)^2}{(2.rs)^2} \qquad x\varepsilon[s+rs,s+2rs]$$
$$= 0 \qquad x\varepsilon[s+2.rs,1]$$

When x=ls or x=rs, $\pi(x,s,ls,rs)=.5$ (Cross-over point).

Suppose that the possibility of failure, Pt, of the standard parallel system (Pa,Pb) is required. Imagine that Pa and Pb are defined by:

$$Pa=\pi(x,sa,lsa,rsa)=\pi(x,.4,.1,.05)$$
$$Pb=\pi(x,sb,lsb,rsb)=\pi(y,.2,.05,.075)$$

Hence, Pt is defined by:

$$Pt=\max\{\min(Pa,Pt/Pa)\} \text{ so that } Pa.(Pt/Pa)=Pt$$

Pt (Exact) is obtained using an especially written computer program (See fig.1). The time taken with this program was excessive and accordingly Dubois and Prade's approach[8] was utilised.

$$\forall x\varepsilon[sa-lsa,sa],\forall y\varepsilon[sb-lsb,sb],\exists w\varepsilon[0,1];w=Pa(x)=Pb(y) \text{ or}$$
$$x=sa+2.lsa\sqrt{(w-1)/-2} \text{ and } y=sb+2.lsb\sqrt{(w-1)/-2}$$
$$\text{but } z=x.y=sa.sb+\sqrt{(w-1)/-2}(sa.2.lsb+sb.2.lsa)-2.lsa.lsb.(w-1)$$

This is a second order equation in w and consequently it cannot be expressed in terms of a π function. However if lsa.lsb is very small compared with sa and sb, then;

$$w = 1-2.\frac{(z-sa.sb)^2}{(2.(sa.lsb+sb.lsa))^2}$$

Therefore;

$$\pi(x,sa,lsa,rsa).\pi(y,sb,lsb,rsb)\approx\pi(z,sa.sb,sa.lsb+sb.lsa,sa.rsb+sb.rsa)$$
and hence $Pt=\pi(z,.08,.04,.04)$
Pt(Approximate) is also plotted in Fig.1.

A similar approach can be utilised for the determination of the membership function of the possibility of failure of a series system.

For Example, $Pa=\pi(x,.4,.1,.05)$
$Pb=\pi(y,.2,.05..1)$

Hence, Pt is defined by:

$$Pt=\max\{\min(Pa,(Pt-Pa)/(1-Pa)\}$$
$$Pt=\pi(z,.52,.11,.10)$$

Both approximate and exact resulting possibility distributions are shown in Fig.2.

APPLICATION OF FUZZY CONTROL TO SAFETY STUDIES

Nowadays, almost every system has instrumentation[9] whose unique role is to protect the system and people involved from the occurrence of undesired events. It is understood that as long as the protective system is working, these events cannot occur. Sprinklers, pop-off valves are examples of some protective systems.

For example, consider an Automatic Fire Detection System (AFDS). The latter consists of two major parts namely fire detectors and control panels. In classical AFDS systems, control panels either give an alarm or do not give an alarm. Intermediate states of the control system such as situations where there is a deviation from normal which is not significant enough to give a full alarm are not usually accounted for. Studies in the U.K. have shown that the ratio of false/unwanted and real alarms was far too large. In such circumstances the concept of fuzzy logic and particularly that of fuzzy implications can be used as a tool to reduce the number of unwanted alarms and hence increase the reliability of the system but still maintain a high degree of safety.

A one dimensional fuzzy implication[9,10] has the following general form:

"if A then B". A and B are fuzzy sets in the Universes U_A and U_B. The membership function of this simple implication is given by:

$$u_{AxB}(x,y)=\min(u_A(x),u_B(y))$$

Given "If A then B" and A' (another fuzzy set of U), the corresponding B' can be computed by the min-max composition.

$$u_{B'}(y)=\max\{\min(u_{A'}(x),u_{AxB}(x,y))\}$$

In order to use this concept in AFDS, the B's are 'crisp' rather than fuzzy. Imagine that the set of required safety actions is defined by:

A={A1,A2,A3}
and A1 : No action required
A2 : Prewarning (Warning light in control panel)
A3 : Full alarm (Fire brigade must attend)

Suppose that heuristic considerations resulted in the following three conditions only:

1. If smoke density is low then Action 1
2. If smoke density is medium then Action 2
3. If smoke density is high then Action 3

Linguistic variables describing smoke densities and safety actions are defined by the following fuzzy sets;

	Units of smoke density				
	1	2	3	4	5
Low	1	.8	.6	.4	.2
Medium	.6	.8	1	.8	.6
High	.2	.4	.6	.8	1

	Safety actions		
	A1	A2	A3
Action 1	1	0	0
Action 2	0	1	0
Action 3	0	0	1

The three conditional statements represent the expert base knowledge from which the fuzzy algorithm S is obtained. In this case;

S = (Low x Action 1) U (Medium x Action 2) U (High x Action 3)

or S = S1 U S2 U S3

$$S1 = \begin{bmatrix} 1 & 0 & 0 \\ .8 & 0 & 0 \\ .6 & 0 & 0 \\ .4 & 0 & 0 \\ .2 & 0 & 0 \end{bmatrix} \quad S2 = \begin{bmatrix} 0 & .6 & 0 \\ 0 & .8 & 0 \\ 0 & 1 & 0 \\ 0 & .8 & 0 \\ 0 & .6 & 0 \end{bmatrix} \quad S3 = \begin{bmatrix} 0 & 0 & .2 \\ 0 & 0 & .4 \\ 0 & 0 & .6 \\ 0 & 0 & .8 \\ 0 & 0 & 1 \end{bmatrix}$$

and hence,

$$S = \begin{bmatrix} 1 & .6 & .2 \\ .8 & .8 & .4 \\ .6 & 1 & .6 \\ .4 & .8 & .8 \\ .2 & .6 & 1 \end{bmatrix}$$

The control action for a given smoke density is determined according to the compositional rule. The procedure for choosing the required control action is determined by the truth value methodology[5].

To evaluate the proposed procedure, a set of three smoke densities is tested against the expert knowledge of the system. The three densities are:

Test No.	Description	1	2	3	4	5
1	Low	1	.8	.6	.4	.2
2	Between low and medium	.8	1	.8	.6	.4
3	More medium than low	.6	.7	.9	.5	.3

The results of the computation are as follows:

Action	Test 1	Test 2	Test 3
Action 1	1	.8	.7
Action 2	.8	.8	.9
Action 3	.6	.6	.6

The results are interpreted so that the safety action that has the highest grade of membership is to be carried out. Note that, in test 2, because the smoke density is between low and medium ('half way'), the algorithm S resulted in either action 1 (no action or action 2 (warning). From this, it is clear that subjective assessment of the smoke density alone is not sufficient and that additional variables have to be included. For example, the rate of change of smoke density may be of interest. Consequently, the conditions may be:

If smoke density is medium AND rate of change of density is low then no action OR
if smoke density is medium AND rate of change of density is high then warning.

The intelligence interface of the expert system decides in exactly the same way as the one dimensional and one variate case.

APPLICATION TO FAILURE MODE AND EFFECT ANALYSIS

The methodology described earlier can also be implemented to model failure mode and effect analysis. FMEA, by definition, is a systematic approach to system fault diagnosis, which is capable of producing a comprehensive model or understanding of system fault behaviour. Each item is considered in relation to its likely modes of failure and the effects of failure. In brief, FMEA is:

Cause $\longrightarrow$ Effect

This can be translated into the following model;

if C_{11} and C_{12} and ...... C_{1n} then E_1
if C_{21} and C_{22} and ...... C_{2n} then E_2
. . . .
if C_{m1} and C_{m2} and ...... C_{mn} then E_m

Where C_{ij} represents a fuzzy cause of the ith conditional statement and the jth variable. E_i represents the corresponding ith effect (n variables and m conditions).

Given a set of causes, the unknown effect can be computed using the min-max method in exactly the same way as the fuzzy controller example. This approach is very useful because it can predict, based on the expert base system, the effect of combination of certain causes that have not occurred before. Furthermore, a separate simulation program can be used with the expert system to create perhaps fictitious causes and compute corresponding effects. This way, many hazards and accidents, that have not been accounted for during the study can be highlighted and subsequently analysed in great detail.

In addition, the approach in FMEA can be reversed; i.e.

Effect - infer - Cause

Here again, fuzzy logic can be applied as a diagnostic tool using a subjective expert base. For example, the relationship between symptoms in the process variables and the corresponding hazards can be expressed as a set of conditional statements such as:

If S_{ij} and S_{ij+1} and ...... then H_i

Where S_{ij} is a fuzzy symptom of the ith conditional statement and the jth variable. The diagnosis of an unknown hazard causing the set of symptoms S is achieved by computing the membership function for that hazard.

CONCLUSIONS

The concept of possibility has a close relationship with every day semantics and is therefore of potential value in supplying information and interpreting results of reliability and safety assessments.

Adding and multiplying fuzzy possibilities of failure expressed as functions requires a moderate extra amount of computation compared with classical methods.

The concept of fuzzy logic controllers to increase system reliability is significant because it handles heuristic rules in a straight forward manner.

Since the fundamental operation (min-max operation) in fuzzy logic is relatively simple, it can be readily implemented on computers for the study of complex systems.

Fuzzy logic could progressively be included in safety studies to guarantee that protective systems would only work when necessary. Unwanted or undesired events could then be kept at a minimum.

The fuzzy approach to failure mode and effect analysis is convenient because knowledge about causes and effects of failures is usually described with a large uncertainty content.

Fuzzy logic is a valuable tool for creating expert bases. These, in turn, can be utilised as a diagnostic means by which hazardous events can be identified.

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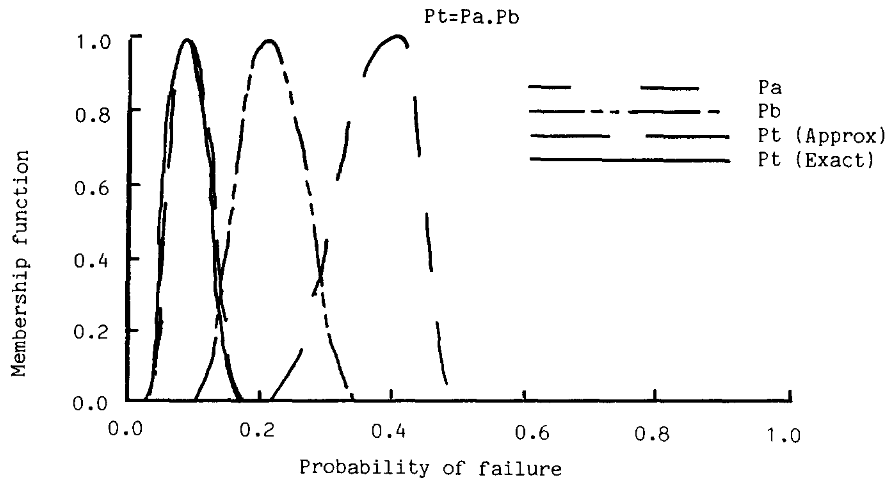

Fig. 1. Representation of Pa.Pb

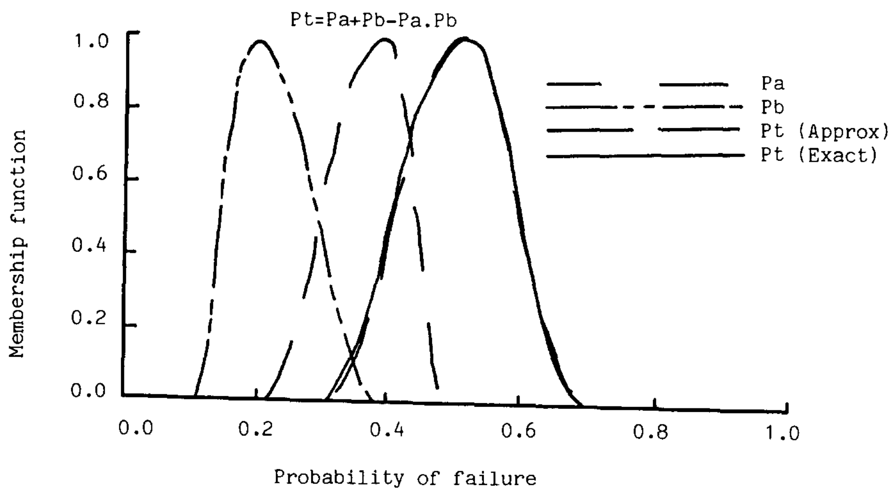

Fig. 2. Representation of Pa+Pb-Pa.Pb