Integrating QRA and SRA Methods Within a Bayesian Framework When Calculating Risk in Marine Operations: Two Examples

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This paper concerns itself with the integration of QRA (quantitative risk analysis) and SRA (structural reliability analysis) methods. For simplicity, we will use the term SRA instead of SRA methods in the paper. The Bayesian (subjective) approach seems to be the most appropriate framework for such integrated analyses. It may, however, not be clear to all what the Bayesian approach really means. There exists alternative Bayesian approaches, and the integration of SRA and QRA is very much dependent on what the basis is. The purpose of this paper is to present two marine operation examples, implementing two different Bayesian approaches: the ''classical Bayesian approach'' and the ''fully Bayesian approach.'' Following the classical Bayesian approach, we estimate a true, objective risk, whereas in the fully Bayesian approach, risk is a way of expressing uncertainty about future observable quantities. In both examples, one initial accidental event is investigated by using a fault tree and by integrating SRA into this fault tree. We conclude that the most suitable framework for integrating SRA and QRA is to adopt the ''fully Bayesian approach.'' [S0892-7219(00)00703-2]

Introduction

In a QRA, risk is quantified in an absolute sense or a relative sense, often in relation to some kind of risk acceptance criteria. The analysis identifies critical activities and systems, and predicts the effect of implementing risk reducing measures. Conducting a QRA also gives us understanding of hazards causation and potential escalation pathways. The purpose of the analysis is to provide a basis for making decisions concerning choice of arrangements and measures. Such decisions could be specified as, for example, change to the installation procedure for a marine structure. It is normal to distinguish between risks threatening human lives and health, the environment, and assets and financial interests.

As to the probability quantification, it seems that most risk analyses being conducted in the offshore petroleum industry today are based on the classical approach, in the sense that the risk analysts see the analyses as a tool for producing estimates of true, unobservable quantities such as probabilities and expected values. A probability is then interpreted in the classical statistical sense as the relative fraction of times the events occur if the situation analyzed were hypothetically ''repeated'' an infinite number of times. The parameters of the models (such as the probabilities of the basic events in the fault trees and the branching probabilities in events trees) are, however, not estimated purely by means of ''hard data.'' In practice, these parameters are estimated by integrating hard data and expert opinions. This integration is usually carried out without using a well-structured procedure. But the interpretation of probabilities is classical—there exists a true (unobservable) risk, and by using risk analyses, we generate estimates of this true risk.

It is often a requirement that the risk analyses should say something about the uncertainty of the estimates. To measure the total gap between the true value and the value obtained in the QRA will be difficult. The classical approach gives an uncertainty measure that only takes a small part (statistical variation) of the uncertainty in the analysis into account.

Lack of information, especially statistics and other experience data, is common when performing a QRA for marine operations. This is mainly due to the uniqueness of the operations. For example, when towing a marine structure or lifting offshore, there will always be some new elements included compared to previous tows or lifts. During a tow, the towing route and the weight of the structure may differ, the draft or the type of tugs and the configuration of the tugs, the weather, and, furthermore, the organization and technical support will normally also vary. Moreover, the installation of a marine structure will vary due to, for example, different locations with different water depths, soil conditions, and weather conditions.

Therefore, carrying out a QRA for a marine operation without including subjective elements is impossible and would not be in the interest of achieving useful results. The analyst should put great efforts in utilizing the information that is available by using experts when assigning probabilities and calculating risk. In this respect, the Bayesian approach [1,2] is considered attractive since it does not break down in the absence of experience data and allows a systematic integration of expert opinions, scientific intuition, and experience data in the analyst's efforts to assign probabilities. Further, it is relatively simple to consistently modify the probabilities of failure when new data become available.

Basically, there are two ways of thinking within the Bayesian context; we refer to these as the ''classical Bayesian approach'' and the ''fully Bayesian approach.'' The classical Bayesian approach is also referred to as a ''combined classical Bayesian approach,'' as it is a combination of the classical statistical approach and the Bayesian approach. For simplicity, we will use the term ''classical Bayesian'' instead of ''combined classical Bayesian.''

In the following we present two integrated approaches for SRA and QRA based on the classical Bayesian approach and the fully Bayesian approach, respectively. Approach 1 integrates SRA and

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QRA within the classical Bayesian framework, whereas Approach 2 integrates SRA and QRA within the fully Bayesian approach [3,4]. In the examples, the software Proban [5] has been used to do this integration. It was not straightforward to use Proban when calculating results within the framework of Approach 1, and therefore some adjustments to the software had to be made.

By applying SRA on QRA, the analyst is able to model the physical system more precisely, handling the uncertainties and parameter correlation separately and systematically. This is ensured by flexible event and system modeling, by logical combination of limit state functions, by uncertainty modeling, and by assigning marginal probability distributions and correlation measures. These properties might enable the analyst to include more knowledge in the analysis than is the case with models traditionally applied in QRA.

Integrating QRA and SRA

SRA [6] is a tool for calculating probabilities of failure of structural systems. Thus, SRA as used here is on the same level as other reliability models, such as lifetime models for mechanical and electronic equipment, reliability models for software, and availability models for supply systems. All models of this kind can be used to calculate single probabilities that are inputs in different methods used in QRA, such as for the basic events in fault tree analysis (FTA) and the branching points in event tree analysis (ETA). A special feature of SRA is, however, that the influence from several random variables and failure modes may be taken into account in a single analysis. Thus, using SRA, the splitting of events into detailed subevents is often not necessary to the same extent as in traditional QRA models, like FTA and ETA. This makes it possible for a whole section of a fault or event tree to be replaced by a single analysis based on SRA. The use of continuous variables is, however, common in SRA, and the ability to treat continuous variables is considered to be one of the main attractions of this technique.

To make use of SRA, the occurrence of at least one event of the system considered must be fully dependent on the outcome of a set of random variables, the basic variables $\boldsymbol{X}=(X_1,X_2,\ldots)$. Further, it must be possible to describe the conditions under which the event will happen, the event space, using one or several limit state functions, i.e. $g_i(\boldsymbol{X})\leq 0$, logically connected by unions and intersections. So, given a limit state function $g(\mathbf{X})$ and a joint distribution function $F_x(\mathbf{x})$ for the random vector $\mathbf{X}$, the probability of failure p can be calculated.

Integrating SRA and QRA requires the establishment of a unified stochastic framework, to treat uncertainties consistently and obtain useful results for decision making. Looking at the alternative probabilistic approaches, it is not obvious how to formulate such a framework. The classical statistical approach to risk analysis is not considered suitable, since there are not sufficient ''hard'' data available to accurately estimate the unknown parameters of the models.

The Bayesian approach is in our opinion the most suitable basis for integrated QRA and SRA. It is necessary to include whatever relevant information is available, and the Bayesian approach provides a consistent tool for combining ''hard data'' and subjective information like expert opinions and engineering judgments.

Classical Bayesian Approach and Integration of SRA (Approach 1)

In quantitative risk analyses we are interested in probabilities of accidental events. For this purpose a model is developed, for example a fault tree, with the basic event probabilities as parameters. In this approach we focus on unobservable quantities, like the true probability p of an accidental event A. Using a fault tree we can establish a model, w, linking p and some parameters $\boldsymbol{q}=(q_1,q_2,\ldots q_n)$ on a lower system level; i.e.

$$p=w(\boldsymbol{q})$$

This model shows the functional relationship between *unobservable* parameters on the basic event level and on the top event level, and the true model w produces the true value of p when the input $\boldsymbol{q}$ is true. According to the classical Bayesian approach, the values of p, $\boldsymbol{q}$, and w are uncertain (unobservable and unknown) and in Approach 1 we use probability distributions to express this uncertainty, i.e., our uncertainty about the true values are expressed as probability distributions. To establish the distributions we start with a priori information (i.e., initial uncertainty) I about $\boldsymbol{q}$, including engineering judgment, that exists before any data are observed. The priori information is then used to establish a priori probability distribution, $H_0(\boldsymbol{q}|I)$, which reflects our initial knowledge concerning the parameter $\mathbf{q}$. If we observe experience data D, we derive a posteriori distribution, $H(\mathbf{q}|I,D)$, (using Bayes theorem), which expresses the updated knowledge of the parameter $\mathbf{q}$ after new data have been observed. From this uncertainty distribution, we obtain an uncertainty distribution, H, for p. Only the entire distribution is a complete measure of our knowledge and H includes both epistemic and stochastic uncertainty. The uncertainty analysis is very often done by using Monte Carlo simulation, a technique that is applied in many risk analysis codes. Mathematically, we can write

$$H_0(p')=P(``p\leq p''')=\int_{\mathbf{q}:w(\mathbf{q})\leq p'}dH(\mathbf{q})$$

where H is either the priori or the posteriori distribution of $\boldsymbol{q}$. Normally, in QRAs we use the best estimate when presenting risk. The best estimate is typically the mean value of the uncertainty distribution, and in Approach 1 we therefore add more information to the risk result by expressing the risk by the entire uncertainty distribution.

Now, to integrate SRA methods in the classical Bayesian setting, consider, for example, a case where one of the q_i's is obtained by SRA, say q_1. Then, we have

$$q_1=P(g_1(\mathbf{X})\leq 0)$$

for a limit state function g_1 and basic variables $\mathbf{X}$. Denoting by F the distribution function of $\mathbf{X}$, we can write

$$q_1=\int_{\{\mathbf{x}:g_1(\mathbf{x})\leq 0\}}dF(\mathbf{x})$$

Assuming the existence of a theoretical, true (but unknown) distribution function F and limit state function g_1, there will also be a true (unknown) value of q_1. Our (the analyst's) uncertainty related to the distribution F and the limit state function g_1 generates the uncertainty distribution on q_1. Assume that g_1 is known and that our uncertainty related to F is restricted to specifying a parameter θ (θ may be a vector); thus, $F(\mathbf{x})=F(\mathbf{x}|\theta)$.

There exists a true, but unknown, value of θ. We write $q_1(\theta)$ and P_θ to show the dependency on the parameter θ. Hence

$$q_1(\theta)=P_\theta(g_1(\mathbf{X})\leq 0)=\int_{\{\mathbf{x}:g_1(\mathbf{x})\leq 0\}}dF(\mathbf{x}|\theta) \qquad (1)$$

From this, an uncertainty distribution for q_1 can be established based on SRA.

Fully Bayesian Approach and Integration of SRA (Approach 2)

The alternative to the classical Bayesian approach is the fully Bayesian approach, which is characterized by focusing on observable quantities, like the occurrence or not of an accidental event, the number of accidental events in a given period of time, or lost production in a period of time. Subjective probabilities are used to express the uncertainties of these quantities. The uncertainties involved are therefore related to whether *the events will occur or not*. The risk result, for example, $P(A)$ is a *total* measure of uncertainty and the probability is used to express our uncertainty related to the accidental event A.

The analyses will provide the probabilities of the uncertain events that are relevant in the specific situation of decision making. Since $P(A)$ expresses our degree of belief (experts' belief), a discussion of the uncertainty of $P(A)$ is regarded irrelevant.

Using various risk analysis models, a functional relationship ν between the occurrence of the event A and the events $\boldsymbol{B} = (B_1, B_2, \ldots)$ on a more detailed level is established, i.e.

$$A = \nu(\boldsymbol{B})$$

The analyst's uncertainty regarding the occurrence of an event B_i is expressed by the subjective probability $P(B_i)$. Using the relationship ν and probability calculus, we can compute the probability $P(A)$ expressing the uncertainty related to whether the event A will occur or not. In most cases, this gives $P(A) = \nu(P(\boldsymbol{B})) = \nu(\boldsymbol{q})$, where $P(\boldsymbol{B}) = \boldsymbol{q}$, $P(\boldsymbol{B}) = (P(B_1), P(B_2), \ldots P(B_n))$.

To illustrate the model, we can use two basic events B_1 and B_2 such that the occurrence of the event A is connected to B_1 and B_2 by an AND-gate. Then

$$P(A) = P(B_1)P(B_2|B_1)$$

where $P(B_2|B_1)$ denotes the conditional probability when it is known that B_1 has occurred. In the fully Bayesian setting, the events B_1 and B_2 are independent if the knowledge of the outcome of B_1 does not make the analyst's change the degree of belief concerning the occurrence of B_2.

The probability $P(A)$ can also be determined by use of a parametric model to quantify the uncertainty whether event A will occur or not, for example, an exponential life time model. Let λ be the model parameter (e.g., the failure rate in the exponential model). Then, by the Bayesian approach and the law of total probability, we can calculate $P(A)$ by

$$P(A) = \int P(A|\lambda)\, dH(\lambda)$$

where $P(A|\lambda)$ denotes the conditional probability of A given λ, and $H(\lambda)$ is a distribution function of λ, apriori or posteriori depending on the availability of experience data. Denoting

$$q_i(\lambda) = P(B_i|\lambda) \quad \text{and} \quad \mathbf{q}(\lambda) = (q_1(\lambda), q_2(\lambda), \ldots)$$

we would usually have

$$P(A|\lambda) = \nu(\mathbf{q}(\lambda))$$

and hence

$$P(A) = \int \nu(\mathbf{q}(\lambda))\, dH(\lambda) \tag{2}$$

The total probability $P(A)$ consists of two elements: $P(\mathbf{B}|\lambda)$ and $H(\lambda)$. Additional information will change $P(A)$ only through its impact on $H(\lambda)$, i.e., $H(\lambda)$ is updated in accordance with Bayes formula, [2].

Now, how should we interpret $H(\lambda)$ and $P(\mathbf{B}|\lambda)$? Does the use of the distribution H mean that we believe in a true value of λ. No, H gives weights to the different λ values according to the confidence we have in the different values (for predicting observable quantities); there exist no true values. Is it consistent with the fully Bayesian approach to assume a true value of λ? No, because, if we believe in a true value of λ, we should also believe in a true value of $P(\mathbf{B})$, and consequently in a true value of $P(A)$. But that is not possible in a full Bayesian setting where $P(A)$ is a total measure of uncertainty.

In the fully Bayesian setting all probabilities are quantifying epistemic uncertainties. The probabilities $P(B_i|\lambda)$ and $P(A|\lambda)$ (when λ varies) represent alternative ''models'' (mathematical expressions) which we consider suitable for expressing our degree of belief concerning the occurrence of B_i and A, respectively. It is a way of standardizing the probability considerations. It is not essential that the parameter λ has a physical interpretation; allowing different values of λ is simply a way of generating a class of appropriate uncertainty distributions for B_i and A.

In this approach the meaning of uncertainty is completely different from uncertainty in the classical Bayesian approach. What is uncertain is the occurrence of the event A, and the probability $P(A)$ expresses this uncertainty. The fact that there could be faults and weaknesses in the model used does not change this interpretation of $P(A)$. There is no sense in speaking about uncertainty of the probability $P(A)$, because such a reasoning would presuppose the existence of a true value of $P(A)$.

When incorporating SRA within the framework of Approach 2 $q_1 = P(g_1(\mathbf{X}) \le 0)$ is a measure of uncertainty, a degree of belief, concerning the occurrence of the event ''$g_1(\mathbf{X}) \le 0$.'' The values of the quantities $\mathbf{X}$ are uncertain (unknown) and the uncertainty is expressed by the subjective probability distribution F, giving

$$q_1 = \int_{\{\mathbf{x}: g_1(\mathbf{x}) \le 0\}} dF(\mathbf{x}) \tag{3}$$

If we consider alternative models $F(\mathbf{x}|\theta)$, we obtain $P(A)$ using Eq. (2) with

$$q_1(\theta) = \int_{\{\mathbf{x}: g_1(\mathbf{x}) \le 0\}} dF(\mathbf{x}|\theta)$$

If SRA replaces more than one of the q_i's, we can proceed along the same lines.

The Two Examples

General Introduction. The objective of this section is to implement the two approaches on one relevant operation in an offshore project [4]. The sub-phase studied here is the towing of a production facility, from a construction site to the offshore field. The production facility could be a production ship, a barge, a concrete or steel structure. The study involves a cause analysis of a navigation failure during towing utilizing both approaches.

A navigation failure is a deviation of the tow of the production facility from its intended towing route. This could, for example, be due to loss of the ability of tugs to hold the production facility, tugs being out of course caused by a failure of the navigation equipment, an error in interpreting navigation data, an error in communication between the tow master controlling the tow and the tug captains, a failure of the tow master to correctly coordinate the actions of the tugs, or a towline failure causing incorrect commands. The navigation failure includes both mechanical failure and human errors.

If the structure drifts out of control, the consequences may be minor or major structural damages to the production facility. A navigation error usually has more severe consequences inshore than offshore. Following a navigation failure, recovery could be obtained by getting signals about the wrong position from the monitors plotting the direction or from any of the operators. If the tow coordinator observes the signals and takes corrective actions, recovery may be obtained.

In the following two examples, we concentrate on the calculation of the probability of navigation failure, $P(\mathrm{NF})$, utilizing Approaches 1 and 2. The fault tree in Fig. 1 illustrates the system considered, and the tree has been broken down to a level where the probabilities of the basic events can more easily be quantified. The reliability block diagram corresponding to the fault tree gives a serial system with four components (minimal cut sets). Component i has the reliability

$$1 - q_i$$

where q_i is the probability of failure of component i.

Assuming independence between the events, the probability of the top event, NF, is then given by

$$p = P(\mathrm{NF}) = 1 - \prod_{i=1}^{n} (1 - q_i), \quad \text{where } n = 4 \tag{4}$$

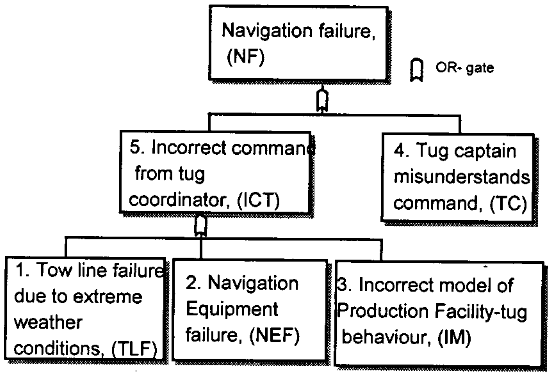

Fig. 1 Fault tree, navigation failure

If all q_i's are small, then an approximation to this formula is given by $P(\mathrm{NF}) \approx \Sigma q_i$.

A description of the base events in the fault tree and the assumptions related to each base event are briefly summarized in Table 1.

Table 1 Assumptions and values of the base events of the fault tree in Fig. 1 (the data are mainly given by experts)

Basic event	Description of event/Assumptions	Value	
		Approach 1	Approach 2
1. Tow line failure due to bad weather TLF	Towline from tug to PF fails because weather exceeds the criteria. Criteria are given by H_s	X_1= Capacity, normally distributed, uncertainty distributions over parameters X_2= load on towline, Weibull distributed	g-function X_1= normally distributed, with parameters(5.5, 1.5) X_2= Weibull distributed (2.269, 1.241)
2. Navigation equipment failure, NEF	Equipment gives no reading or the readings are wrong, but not identified Duration of tow about 20 hours -	Triangle distr.. $10^{-4} < q_2 < 5\times10^{-3}$	5×10^{-4} per tow
3. Incorrect model of platform-tug behaviour, IM	Tug co-ordinator gives incorrect command to tug captains due to incorrect mental model of the tug-platform system Tug-PF system is complex and feedback of effects of tow line failure is delayed. Knowledge based task. There is few procedure	fixed value: q_3 = 5×10^{-3}	q_3 = 5×10^{-3}
4. Tug captain mis-understand command, TC	Misunderstands a correct command and hence makes manoeuvring errors Skilled bases task Unlikely error if captain is experienced	Uniform distributed; $10^{-3}<q_4<10^{-5}$	5×10^{-3}

To utilize the two approaches when calculating $P(\mathrm{NF})$, the occurrence of at least one event must be described using one or several limit state functions. From the fault tree we identify two continuous random variables, X_1 and X_2 for the event TLF, each expressed by their density functions, $fX_i(x_i)$. Given a limit state function $g(\mathbf{X})$ and using the joint density function $f_X(\mathbf{x})$ for the random vector $\mathbf{X}=(X_1,X_2)$, we get

$$P[\mathrm{TLF}]=P(\mathrm{g}(\mathbf{X})\leqslant 0)=\int_{x:g(\mathbf{x})\leqslant 0} f_X(\mathbf{x})d\mathbf{x}$$

The variable X_2 represents the load on the towline, whereas X_1 represents the capacity of the towline. Variable X_2 is expressed in terms of significant wave height, H_s.

The other basic events shown in the fault tree are assigned directly by experts, and their probabilities are listed in Table 1.

In examples 1 and 2, only one limit state function with just two variables has been analyzed. The SRA analysis could have been more detailed using more variables and limit state functions. For example, we could extend the event ''towline failure'' with one more failure function: ''mooring failure'' including two variables X_5 and X_6. Then, event TLF occurs if one of the failure modes occurs, and could be modeled as a parallel system, i.e.

$$\mathrm{TLF}=((X_1-X_2)\leqslant 0)\cup((X_6-X_5)\leqslant 0)$$

where X_6 represents the capacity of the mooring and X_5 represents the load on the mooring.

Example 1, Using Approach 1. In this example, $P(\mathrm{NF})$ is expressed within a classical Bayesian framework (utilizing approach 1). Let us start by assigning an uncertainty distribution to the probability of a ''towline failure,'' (TLF).

We assume the existence of a theoretical, true (but unknown) distribution function $F_{X_1}(x_i|\theta)$ for the variables X_1 and X_2 and a limit state function g_1. There will also be a true (unknown) value of $q_1=P(\mathrm{TLF})$. We ignore any uncertainty related to g_1. Our uncertainty related to F is related to the parameters θ of the distribution functions. There exist true, but unknown, values for θ, and our uncertainty related to these values is expressed by uncertainty distributions for the parameters θ. Let us now assume the following:

- The random variable X_1 is expressed in terms of significant wave heights, H_s, and is normally distributed with parameters, μ and σ. The density function is given by

$$f(x_1;\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x_1-\mu/\sigma)^2}$$

for

$$0<x<\infty,\quad -\infty<\mu<\infty,\quad \sigma^2>0$$

- The random variable X_2 has a Weibull distribution with parameters λ and β. The cumulative distribution function is given by $F_{X_2}(x_2)=1-\exp(-(x_2/\lambda)^{\beta})$

Within the classical Bayesian framework we focus on the unobservable and unknown parameters of the mathematical models (distribution functions) and express our uncertainty about where the true values of these parameters are by introducing uncertainty distributions. Our uncertainty related to F is restricted to specifying the parameters μ, σ, β, and λ. There exist true, but unknown, values of these parameters, and our uncertainty about the true values is expressed by uncertainty distributions defined by experts.

Our uncertainty related to the value of σ is expressed by a triangle distribution, $H_\sigma(\sigma)$, in the interval (1.5, 2.0). Further, our uncertainty about the value of μ is expressed by a uniform distribution, $H_\mu(\mu)$; i.e., our uncertainty related to the mean value, μ, is uniformly distributed on the interval (5.0, 6.0). Hence, μ has a density distribution

Table 2 Probability of exceeding wave heights, during the towing operation

Significant Wave Height, H_S	Probability of exceeding given criteria in a 20 hr tow = $P(H_S > p_H)$
2	0.28
2.5	0.11
3	3.3×10^{-2}
3.5	7.2×10^{-3}
5	1.9×10^{-5}

$$h(\mu)=1/(6.0-5.0) \text{ if } 5.0<\mu<6.0, \text{ and } 0 \text{ otherwise}$$

We assume statistical independence between the uncertainty distributions $h_\mu(\mu)$ and $h_\sigma(\sigma)$, giving a joint uncertainty distribution: $h_{\mu\sigma}(\mu,\sigma)=h_\mu(\mu)h_\sigma(\sigma)$.

Our uncertainty regarding the parameters of the Weibull distribution is ''minimum'' due to a large amount of data from the North Sea, and therefore they are presented as fixed values. Using the data from the classical statistical analysis in Table 2, the parameters λ and β of the Weibull distribution can be determined

$$p_H=P(X_2>x_2)=1-F_{X_2}(x_2)=\exp\left(-\left(\frac{x_2}{\lambda}\right)^\beta\right); \quad -\ln p_H=\left(\frac{x_2}{\lambda}\right)^\beta$$

$$\ln(-\ln p_H)=\beta\ln x_2-\beta\ln\lambda,$$

where $x_2=5m$ and $3m$ with corresponding values $p_H=3.3\times10^{-2}$ and 1.9×10^{-5}, respectively. This gives:

$$\beta\ln 3-\beta\ln\lambda=\ln(-\ln 3.3\times10^{-2})=1.227,$$

$$\beta\ln 5-\beta\ln\lambda=\ln(-\ln 1.9\times10^{-5})=2.386$$

Thus

$$\beta=\frac{1.159}{\ln 5-\ln 3}=2.269$$

$$\ln\lambda=\frac{0.492}{2.269}$$

$$\lambda=e^{0.216}=1.241$$

The *uncertainty distribution* over q_1, where $q_1=P((X_1-X_2)\leq 0)$, is established by drawing numbers from the uncertainty distributions H_μ and H_σ and using the limit state function $g(\mathbf{X})$. The software Proban was used for this purpose. The results fit a beta distribution

$$f_X(x)=\frac{1}{(b-a)^{t-1}B(r,t-r)}(x-a)^{r-1}(b-x)^{t-r-1}$$

The following parameter values for the Beta distribution were calculated by Proban [5]:

- Mean value: $\mu=a+(b-a)\frac{r}{t}=4.3317\times10^{-3}$
- Standard deviation: $\sigma=(b-a)\frac{r}{t}\frac{\sqrt{(t-r)}}{\sqrt{r(t+1)}}=4.4091\times10^{-3}$
- Lower bound: $a=0.0$
- Upper bound: $b=1.0$

The parameters r and t of the distribution of the variable q_1 are

$$r=\mu t$$

$$\sigma=\frac{r}{t}\frac{\sqrt{t-r}}{\sqrt{r(t+1)}}\Rightarrow\sigma^2=\frac{r^2}{t^2}\frac{(t-r)}{r(t+1)}$$

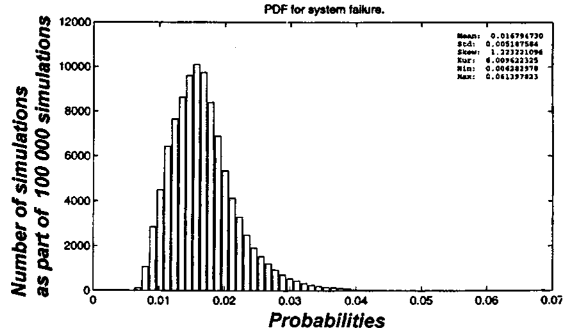

Fig. 2 Density distribution of the probability of navigation failure, *P*(NF)

$$\sigma^2=\frac{(\mu^2t^2)}{t^2}\frac{(t-\mu t)}{\mu(t+1)}=\frac{\mu-\mu^2}{t+1}\Rightarrow t=\frac{\mu-\mu^2}{\sigma^2}-1=220$$

$$r=\mu t=0.96$$

The beta function will now, together with the uncertainty distributions over the probabilities of the remaining basic events in the fault tree, be used in Eq. (4) to calculate the overall probability of navigation failure; see overall results in the forthcoming.

For the remaining base events in the fault tree, the following uncertainty distributions over the probabilities are expressed by experts:

- Navigation equipment failure (NEF): The probability q_2 is expressed by an uncertainty distribution $H_2(q_2)$. $H_2(q_2)$ is taken to be triangle distributed on the interval $(10^{-4},5\times10^{-3})$ with peak value at 10^{-3}. Hence, q_2 has a density $h(q_2)=aq_2+b$, for $10^{-4}<q_2<10^{-3}$ and $h(q_2)=cq_2+d$ for $10^{-3}<q_2<5\times10^{-3}$.
- Incorrect model of production facility tug behavior, (IM): Our uncertainty about the value of q_3 is ''minimal'' and is expressed by a fixed value, $q_3=5\times10^{-3}$.
- The tug captain misunderstands command (TMC): The probability q_5 is by an uncertainty distribution, $H_4(q_4)$. $H_4(q_4)$ is uniformly distributed on the interval $(10^{-5},10^{-3})$. Hence, q_4 has a density $h(q_4)=1/(10^{-5}-10^{-3})$ if $10^{-5}<q_4<10^{-2}$, and 0 otherwise.

Overall Results. Using the uncertainty distributions for q_1, q_2, q_3, and q_4 and probability calculus, (*4*), the probability $p=P(\text{NF})$ and associated uncertainty distribution was calculated by Monte Carlo simulations using the software program Matlab (version 4);

$$p=1-\left[(1-P(g_1(\mathbf{X})<0))\left(1-\prod_{i=2}^{4}q_i\right)\right]$$

Matlab calculates the uncertainty distributions of p by simulation runs, i.e., drawing numbers from the probability distributions of q_1, q_2, q_3, and q_4. Mathematically, the posteriori distribution H^* of p is given by

$$H^*(p')=P(\text{``}p\leq p'\text{''})=\int_{\{\mathbf{q}:w(\mathbf{q})\leq p'\}}dH(\mathbf{q})$$

where H^* is the uncertainty distribution over p, and $\mathbf{q}$ is the input parameter, H is the uncertainty distribution over $\mathbf{q}$, and w is the function linking $\mathbf{q}$ to p.

With a large number of simulation runs we obtained in this way the uncertainty distribution of p, as shown in Fig. 2.

Example 2, Using Approach 2. In this example the fully Bayesian approach is used to assign the probability of navigation failure, P(NF), as illustrated in the fault tree, in Fig. 1. In Approach 2 the probability is calculated within a full Bayesian framework, and in this context $P(\mathrm{NF})$ expresses our degree of belief about the occurrence of a navigation failure. Hence, the uncertainty involved here is related to whether a navigation failure will occur or not.

The probability of navigation failure, assuming the events are judged independent, is given as

$$P(\mathrm{NF})=1-\left(1-\prod_{i=1}^{4} q_i\right)$$
$$=1-[(1-P(\mathrm{NEF}))(1-P(\mathrm{PF}))(1-P(\mathrm{IM}))(1-P(\mathrm{TC}))]$$

The probabilities of $q_2=P[\mathrm{NEF}]$, $q_3=P[\mathrm{IM}]$ and $q_4=P[\mathrm{TC}]$ are assigned directly by experts and listed in Table 1, whereas $q_1=P[\mathrm{TLF}]$ is established utilizing SRA.

When assigning $P(\mathrm{TLF})$ within a fully Bayesian framework, the limit state function, $g(\boldsymbol{X})$, is utilized within the fully Bayesian framework, where attention is put on the event TLF and our uncertainty about the occurrence of TLF is expressed by a probability. Within this framework we focus on the observable and unknown quantity X_i with the uncertainty distribution $f_{X_i}(x_i)$. Hence, $q_1=P[\mathrm{TLF}]=P(g_1(\mathbf{X})\leqslant 0)$ is a measure of uncertainty, a degree of belief, concerning the occurrence of the event $g_1(\mathbf{X})\leqslant 0$.

We have chosen to express the subjective distributions $F_{X_i}(x_i)$ by standard mathematical models. It seems reasonable to use the same standard distributions used in the previous example (e.g., Normal and Weibull). In the fully Bayesian framework, we do not focus on the parameters of the distributions and we do not think of them as having true values as in Approach 1. They are simply needed to describe the shape of the distributions and are expressed as fixed values.

The unknown and observable quantities $\boldsymbol{X}$ are expressed by the following two uncorrelated variables:

- X_1 is normally distributed with the parameters (μ,σ^2), where μ is equal to 5.5 m and σ is equal to 1.5 m.
- X_2 is Weibull distributed, with parameter values as calculated previously, i.e. $\beta=2.27$ and $\lambda=1.24$.

It should be noted that the parameter values are not the ''best estimates'' of the uncertainty distributions for the parameters in the distribution of X_1. The parameter values are chosen to obtain good predictions of $\mathbf{X}$ and reflect our uncertainty of their values. The software Proban [5] has also here been used to calculate the probability of towline failure. By using FORM analysis, the result is given as

$$P[\mathrm{TLF}]=2.8\times 10^{-3}$$

Overall Results (Probability of Navigation Failure). The *overall probability* of the top event, navigation failure is now calculated using probability calculus, giving

$$P[\mathrm{NF}]=1-\prod_{i=1}^{n}(1-q_i)$$
$$=1-[(1-P(g_1(X)\leqslant 0))(1-q_2)(1-q_3)(1-q_4)$$
$$=1-[(1-2.8\times 10^{-3})(1-5\times 10^{-4})(1-5\times 10^{-3})\times(1-5\times 10^{-3})]$$
$$=\underline{1.3\times 10^{-2}}$$

Discussion

The classical statistical approach to risk analysis is not considered suitable for integrating QRA and SRA. There are not sufficient ''hard'' (historical) data to accurately estimate unknown parameters of the models, and therefore a Bayesian approach is preferable. During a QRA for construction projects it is necessary to include whatever relevant information is available, and the Bayesian approach provides a consistent tool for combining ''hard data'' and subjective information. The common practice today, when interpreting risk results, is conceptually quite similar to the classical Bayesian approach. They both want to say something about true, unobservable quantities (probabilities and statistically expected values). The main difference between them is related to the treatment of uncertainty. Common practice allows for subjective point estimates (''best estimates'') of parameters, but the uncertainty associated with these estimates are seldom quantified. In the classical Bayesian approach, however, uncertainty related to the true parameter values and model are expressed by subjective uncertainty distributions, which then generate uncertainty distributions for the output risk results.

Within the fully Bayesian framework, however, the uncertainty element is totally different from the classical Bayesian. The fully Bayesian approach which forms the framework for Approach 2, will provide the probabilities of the uncertain events that are relevant in the specific situation of decision making. The probabilities express our degree of belief concerning the occurrence of the events. Thus, the result itself is a total measure of uncertainty, and does not require any further discussion of ''uncertainty of the probabilities.''

The advantages by integrating SRA into QRA are mainly the ability to easily handle continuous variables appearing in the accidental events and the possibility to include several random variables and failure modes into one single analysis. The use of probability distributions in SRA enables the analyst to give a detailed description of one's knowledge about an uncertain quantity. This is not possible to the same extent if the description's restricted to central measures like mean (as normally done in traditional QRAs) or median values. In both approaches, the system considered is modeled by one or several failure functions, g of the stochastic variables representing ''load'' and ''capacity'' quantities. By using continuously distributed quantities, a full probabilistic description of the experts uncertainty regarding each of the quantities influencing the occurrence of an event, i.e., the random variables appearing in the limit state functions, are given. Thus, the experts' knowledge about the event is put into the design as well as the interconnection of limit state functions, distribution functions, and correlation measures (if present), reflecting the uncertainty or knowledge related to the basic variables and the event.

Depending on the Bayesian framework of the analysis, the interpretation of the distribution functions, F, differs. In the classical Bayesian approach we focus on the parameters of the distribution function, F, and express our uncertainty about the true values of these parameters by subjective uncertainty distributions when utilizing SRA. In the fully Bayesian setting, on the other hand, we focus on the observable quantity, X_i. The value of X_i is uncertain and the uncertainty is expressed by a subjective probability distribution F (where the parameters only serve as a mathematical input to describe the functions). Normally, $\mathbf{X}$ is expressed by standard distribution functions, e.g., normal, lognormal, Weibull, or beta.

How should the results in example 1 be presented and interpreted? Is it sufficient to use the standard deviation and the expected values from the uncertainty distributions? Or, should we use the full uncertainty distribution when presenting and interpreting the output results? By using expected values and standard deviations, some information will be lost. This information may be interesting if the total uncertainty was included in the uncertainty distributions. Prior to an analysis, such questions should be addressed and it is the responsibility of the decision makers to answer them.

Comparing the resulting uncertainty distributions following Approach 1 is not straightforward. It is, of course, convenient from a practical point of view to focus on the mean value in the uncertainty distributions since then we can more easily compare the results. However, such an approach means that we lose valuable information about the risk. The mean could be a poor estimate of the true risk. Within this framework the interesting quantity is the true risk which objectively characterizes the performance of the system. Perhaps, in some cases the uncertainty distributions would imply high probabilities for some rather extreme situations, even though the mean values are relatively low. So in addition to the mean, attention should also be given to the probabilities of extreme risk values.

To be more specific, consider in the classical Bayesian approach an exponential lifetime model

$$P(T\leqslant t)=1-e^{-\lambda t}$$

Then, we can write

$$P(T\leqslant t)=\int (1-e^{-\lambda t})dH(\lambda) \quad (5)$$

where $H(\lambda)$ is the marginal uncertainty distribution of λ. The distribution $P(T\leqslant t)$ is referred to as the predictive distribution for the lifetime T.

By this formula, the uncertainty is divided into two: the stochastic (aleatory) uncertainty expressed by $(1-e^{-\lambda t})$ and the state-of-knowledge (epistemic) uncertainty expressed by $H(\lambda)$. Now, using the fully Bayesian approach we would normally use a specific value of λ, and this gives a specific distribution $1-e^{-\lambda t}$. If we choose to use the predictive formula (2), the mathematics seem to be similar. Yes the computation of the predictive distribution is the same as the uncertainty distribution given by formula (5) with $A=T\leqslant t$. The important points here are not mathematics, but ideas and interpretation. Following the fully Bayesian approach, the uncertainty measure $P(T\leqslant t)$ is the interesting quantity, and there is no true value of λ and $F(\lambda)$. In the classical Bayesian approach, the interesting quantity is the true value of the lifetime distribution, since this distribution is supposed to be a property of the system being analyzed. The probability $P(T\leqslant t)$ given by (5) is a measure of uncertainty, but it is not total as it does not reflect uncertainty related to the choice of lifetime distribution class; and it is equal to the mean of the uncertainty distribution related to the true underlying lifetime distribution, and the mean can in many cases give a poor picture of this distribution, as mentioned in the foregoing.

Example 1 discussed herein has shown that the determination of uncertainty distributions and the establishment of the input to the simulation runs are time consuming, and may be complex compared to the assignment of probabilities within a fully Bayesian framework. Assigning hundreds of uncertainty distributions for the parameters in a typical QRA will most certainly be difficult to handle.

In order for the decision makers to choose between the models, they need to understand the two approaches and go back to the philosophies behind the models and choose the one which is considered to best suit the purpose of the analysis. Most analysts and decision makers have a background in classical statistics, i.e., they believe in a true value of p, and therefore they may have more confidence in Approach 1. Further, the uncertainty distributions indicate uncertainty in the results, and are therefore encouraging the decision makers and analysts to a discussion about uncertainty, as normally done in classical risk analysis. Finally, the Monte Carlo simulations may indicate that there is advanced mathematical modeling behind the results, which may indicate a good knowledge of the risk concept and input data. If the decision makers do not fully understand the philosophy behind Approach 2, they will, because of the aforementioned circumstances, most probably be in favor of Approach 1!

The total uncertainty related to the analysis (model and parameter space) should theoretically be covered within the classical Bayesian framework of the uncertainty distributions, but it is impossible to do this in practice, especially for large systems. So, in practice, only some distributions for some few parameters are assigned, and consequently, the uncertainty distributions of the output probabilities (results) just reflect some aspects of uncertainty.

Since the main task when performing QRA is decision making, the most relevant question when comparing the two approaches must be: ''which model is most efficient for making decisions?'' We believe that presenting results, as in example 2, will give the decision makers a more clear message about the risk involved in a project than by using Approach 1. The fully Bayesian approach means that we consider risk analysis as a tool for debate over safety, rather than an attempt to say something about objective risk values [6].

Using Approach 2, the message of the analysis is not ''disturbed'' by a discussion of uncertainty of the output probabilities, as in the classical Bayesian approach. In our opinion it is often difficult to use Approach 1 in decision making, as the resulting uncertainty intervals are large. What are the conclusions if two options are compared and the uncertainty bands are (0.001, 0.01), and (0.002, 0.1), respectively? The risk analysis group is consulted as an expert team to help the decision maker, but the message when adopting the foregoing approach is not very informative and gives the impression that risk analysis results are extremely uncertain. The integration of SRA into the QRA models reduces this problem, but does not remove it. If Approach 2 is adopted instead, the output results are expressing the analysis group's total uncertainty related to observable quantities, and it is possible to present a clear message, without a discussion of uncertainties of the risk figures.

To eliminate unwanted variability in results from one analysis to another, guidelines/standards related to methods and data are required. Of course, standardized input data cannot be used when facing new types of problems and situations. Such guidelines/standards should, however, not reduce the flexibility and freedom of choosing the analysis group too much. Remember that, in a Bayesian setting, the results of the analysis expresses the *best judgments* of the analysis group. Of course, all elements of the analysis must be properly documented.

The decision maker will take into account a number of factors when deciding to implement or not to implement risk-reducing measures. The information that the results are based on, the presuppositions and assumptions made in the analysis, the confidence in the analysis, and the *acceptance criteria* will also be taken into account before making the final decision.

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