MATHEMATICAL CHARACTERIZATION OF HUMAN RELIABILITY FOR MULTI-TASK SYSTEM OPERATIONS

Ronald E. Giuntini
Wyle Laboratories

1.0 ABSTRACT

The purpose is to provide a mathematical reliability methodology for estimating the contribution of the human operator in a human-machine system. The methodology is applicable to high value, high risk human-machine operations where failures can be catastrophic and costly such as rendezvous and docking of a space craft at the International Space Station, landing of a new generation of space vehicles, the operation of piloted dynamic flight simulators, critical control activities in a nuclear power plant, and other potentially risky human-machine interfaces.

The metholology consists of a mathematical probabilistic conceptual approach for the analytical characterization of human reliability for multi-task system operation. From a system perspective, reliability for a system includes three different reliability elements: (1) hardware reliability, (2) software reliability, and (3) human reliability. Therefore, the reliability of a system is just as constrained by the reliability of the human operator as it is by the reliability of the hardware and software. Seldom is the human component considered in the mix when the total reliability of a system is being planned. This is tantamount to the assumption that the human reliability is 1.0 or perfect [Giuntini and Wessels, 1997].

Characterization of human reliability most often has been relegated to non-mathematical subjective factors; i.e., levels of training, workplace layout, ergonomics issues, etc. Some researchers have sought to characterize human reliability in terms different from those used for hardware reliability [Meister and Rabideau, 1967]. In this paper, the elements of the bathtub curve for human reliability have been constructed analogous to comparable elements of the bathtub curve for hardware reliability. This human error rate bathtub curve concept enables the formulation of a useful, workable mathematical modeling process for human reliability [Giuntini, 1970].

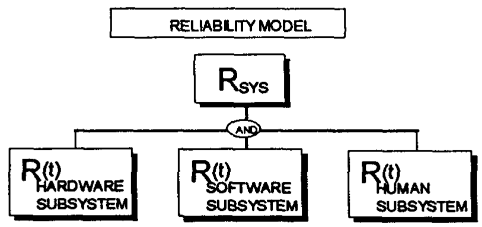
The results consist of a fully developed method that can be applied to quantify human reliability in a human-machine system. A detailed mathematical process of each of the three error rate phases: (1) learning error rate phase, (2) stabilized error rate phase, and (3) fatigue error rate phase is presented in terms of the Weibull probability distribution to characterize the human reliability process.

2.0 INTRODUCTION

The most widely accepted definition of reliability is given below:

> *Reliability is the probability that a system will perform satisfactorily for at least a given period of time when used under stated conditions [Von Alven, 1965].*

A reliability function is this probability expressed as a function of the time period and thus relates to the frequency with which failures occur. Most systems consist of three diverse functioning subsystems. These are the hardware subsystems, the software subsystem, and the human subsystem. The Space Shuttle and the International Space Station are examples of such a system. From a reliability perspective, the reliability of a system is illustrated in Figure 1.



$$R(t)_{SYSTEM} = R(t)_{HARDWARE} \cdot R(t)_{SOFTWARE} \cdot R(t)_{HUMAN}$$

Figure 1. Dimensions of System Reliability

Bazovsky suggested two aspects where the human element affects the system operational reliability that could be included in a total system reliability: 1. Probability that operating personnel will not inadvertently operate the system, and 2. Probability that the maintenance personnel have not made errors during the last maintenance operation [Bazovsky, 1961]. Item 1 does not address the situations of errors committed during performance. The human reliability persented herein is the treatment of the operator as a subsystem in the operation of the system which is independent of the maintenance process.

Each of the three subsystems provides unique nuances not found in the other two, but all have similarities in their respective modeling processes and their mathematical treatments. The remainder of this paper addresses the logic for the human reliability.

3.0 COMBINED ERROR RATE THEORY

Human error rates are functions of many variables, one of which is time. People, unlike machines, tire from work and require rest. People, unlike machines, have to learn a job. There is no instantaneous human programming that will enable a person to perform a complex job, task, or activity without training and practice. Further, it is known that even with training, people commit errors from time to time on tasks they have performed flawlessly for months or years.

These facts have to be put into a time-related function that we refer to as the error rate. The error rate will vary with time as a function of some of the variables mentioned. It will increase, decrease, or remain constant over time.

Symbolically, the error can be represented by E(t). An examination of error rate behavior over time yields three phases. These three phases are defined as follows:

1. **Learning Phase (T_L)**
During the learning phase of system operations, the rate at which human errors occur decreases with time. As the operator learns the task, there is less likelihood that errors will occur. During the learning phase time, T_L, the error rate, E(t), will be a decreasing function as shown in the following diagram. The length of the learning phase will vary from minutes, to hours, to days as the complexity and size of the task increases.

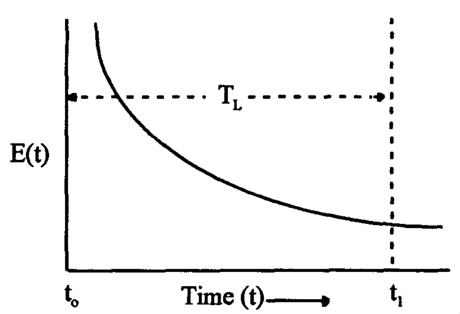


Figure 2. Learning Phase Curve

2. **Stabilized Error Phase (T_S)**
After the operator has learned the task, the error rate will stabilize at some relatively constant value and will no longer be a decreasing function of time. The E(t) will simply become E_k, a constant, and human errors will have the same likelihood of occurrence at any point in the stabilized region from t_1 to t_2. This is the random error region [Hammer, 1972].

The time between t_1 and t_2 would be a function of the difficulty and complexity of the task and other stress-inducing and stress-reducing factors. Breaks and short rest periods may be essential to maintain efficiency during the t_1 to t_2 interval.

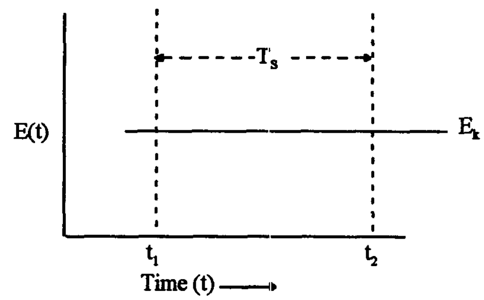


Figure 3. Stabilized Error Rate Curve

3. **Fatigue Phase (T_F)**
The fatigue phase is characterized by an increasing E(t). The error rate, E(t), is no longer constant but increases with time. The impact of the increasing error rate can be offset by periods of rest with the effect of placing the human back into the stabilized error phase.

The duration of the rest period is also a function of a number of stress-inducing factors and could range from minutes to many hours.

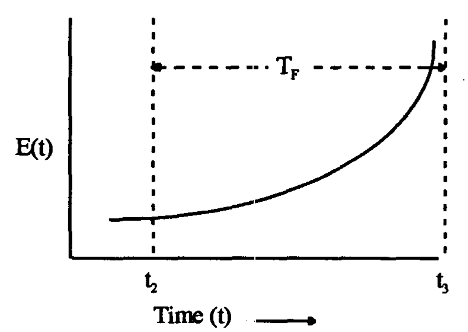


Figure 4. Fatigue Phase Curve

4.0 COMBINED ERROR RATE CURVE

Consolidating the three (3) phases (T_L, T_S, T_F) into a single diagram yields the following curve.

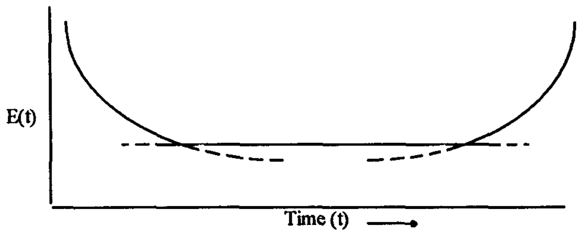


Figure 5. Consolidation of the Three Error Rate Phase Curves

The final error rate curve showing the three (3) phases is depicted in the next curve.

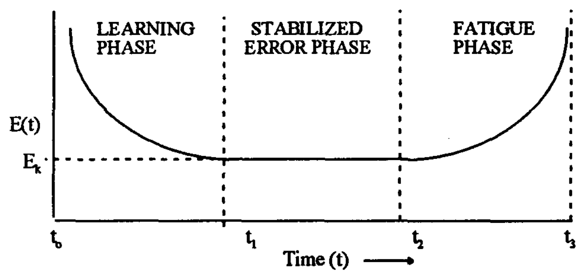


Figure 6. Combined Error Rate Curve

5.0 DEVELOPMENT OF THE HUMAN RELIABILITY FUNCTION

The reliability function, R(t), can be derived from the probability density function for human errors , f(t). The density function , f(t), if integrated from minus infinity to some t, yields the cumulative probability distribution, F(t), which is the probability that in a random trial, the random variable is not greater than t [Von Alven, 1965]. In the diagram below, since the mass or density function is positive with time, then the integration is from $t = t_o$ to $t = t_m$ where F(t) approaches 1.0 asymptotically as in the equation that follows.

$$F(t)=\int_{t=t_o}^{t=t_m} f(t)\,dt$$

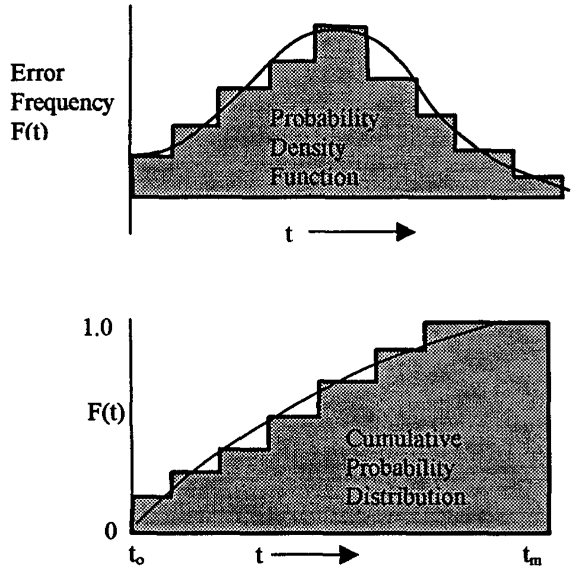


Figure 7. Derivation of the Cumulative Probability Distribution From the Probability Density Function for Human Errors

F(t) is the unreliability function. The reliability function is:

$$R(t)=1-F(t)$$

$$R(t)=1-\int_{t=t_o}^{t=t_m} f(t)\,dt$$

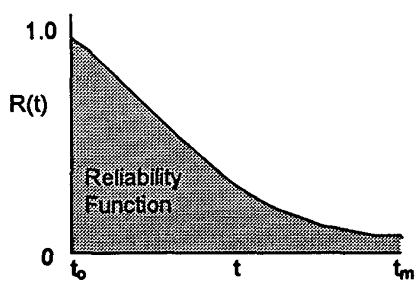


Figure 8. The Human Reliability Function

The section entitled "Combined Error Rate Theory" describes three phases of the error rate, E(t), but stops short of providing mathematical models for these three phases. Several of the probability density functions could be used to model the behavior of these phases. For example, the Weibull can model any of the phases by variation in the shape or slope parameter (β). This is illustrated in Figures 9, 10, and 11.

1. **Learning Phase (T_L)**

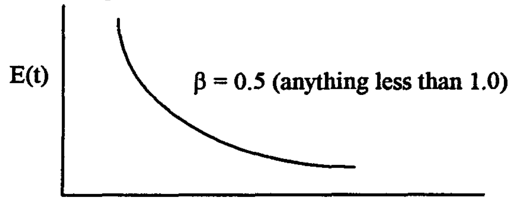


Figure 9. Learning Phase Error Rate Curve Modeled with a Weibull Function

2. **Stabilized Error Phase (T_S)**

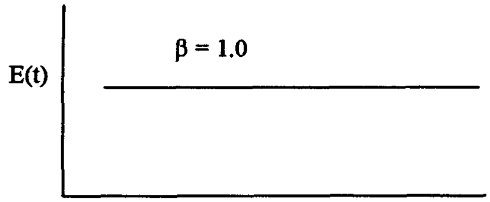


Figure 10. Stabilized Error Rate Curve Modeled with a Weibull Function

3. **Fatigue Phase (T_F)**

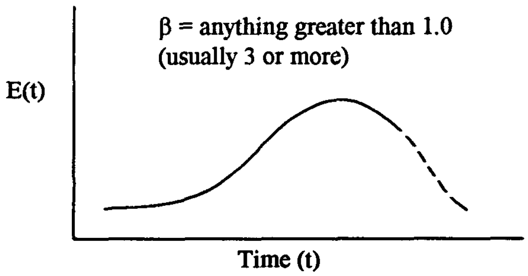


Figure 11. Fatigue Phase Error Rate Curve Modeled with a Weibull Function

There are several ways for deriving the human reliability model as a function of the error rate and time. Since the discussion of the three error rate phases used the Weibull probability distribution, the Weibull will be used to derive the reliability equation for the human reliability process discussed in this paper.

The Weibull probability density function is as follows [Abernethy et al, 1983]:

$$f(t)=\frac{\beta}{\eta}\left(\frac{t}{n}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

Where η is the characteristic life and β is the shape or slope parameter. As was illustrated in the discussion of the three error rate phases, it is the second error rate phase where E(t) = Ek that is the basis for a human reliability program. In that phase, $\beta=1$. If $\beta=1$ is put into the following equation, it reduces to

$$f(t)=\frac{1}{\eta}\left(\frac{t}{\eta}\right)^{o} e^{-\left(\frac{t}{\eta}\right)^{1}}=\frac{1}{\eta}e^{-\left(\frac{t}{\eta}\right)}$$

which is the exponential probability density function.

Integrating f(t), gives F(t).

$$F(t)=\int_{-\infty}^{t}\frac{1}{\eta}e^{-\frac{t}{\eta}}=-e^{-\frac{t}{\eta}}\Big]_{-\infty}^{t}=\left[-e^{-\frac{t}{\eta}}+1\right]$$

Since R(t) = 1 - F(t)

$$R(t)=1-\left[-e^{-\frac{t}{\eta}}+1\right]=e^{-\frac{t}{\eta}}$$

The Weibull characteristic life, η, becomes the mean-time-between-errors (MTBE) in the exponential distribution which is the reciprocal of the steady-state or stabilized error rate E_k. Therefore, the reliability equation for the stabilized error phase (i.e. useful working phase) is as follows:

$$R(t)=e^{-E_k\cdot t}$$

5.0 CONCLUSIONS

The effectiveness of the human in the performance of a task goes through several phases. The first phase begins with the introduction of the new task and ends with learning and proficiency in the performance of the task. Initially the error rate is high and mistakes are caused by a wide variety of reasons. The error rate diminishes over time until ultimately the task is mastered to an acceptable performance and an acceptable error rate. The second phase is manifest by a steady performance of the task at a consistent level of proficiency and a constant error rate which is largely absent of special causes. The third phase begins as the error rate ceases to be constant. It increases to a level which is economically unacceptable. This phase is viewed as the fatigue limit. It is also indicative of boredom, burn out, and loss of attention, but largely fatigue. The pattern of the error rate can be characterized by a human error rate 'bathtub' curve which is analogous to behavior of the failure rate in hardware reliability analysis. This parallel behavior between human error rate and hardware failure rate is not directly one-to-one, but does lead to a similar approach to reliability modeling and makes the two compatible. A steady-state, or stabilized, error rate phase can be defined where a constant error rate is analogous to the constant failure rate for hardware reliability analysis.

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