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POTENTIAL APPLICATIONS OF FUZZY SETS IN INDUSTRIAL SAFETY ENGINEERING

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This paper discusses potential applications of fuzzy set theory to risk analysis in the area of industrial safety engineering. Vagueness and imprecision in mathematical quantification of risk are equated with fuzziness rather than randomness. The concept of risk evaluation, using linguistic representation of the likelihood of the occurrence of a hazardous event, exposure, and possible consequences of that event, is proposed. The approximate reasoning technique based on fuzzy logic is used to derive fuzzy values of risk.

Keywords: Risk analysis, Linguistic variables, Approximate reasoning.

1. Introduction

An assessment of risk in industrial and urban environments is essential in the prevention of accidents and in the analysis of situations potentially hazardous to public health and safety [1, 2]. The problems of risk, broadly defined as an undesirable implication of uncertainty [3], and its analysis can be classified into two distinct categories. The first category of problems deals with uncertainties which are essentially random and, therefore, probabilistic. The second category involves problems which are not really probabilistic, but cause uncertainty due to imprecision associated with complexity of the systems under investigation as well as vagueness of human thought and perception processes.

Mathematical evaluations for controlling hazards and calculating risk in the area of systems safety equate imprecision with randomness only. According to Malasky [4], the probability distribution is used in order to "compensate for the fact that the given nominal value of any parameter, even if explicitly defined, is rarely known with absolute precision". Therefore, the quantification is usually obtained by assigning to each set of events a probability measure.

However, in many cases it is virtually impossible to determine precisely the value of the probability of a given event. Such a situation may be due either to lack of evidence or to the inability of the safety engineer to make a significant

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assessment of the frequency of occurrence of an event [5]. In other words, the probability of an event may be ill-defined, and instead of specifying its numerical value (e.g., 0.5), one would simply say that a given event is 'more or less likely', 'remotely possible', etc., where terms such as 'likely' and 'possible' are vague and imprecise descriptors which are frequently used by risk analysts [6]. Use of such value judgments introduce uncertainty which is the result of fuzziness, not randomness.

Recent advances in the theory of fuzzy sets make it possible to study the complex and ill-defined systems (and concepts) where uncertainty is due to fuzziness, or degree vagueness [7]. In this paper, a fuzzy set theoretic approach to risk analysis is proposed as an alternative to the techniques currently used in the area of systems safety, and linguistic variables [8] are introduced to analyze potentially hazardous situations using approximate reasoning methods.

2. The use of verbal descriptions in risk analysis

According to Fine [6], the risk (or uncertainty of loss) imposed by a particular hazard increase with the *likelihood of occurrence* of the event (L), *exposure* (E), and the possible *consequences* (C) of that event. Hazard is defined as "some potential danger beyond one's immediate control" [2], and it is assumed that although all hazards can never be completely eliminated, the associated risks from the hazards can be reduced. In a traditional approach, the calculation of a quantitative value of *risk* (S) is usually based on an assignment of numerical values of each of the above factors. The product of the values of likelihood, exposure and consequences, called 'risk score', is then derived. Using experts' judgements, several risk scores for many different hazardous situations can be obtained and ordered with respect to seriousness of their risks. Such a risk score summary is then presented to safety managers in order to undertake specific action and reduce the existing hazards.

Development of a practical safety analysis system for hazard control [2] indicates that engineers have long recognized the imprecise nature of risk evaluations and the importance of judgments based on sound experience [9, 10]. Since risk is a concept which is not absolutely objective in nature [11], but rather relative and subjective, the notion of risk must be looked upon in terms of the interaction between the object (environment) and the subject (individual assessor). Furthermore, as stated by Feagans [12] risk is a fuzzy concept in the sense that there does not exist a unique risk that a hazardous event will occur in a given period of time.

Risk analysis deals then with quantities which are inherently imprecise and whose future values are uncertain. Therefore, such values may be based on subjective judgments, especially when one deals with non-numerical data. As suggested by Zimmer [13], although humans are quite unsuccessful in quantitative predictions, they may be comparatively efficient in qualitative forecasting. In fact, the knowledge of experts usually consists of qualitative variables stated verbally, as evidenced by recent developments in knowledge-based expert systems [14].

Moreover, people are more prone to interference from biasing tendencies if one forces them to give numerical estimates. This is due to the fact that by eliciting numerical estimates one is forcing people to operate in 'a mode' which requires more mental effort [15].

The meaning of verbal descriptors is usually vague and it may be difficult to find their numerical representations [15–19]. Nevertheless, in the area of systems safety, analysts have worked out a method for risk assessment which is primarily based on human judgment and experience. Through trial and error, several verbal descriptors of the risk score were derived, and their approximate numerical correspondents were found. Such descriptors relate to all three factors, i.e.: (1) the *likelihood* that some hazardous event will occur, (2) the *exposure* to that particular hazardous situation, and (3) the possible *consequences* should the hazardous event actually occur [2]. These descriptors are shown in Table 1.

Table 1

Descriptors used in risk analysis after [2]	Corresponding fuzzy linguistic values
1. Likelihood (of the accident-sequence)	
might well be expected	[very likely]
quite possible	[likely]
unusual but possible	[more-or-less likely]
only remotely possible	[unlikely]
conceivable, but highly unlikely	[very unlikely]
practically impossible	[very very unlikely, impossible]
2. Exposure (to the hazardous event)	
continuous (many times daily)	[very high]
frequently (once a day)	[high]
occasionally (one per week or month)	[moderate]
monthly (one per month/year)	[more-or-less low]
rarely	[low]
very rare	[very low]
3. Consequences (of the accident)	
catastrophe (extensive damage, over $\$10^7$; many fatalities)	[extremely high]
disaster ($\$10^6$–$10^7$, fatalities)	[very high]
very serious ($\$10^5$–$10^6$, a fatality)	[high]
serious ($\$10^4$–$10^5$, serious injury)	[medium]
important ($\$10^3$–$10^4$, disability)	[more-or-less medium]
noticeable ($\$10^2$–$10^3$, first aid)	[low]

Since fuzzy set models of human judgment permit translation of verbal expressions into numerical ones [15], and deal with imprecisions in the expression of the occurrence of events, in this paper an attempt was made to develop the fuzzy linguistic model of the above practical risk analysis system.

3. Fuzzy linguistic variables and risk factors

The overall risk score is obtained in a traditional approach as a product of exposure, likelihood, and consequences of a possible accident due to the hazard. An assignment of numerical values to any of the above components of the 'risk score' is subjective by the nature of human judgment. For example, although the concept of probability as suitable for risk analysis is well defined, it does not provide for the sharp probability estimates needed to generate adequate risk estimations. Instead, the quantification of 'likelihood' is imprecise since it uses linguistic descriptors like: *quite possible, unusual but possible, very unlikely*, etc. An *event* here is clearly stated, but its probability is vaguely defined, and therefore, it is also imprecise. Therefore, the probability P is treated here as a linguistic variable [8] with the typical values (P_i) such as: *likely, very likely, more or less likely, very unlikely*, etc., with the understanding that *likely* is synonymous with probable.

Since the likelihood of occurrence of the hazardous event is related to the probability that it might actually occur [2], the numerical variable 'probable', with values $0 \leqslant P_i \leqslant 1$, is the base variable for the 'likelihood'. A linguistic value such as *likely* is interpreted as a label for the fuzzy restriction (characterized by its compatibility function) on the values of the base variable. Typical values of the linguistic variable contain primary terms, such as *likely* and *unlikely*; hedges such as *very, more or less, quite, extremely*, and somewhat; and fuzzy connectives such as *and, or*, and *either*; as well as negation, *not*. The connectives, negation and hedges are treated as modifiers of the operands (primary terms) in a context-dependent situation [8].

In a practical safety analysis system [2], two likelihoods are established as defined reference points with arbitrarily assigned values of likelihood. These are (1) 'a completely unexpected and unanticipated (but remotely possible) event' (value of 1), and (2) an event that 'might well be expected at some future time', (value of 10). There are also event likelihoods perceived as 'highly unlikely', 'practically impossible' and 'virtually impossible' with the numerical values of 0.5, 0.2 and 0.1, respectively. The above information can be used to develop context specific values of likelihood.

3.1. *Interpretation of the linguistic values*

It should be emphasized here, that although the meaning of the proposed linguistic values are open to individual interpretation, the differences in subjective assessments can be resolved by extending the precision of associated verbal definitions through discussion among the experts in the field of risk analysis. It is very important that the structure of verbal descriptors does not cause misunderstanding [3, 12], and this can be prevented if the agreed upon definitions are provided. As indicated by Cooley and Hicks [20], primary linguistic values should have an intuitive appeal and be easily differentiated. For that reason, the values 'likely' and 'unlikely' were proposed to represent verbal descriptors which are most frequently used by risk analysts, i.e. 'quite possible' and 'only remotely possible', respectively. The other values of likelihood were derived by the use of the appropriate hedges (see Table 2).

Table 2

Individual term	Compatibility function						
high	0	0	0.1	0.3	0.7	0.9	1
medium	0	0.2	0.7	1.0	0.7	0.2	0
low	1	0.9	0.7	0.3	0.1	0	0
unknown	1	1	1	1	1	1	1
undefined	0	0	0	0	0	0	0
more or less high	0	0	0.3	0.5	0.85	0.95	1
very high	0	0	0	0.1	0.5	0.8	1
likely	0	0.1	0.5	0.7	0.9	1	1
unlikely	1	1	0.9	0.8	0.5	0	0
not likely	1	1	0.5	0.3	0.1	0.1	0

According to Narasimhan [21] a method of empirical definitions of the most important linguistic values, and use of the concepts of fuzzy set theory to define the other values, seems to be an advantageous one. Clements [22] reported satisfactory results when applying the predefined linguistic values (for the users who were trained as to their meaning) to the analysis of computer security systems. One must still keep in mind that the assessor himself is an essential source of fuzziness, since the same hazardous event may be perceived differently depending upon the experience and individual preferences in risk acceptability [11].

The compatibility functions for the chosen linguistic values are represented by a string of numbers rather than a continuous function [23, 24]. In the computerized version of the proposed systems, the user will be able to derive the representations of the primary linguistic terms by using the canonical forms of the S and π functions [25], and adjusting the appropriate parameters.

3.2. *Definitions of risk factors*

The degree or severity of *consequences* (C) of the particular event due to some hazard conditions, and the *exposure* (E) to such hazard were defined in the similar way as the *likelihood* (L) of the event. The base variable for the degree of consequences was represented by the extent of property damage, and/or by the seriousness of the injuries (ranging from minor cuts of one individual to numerous fatalities), expressed by the amount of loss, in a range from $\$10^2$ to $\$10^7$ (see Table 1). The primary terms of the variable consequences (C) are *high*, *low*, and *medium*, with the graphical interpretations depicted in Figure 1.

Similarly, the primary terms for the variable *exposure* (E) were defined. The base variable was defined numerically by the relative frequency of occurrence of the hazard events, in days of operation. Graphical interpretation of these values is depicted in Figure 2.

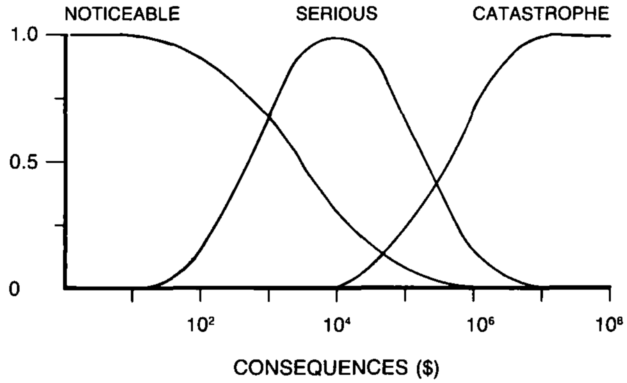

Fig. 1. Linguistic values for 'consequences'.

In choosing the base variable for the risk factors, the following guidelines were taken into consideration [20]: (1) the base variable should accurately reflect the meaning of the linguistic value, (2) the values associated with a particular linguistic value should not change because of low or moderate judgment uncertainties, (3) strong judgment changes should be recognized by the appropriate movements along the base scale, and (4) small changes in judgment should not significantly affect the results of the model.

As indicated above, most of the primary terms for the linguistic variables can be derived based on empirical data and the experience of safety experts, as evidenced by the numerical reference points associated with each of the descriptive (verbal) estimates of the magnitude of likelihood, exposure, and consequences. According to Shinochaura [11], the successful application of the linguistic approach largely depends as much upon the skill of the analyst as the nature of the problem itself.

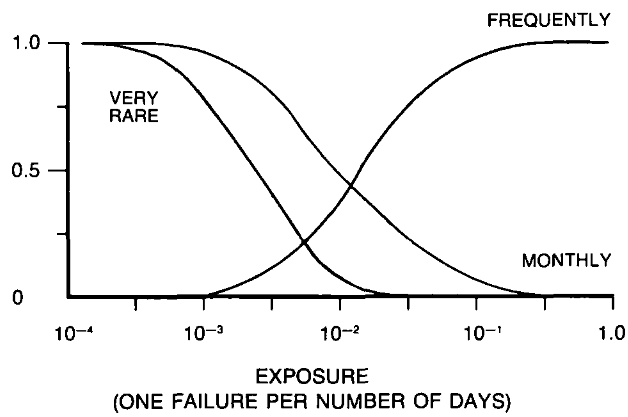

Fig. 2. Linguistic values for 'exposure'.

4. Approximate reasoning and fuzzy risk scores

In a traditional approach to risk analysis [2, 6], the seriousness of the risk due to a recognized hazard (risk score) is calculated as the product of numerical ratings assigned of each of the three factors. The risk score is then compared with the Risk Score Summary. This is done to make a recommendation with respect to an appropriate action to be undertaken in order to reduce or eliminate existing hazards. For example, for the hazards with *higher* risk scores (high risk zone) the action columns calls for 'immediate' corrective action, while *medium* risk scores are in the action category called 'urgent'.

Although in the first step of the above procedure the analyst is required to select some verbal descriptors of each of the three risk factors, in the second step these are translated into single numbers, and their product (risk score) is translated back into the action column, with verbal description of the seriousness of risk. Also, the risk scores do not have unique recommended actions, but a broad range of risks, for example between 270 and 1500 is considered to constitute the highest hazards for which an immediate corrective action is required. Clearly, there must be some difference in seriousness of perceived risk, and different actions (if any) may be required for the hazards with the risk scores of 270 and 1500.

4.1. *Vagueness in risk analysis*

Logically, the risk analysis can be based on the following premise: IF *exposure* = (e) and *likelihood* = (l) and *consequences* = (c), THEN *risk* = (s), where s stands for the linguistic variable with such values as *very high*, *high*, *substantial* risk, *possible* risk, and *some slight* risk. The above linguistic values are based on the numerical magnitude of the risk score. Still, the meaning of the above propositions are vague, and therefore the rules of classical logic are not applicable.

People usually organize their world knowledge by causal relationships, and in reasoning people apply what they are most familiar with, i.e. the rules underlying conversation and language [26]. However, the application of rules of classical logic implies that the meaning of propositions is unambiguous [13]. Because all the factors in risk analysis are vague and imprecise, an alternative method must be used, allowing for approximate reasoning from the vague inputs. The method used here is based on fuzzy reasoning [25, 27]. In the following section the exploratory usage of fuzzy (or approximate) reasoning is presented.

4.2. *Fuzzy models in risk analysis*

As stated by Zadeh [28] the approximate (fuzzy) reasoning refers to the process by which an imprecise conclusion is deduced from a collection of imprecise premises, and such reasoning is qualitative rather than quantitative in nature. As evidenced by the published literature [10, 16, 18, 24, 29] there have been numerous applications of fuzzy logic and approximate reasoning techniques in many different areas of interest. In this paper, we propose to base the risk analysis on the method of approximate reasoning [24, 27] which utilizes fuzzy conditional statements and compositional rules of inference.

A fuzzy conditional statement: IF A THEN B, or $A \to B$, where the antecedent (A) and consequent (B) are fuzzy sets, describes a fuzzy relation R between two fuzzy variables A and B. If A is a fuzzy subset in a universe of discourse U, and B is a fuzzy subset of a universe of discourse V, then the Cartesian product of A and B is defined as a fuzzy relation R:

$$A \times B = \sum_{R=U\times V} (f_A(u) \wedge f_B(v))/(u, v)), \tag{1}$$

where R is usually given in the form of a matrix, and $\sum$ stands for the union.

According to Mamdani's conjunctive logic [24], if the fuzzy relation R, from U to V, is known, and A is a fuzzy subset of U, then the fuzzy subset B of V, which is induced by A, is given by the composition of R and A as follows:

$$B = A \circ R, \tag{2}$$

where B is given by the max–min product of A and R:

$$B = A \circ R \Rightarrow f_B(v) = \sup_u (f_A(u) \wedge f_R(u, v))/(v). \tag{3}$$

For example, if it is known that *exposure* = 'high', and the relation between *exposure* and *risk* (where risk is also defined as a linguistic variable) is R, then the value of *risk* can be found using the compositional rule of inference.

Although, there are many different ways of interpreting conditional propositions for the purposes of fuzzy reasoning (for a review see [30] and [31]), in this paper only Zadeh's maximin rule [28] was used. The elementary models of approximate reasoning from conditional propositions were taken from Baldwin and Pilsworth [30].

4.3. *Expert derived verbal rules for risk assessment*

A number of simple production rules, which would be most likely perceived in a similar way by a majority of risk analysts, can be relatively easily identified. For example, if *exposure* is known to be 'very high', *likelihood* is 'very likely', and *consequences* are 'very high', then *risk* could be defined as 'very very high' or 'extremely high'. However, in many other cases derivation of the risk value is not that obvious and hence derivation may be very difficult.

For the purpose of this study, two different examples of risk estimation in hypothetical situations are considered. The first example refers to the situation in which both *exposure* and *likelihood* are more or less constant and can be easily estimated, but the potential *consequences* may vary considerably, and therefore the value of *risk* will also change. The second example deals with the estimation of risk in a situation where originally the relationships between the given factors and risk, and risk values are known, and where the risk factors change inducing a change in the original value of risk.

Example 1. Suppose the *exposure* or frequency of occurrence of the hazard event that could start an accident sequence is 'high', meaning that the hazard event occurs daily. The *likelihood* that once the hazard event occurs, the complete accident sequence of events will follow, is perceived as 'likely' or 'quite possible'. Although these two factors are considered to be relatively constant, the *consequences* of the hazard event may change, depending upon the time of the day, and therefore may be perceived differently at different times. The question to be answered is how would the value of *risk* (S) be affected by changes in potential *consequences* from 'more-or-less medium' to 'high'?

Suppose the following universes of discourse and relevant propositions (see Table 2 for the definitions of linguistic values) are defined:

Exposure $X_E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$,
Likelihood $X_L = \{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$,
Consequences $X_C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$,
Risk $Z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7\}$,

and

P_E: E = 'high' = (0, 0, 0.1, 0.3, 0.7, 0.9, 1.0),
P_L: L = 'likely' = (0, 0.1, 0.5, 0.7, 0.9, 1.0, 1.0),
P_{C1}: C_1 = 'more-or-less medium' = (0, 0.45, 0.84, 1.0, 0.84, 0.45, 0),
P_{C2}: C_2 = 'very high' = (0, 0, 0, 0.1, 0.5, 0.8, 1.0),
P_S: S = '?'

where $P_E \subset X_E$, $P_C \subset X_C$, $P_L \subset X_L$, and $S \subset Z$. P_i for $i = (E, C, L$ and $S)$ are fuzzy propositions, and X_E, X_C, X_L and Z are universes of discourse.

The fuzzy relation R between $E = P_E$ and $L = P_L$ is the product of P_E and P_L in the following form:

$$R_{E \times L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0 & 0.1 & 0.5 & 0.7 & 0.7 & 0.7 & 0.7 \\ 0 & 0.1 & 0.5 & 0.7 & 0.9 & 0.9 & 0.9 \\ 0 & 0.1 & 0.5 & 0.7 & 0.7 & 1.0 & 1.0 \end{bmatrix}.$$

Since the risk $S = P_C \circ R_{E \times L}$, then according to equation (3),

$$S_1 = (0, 0.1, 0.5, 0.7, 0.7, 0.7, 0.7),$$

$$S_2 = (0, 0.1, 0.5, 0.7, 0.9, 1.0, 1.0) \quad \text{or 'high'}.$$

In other words, if the consequences are 'very high' then the risk is 'high'. An interpretation of S_1 when consequences are 'more-or-less medium' will be discussed below.

Example 2. Consider the hazardous situation in which the following fuzzy conditional statement, given by a human expert, applies: IF *exposure* = 'very high', *consequences* = 'more or less high', *likelihood* = 'likely' THEN risk = 'high'. We will define the following propositions:

P_E: E = 'very high' = (0, 0, 0, 0.1, 0.5, 0.8, 1.0),
P_C: C = 'more or less high' = (0, 0, 0.3, 0.5, 0.85, 0.95, 1.0),
P_L: L = 'likely' = (0, 0.1, 0.5, 0.7, 0.9, 1.0, 1.0),
P_S: S = 'high' = (0, 0, 0.1, 0.3, 0.7, 0.9, 1.0).

The above statement, concerning the value of risk under the described hazardous condition, can be written using fuzzy logic [30] as follows:

$$\text{IF } (E = P_E) \cap (C \times P_C) \cap (L = P_L) \text{ THEN } S = P_S \tag{4}$$

subject to a propositional assertion $P = P_E \cap P_C \cap P_L$. Such a conditional proposition defines a relation D, where $D \subset X_E \times X_C \times X_L \times Z$. This can be expressed using the max–min definition as

$$D = D_E \cap D_C \cap D_L \tag{5}$$

and primitive conditional propositions are in the form of:

$$\left.\begin{array}{l} \text{IF } E \text{ THEN } S \ (D_E) \\ \text{IF } C \text{ THEN } S \ (D_C) \\ \text{IF } L \text{ THEN } S \ (D_L) \end{array}\right\} \quad D_i \subset X_i \times Z \quad \text{for } i = (E, C, L), \tag{6}$$

or equivalently:

$$\text{IF } E = P_E \text{ THEN } S = P_S \Rightarrow D_E = P_E \times P_S,$$
$$\text{IF } C = P_C \text{ THEN } S = P_S \Rightarrow D_C = P_C \times P_S,$$
$$\text{IF } L = P_L \text{ THEN } S = P_S \Rightarrow D_L = P_L \times P_S.$$

The relationship between likelihood and risk ($X_L \times Z$), exposure and risk ($X_E \times Z$), and consequences and risk ($X_C \times Z$) can be established by human experts, as is done in the traditional approach. Considering the primitive conditional propositions given by (6), we derive the following fuzzy relations D_E, D_C, and D_L using formula (3):

$$D_E \subset X_E \times Z = \begin{bmatrix} 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0 & 0 & 0.1 & 0.3 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.8 & 0.8 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 1.0 \end{bmatrix},$$

$$D_C \subset X_C \times Z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0 & 0 & 0.1 & 0.3 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.85 & 0.85 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 0.95 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 1.0 \end{bmatrix},$$

$$D_L \subset X_L \times Z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.3 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.7 & 0.7 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 0.9 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 1.0 \\ 0 & 0 & 0.1 & 0.3 & 0.7 & 0.9 & 1.0 \end{bmatrix}.$$

Let us assume now that the conditions specified above have changed, and one has to deal with a new hazardous situation which induces a different value of risk. We will use the above model of approximate reasoning (fuzzy compositional rule of inference) to answer the following question: given (E) *exposure* = 'medium', (C) *consequences* = 'very high', and (L) *likelihood* = 'unlikely', what is the value of *risk* (S)?

The compositional rule of inference [30] (as the least restrictive inference $S \subset Z$ of P from D), given by $S = P \circ D$, becomes now

$$S = \bigcap_{i=(E,C,L)} (P_i \circ D_i) \tag{7}$$

where P_E = 'medium', P_C = 'very high', and P_L = 'unlikely' (see Table 2), and the above formula can be written as follows:

$$s(z) = \bigwedge_{i=\{E,C,L\}} \left\{ \bigvee_{x_i \in X_i} [p_i(x_i) \wedge d_i(x_i; z)] \right\}. \tag{8}$$

According to (8) the value of risk will then be equal to

$$S = (P_E \circ D_E) \cap (P_C \circ D_C) \cap (P_L \circ D_L). \tag{9}$$

After performing relevant computations, one can obtain a numerical interpretation of the linguistic value of risk (S) in a new hazardous situation. From [9] we have:

$$\begin{aligned} S &= (0\ \ 0\ \ 0.1\ \ 0.3\ \ 0.5\ \ 0.5\ \ 0.5) \cap (0\ \ 0\ \ 0.1\ \ 0.3\ \ 0.7\ \ 0.9\ \ 1.0) \\ &\quad \cap (0\ \ 0\ \ 0.1\ \ 0.3\ \ 0.5\ \ 0.7\ \ 0.7) \\ &= (0\ \ 0\ \ 0.1\ \ 0.3\ \ 0.5\ \ 0.5\ \ 0.5). \end{aligned}$$

which could be approximated by '[(more or less high) and (more or less medium)]'.

5. Linguistic approximations and interpretations of the risk scores

Verbal models [16, 32], or quantitative models with linguistic variables and verbally formulated relations between the variables, may be very useful in the area of systems safety in general [9, 10], and, as shown above, in risk analysis in particular. This is primarily due to the fact that human experts are capable of making knowledgable and reliable verbal statements about intuitively perceived phenomena of the real world [34]. Since the linguistic values are naturally vague, they allow greater flexibility than single numbers.

According to Wenstøp [16] by introducing linguistic variables as the inputs into models, one takes a step toward meeting the demand for validity. The problem remains, however, how to translate the fuzzy outputs into a meaningful set of linguistic values. Such a process, called *linguistic approximation*, is essential in risk analysis to decide on the corrective action that may need to be associated with a particular fuzzy value of risk. As seen in the example stated above, it is not a trivial task to find a label for such a fuzzy set (of risk) at $S = (0, 0.1, 0.5, 0.7, 0.7, 0.7)$.

Fortunately, some methods are presently available to deal with this problem. The simplest method, called the 'best fit' method is usually applied when the set of possible linguistic expressions is small, and it is computationally easy to calculate a distance from the fuzzy output to the fuzzy sets representing the available linguistic values. The natural term whose fuzzy set is the closest to the output is then selected as its meaning. Such a method was used in development of the Fuzzy Risk Analyzer [33] in the domain of computer systems security.

Clements [22] proposed a more advanced method of 'successive approximations' based on the evaluation of the endpoint 'brackets' and hedges which are being replaced as the expression 'in progress' gets closer to the fuzzy set being approximated. Although a large number of natural expressions can be efficiently evaluated, the method requires that all fuzzy sets be convex.

The LAMS system developed by Eshragh and Mamdani [18] does not require normality of fuzzy sets, and allows an assignment of linguistic values to a fairly complex fuzzy spread by labelling its segments. In this 'piecewise decomposition' technique, the linguistic expressions chosen for each interval are then combined using the fuzzy connectives 'and' or 'or'.

Wenstøp [16, 23] proposed a context independent, quantitative analysis with linguistic values utilizing an APL auxiliary language. The linguistic approximation method, implemented by an APL's LABEL function is based on two parameters of the fuzzy set to be labeled, i.e. its *imprecision* (the sum of its membership values) and its *location* (the center of gravity). One of the 56 linguistic labels (spread out in a location–imprecision system) with the shortest distance to the coordinates of the fuzzy set to be labeled is chosen as the representation of the fuzzy output from the model. The main concern of this method is to ensure that the input–output values are acceptable by the standards of natural language, provided that the linguistic expressions are used systematically and diligently.

This short review of linguistic approximation techniques indicates that the meaningful interpretation of the fuzzy outputs of approximate reasoning models are not only possible, but can be efficiently performed with assistance of computers. Technical feasibility to successfully use verbal models is of utmost importance with respect to the future work of developing domain oriented expert system for risk analysis.

6. Psychophysical judgment and measurement

The important issue in developing a fuzzy model of the risk analysis system is practical derivation of the numerical representations for linguistic values of the risk factors. This problem is one of measurement, i.e. the assignment of numbers to represent properties of the involved events, objects or situations. Although measurements of subjectivity perceived phenomena is not an easy task, modern psychophysics offers methods that allow us to do just that. In fact, the psychophysical scaling techniques, extensively used in the field of experimental psychology [35, 36], are credited in contributions to the solution of problems in sensory processes, memory, learning, social behaviour and ergonomics/human factors.

In psychophysical measurement, the extent to which the number system reflects the properties of objects or events define one of the three basic scales: ordinal, interval, or ratio. The conclusions that can be drawn about differences among numbers are restricted to proper recognition of the type of scale the particular measurement constitutes [36]. Otherwise, serious errors in data analysis and interpretation may result.

While ratio scales are the most desirable ones, they are often difficult, if not impossible, to develop. The scale of most practical use in industrial research is the interval scale. This scale measurement requires that the numbers are assigned to properties (situations) in such a way that the differences among numbers reflect the differences among properties or situations being measured. Since this is a sufficient condition in risk analysis, interval scaling can be used to develop numerical bases for the linguistic values of risk factors.

One of the commonly applied methods to develop an interval scale is the 'categorical judgment' technique. Though there are many different ways through which category judgments may be derived, the most extreme case is when the subject makes his judgment on a continuous line, and the marks are later categorized with a ruler [37, 38].

In order to provide for reliable distinction between different categories (linguistic descriptors in our case), it is often recommended that the number of categories be restricted to no more than seven. However, as indicated by Jones [37] even though apparent reliability may go down, it sometimes is desirable to have more categories since one intends to establish category boundaries. Thus, there may be some overlapping in stimulus placement. This way the 'end-effect' or a situation in which the subjects tend to avoid the use of one of the end categories can be diminished.

The use of categorical judgments have been quite successful in many practical situations. The scales are usually easy for experts to use and give a lot of information in a relatively short time. Rodgers and Shealy [39] utilized an interval scaling technique to construct the degree of *consequences* of a hazardous event. A word scale with linguistic descriptors ranging from 'minor cuts and bruises' to 'catastrophe/multiple fatalities' was presented to a group of 18 safety engineers (see Table 1). The subjects were asked to locate the linguistic descriptors, proposed originally by Fine [6], along a vertical line marked 0 (at the bottom) and 100 (at the top). The average values and standard deviations measuring response variability were then plotted on the 0–100 scale (see Figure 3). The authors concluded that contrary to the original scale where the descriptors were equally spaced along the scale (with the exception of the 'minor cuts and bruises' category), there was 'a considerable' overlap in different subjects' judg-

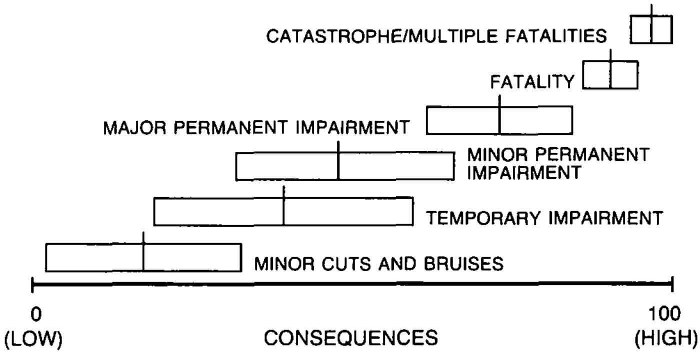

Fig. 3. Psychophysical scaling of 'consequences' (based on Rodgers and Shealy [39]).

ments related to placement of 'temporary impairment' and 'minor temporary impairment' categories.

The above interval scaling technique can be used not only to determine the location of different verbal descriptors on the interval scale, but also to derive a numerical representation of the compatibility functions for the linguistic values. This can be achieved by treating the derived scale as the base variable for *consequences*, and having the derived averages and their spreads as representations of the peak and border values of the respective linguistic descriptors. This way the differences in subjective opinions about judgmental factors of risk can be incorporated into the model, and more objective representations of the linguistic values can be derived.

7. Final remarks

The theoretical considerations presented here are of a preliminary nature. Since human reasoning is intrinsically fuzzy, it is believed that the proposed approach will be very useful in the analysis of hazards and risks in many industrial environments. The advantage of the fuzzy approach lies in the fact that fuzzy reasoning can be computerized, thereby opening the possibility for creation of a fuzzy expert system in the area of risk analysis.

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References

[1] W.H. Heinrich, D. Peterson and N. Ross, Industrial Accident Prevention (McGraw-Hill, New York, 1980).
[2] K.T. Graham and G.F. Kinney, A practical safety analysis system for hazards control, J. Safety Research 12 (1980) 13–20.
[3] C.B. Chapman and D.F. Cooper, Risk analysis: Testing some prejudices, European J. Oper. Res. 14 (1983) 238–247.
[4] S.W. Malasky, Systems Safety (Heyden, New Jersey, 1974).
[5] C.B. Brown, A fuzzy safety measure, J. Engrg Mech. Div. 5 (1979) 855–872.
[6] W.T. Fine, Mathematical evaluations for controlling hazards, in: Selected Readings in Safety (Academic Press, Macon, GA, 1973) 68–85.
[7] M. Smithson, Applications of fuzzy set concepts to behavioral sciences, Math. Social Sci. 2 (1982) 257–274.
[8] L.A. Zadeh, The concept of linguistic variable and its application to approximate reasoning, Inform. Sci., part I: 8 (1975) 199–249, part II: 8 (1975) 301–357, part III: 9 (1975) 43–80.
[9] D.I. Blockley, Analysis of subjective assessments of structural failures, Internat. J. Man–Machine Stud. 19 (1978) 185–195.
[10] T.T.P. Yao, Damage assessment of existing structures, J. Engrg. Mech. Div. 4 (1980) 785–799.
[11] Y. Shinohara, Fuzzy set concepts of risk assessment, Unpublished Report WP-76-2, International Institute for Applied Systems Analysis, Laxenburg (1976).
[12] T.B. Feagans and W.F. Biller, Fuzzy concepts in the analysis of public health risks, in: P.P. Wang, S.K. Chang, Eds., Fuzzy Sets (Plenum Press, New York, 1980) 391–404.
[13] A.C. Zimmer, Verbal versus numerical processing, in: R. Scholz, Ed., Individual Decision Making Under Uncertainty (North-Holland, Amsterdam, 1983).
[14] C.V. Negoita, Expert Systems and Fuzzy Systems (Benjamin/Cummings, Menlo Park, CA, 1985).
[15] A.C. Zimmer, A model for the interpretation of verbal predictions, Internat. J. Man–Machine Stud. 20 (1984) 121–134.
[16] F. Wenstop, Quantitative analysis with linguistic values, Fuzzy Sets and Systems 4 (1980) 99–115.
[17] A.K. Nath and T.T. Lee, On the design of a classifier with linguistic variables as inputs, Fuzzy Sets and Systems 11 (1983) 265–286.
[18] F. Eshragh and E.H. Mamdani, A general approach to linguistic approximation, Internat. J. Man–Machine Stud. 11 (1979) 501–519.
[19] F. Wenstop, Exploring linguistic consequences of assertions in social sciences, in: M.M. Gupta, R.K. Ragade, R.R. Yager, Eds., Advances in Fuzzy Set Theory and Applications (North-Holland, Amsterdam, 1979) 501–518.
[20] T.W. Cooley and T.O. Hicks, Jr., A fuzzy set approach to aggregating internal control judgment, Management Sci. 29 (1983) 317–334.
[21] F. Narashimhan, Goal programming in a fuzzy environment, Decision Sci. 11 (1980) 325–336.
[22] D.P. Clements, Fuzzy ratings for computer security evaluation, Ph.D. Dissertation, University of California, Berkeley (1977).
[23] F. Wenstop, Quantitative analysis with linguistic values, Fuzzy Sets and Systems 4 (1980) 99–115.
[24] E.H. Mamdani, Advances in the linguistic synthesis of fuzzy controllers. Internat. J. Man–Machine Stud. 8 (1976) 669–678.
[25] L.A. Zadeh, A fuzzy-algorithmic approach to the definition of complex and imprecise concepts, Internat. J. Man–Machine Stud. (1976) 249–291.
[26] I. Begg, On the Interpretation of syllogisms, J. Verbal Learning and Verbal Behavior 21 (1982) 595–620.
[27] L.A. Zadeh, Fuzzy logic and approximate reasoning, Synthese 30 (1975) 407–428.
[28] L.A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. Systems Man Cybernet. 3 (1973) 28–44.
[29] Y. Tsukamato and T. Terano, Failure diagnosis by fuzzy logic, in: Proceedings of the IEEE Conference on Decision and Control (1977) 1390–1395.

[30] T.F. Baldwin and B.W. Pilsworth, A model of Fuzzy reasoning through multi-valued logic and set theory, Internat. J. Man-Machine Stud. 11 (1979) 351–380.
[31] T. Whalen and B. Schott, Alternative logics for approximate reasoning in expert systems: A comparative study, Unpublished Report, Georgia State University, Atlanta, GA (1984).
[32] G.C. Oden, Integration of fuzzy linguistic information in language comprehension, Fuzzy Sets and Systems 14 (1984) 29–41.
[33] K.T. Schumucker, Fuzzy Sets, Natural Language Computations, and Risk Analysis (Computer Science Press, Rockville, MD, 1984).
[34] H.M. Hersh and A. Caramazza, A fuzzy set approach to modifiers and vagueness in natural language. J. Experiment. Psychology: General 105 (1976) 101–107.
[35] M. Kochen, Applications of fuzzy sets in psychology, in: L.A. Zadeh, Ed., Fuzzy Sets and Their Applications to Cognitive and Decision Processes (Academic Press, New York, 1975) 395–408.
[36] G.A. Gescheider, Psychophysics: Method and Theory (Lawrence Erlbaum Associates, Hillsdale, NJ, 1976).
[37] F.N. Jones, Overview of psychophysical scaling methods, in: E.C. Carterette, M.P. Friedman, Eds., Handbook of Perception, Vol. II, Psychophysical Judgment and Measurement (Academic Press, New York, 1974).
[38] S.S. Stevens, Perceptual magnitude and its measurement, in E.C. Carterette, M.P. Friedman, Eds., Handbook of Perception, Vol. II, Psychophysical Judgment and Measurement (Academic Press, New York, 1974).
[39] S.H. Rodgers and T. Shealy, Psychophysical scaling methods, In: Ergonomic Design for People at Work, Vol. I (Lifetime Learning Publications, Belmont, CA, 1983).