

Felles-lab RE7: Residence Time distribution  
Report

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**Gruppe B16**

October 4, 2013



### **Abstract**

This experiment has shown that the flow in a packed tubular reactor behaves closer to ideality when the flow rate is high with respect to residence time. Residence time distributions have been calculated for cases of several concentrations of tracer and flow speeds, and the trends are consistent for all of the cases.

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## 1 Introduction

The main target of this experiment is to measure the average residence time ( $t_m$ ) in a packed tubular reactor by first conducting an experiment using a tracer (NaCl) and then repeat the test using a dye for visual inspection. The concentration in the flow will be measured by a computer and then analysed in order to produce a probability density ( $E(t)$ ) for the residence time. Comparing the calculated residence time to the ideal case is also an important task in this experiment.

## 2 Theory

The theory in this chapter is gathered from Fogler [1].

Residence time distribution (RTD) is used in chemical engineering as a probability density distribution that describe the amount of time a given fluid element is spending inside a reactor. A fluid element is defined as a small volume of a fluid with continuous properties such as concentration. RTD is used to find the deviation from the ideal case. For the ideal case it is possible to calculate RTD, but for the real case, the RTD must be measured experimentally. To calculate the RTD there are three assumptions must to be fulfilled:

- The reactor is at steady-state
- Transport at the inlet and the outlet takes place only by advection.
- The fluid is incompressible

The last assumption is not really necessary, but compressible flow and complexity. Complexity also increases if the flow is multi-phase.

### 2.1 External residence time distribution

The external residence time distribution or exit residence time distribution ( $E(t)$ ) is a probability density function that when integrated over all time equals 1.

$$\int_0^{\infty} E(t) dt = 1 \quad (2.1)$$

The term also says how large fraction that spends time  $\Delta t$  inside the reactor by setting up the closed integral so that (2.1) equals:

$$\int_{t_1}^{t_2} E(t) dt \quad (2.2)$$

In the same way the cumulative distribution over a time from  $t = 0$  to  $t = t_1$  equals:

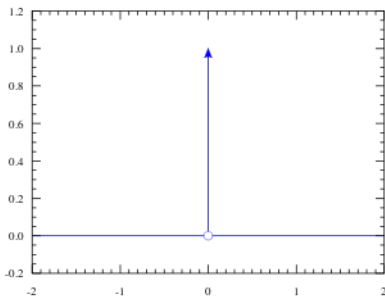
$$F(t_1) = \int_0^{t_1} E(t) dt \quad (2.3)$$

## 2.2 Measurement of the RTD

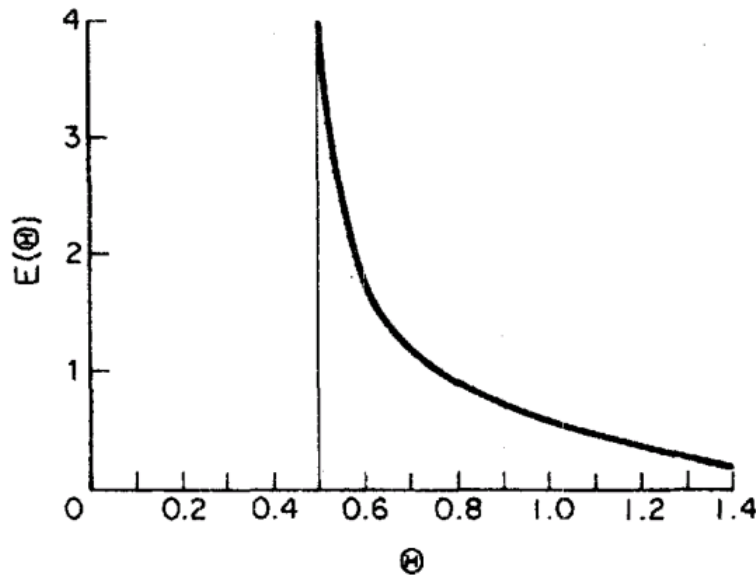
There are two commonly used methods to experimentally determine the RTD of a reactor: the *pulse input method* and the *step input method*. In both cases a tracer is sent through the reactor making it possible to measure how long time it takes for the particles to travel through a designated volume. In this experiment the experimental procedure will follow that of the pulse method.

### 2.2.1 Pulse input method

In a pulse input all of the tracer is briefly injected in one shot into the feedstream giving a concentration peak of the tracer at the reactor inlet. Theoretically the injection is described mathematically by the Dirac delta function, and even though it is not possible to undertake an injection lasting for an infinitely short period of time, the injection time is so short compared to the total time the tracer spends in the reactor that it is an acceptable approximation. The properties of the Dirac delta function are described in appendix D.



**Figure 1:** Graphic representation of the Dirac delta function gathered from Wikimedia [2].



**Figure 2:** The figure is showing an illustration of the concentration of the tracer at the outlet and when inserted with a pulse injection. The figure is gathered from [4]

If the reactor in question is an ideal reactor, the concentration impulse exiting the reactor will be identical to the injection pulse, but in reality this is not the case. The case of a real reactor will be further explored in section A.4. A graphical illustration of the pulse response is found in figure 2 (figure 13.9 in [?]).

### 2.3 Equations for calculating residence time distribution

To set up equations for the outlet we first define  $\Delta N$  as the amount of tracer leaving the reactor in the time interval from  $t = t$  to  $t = t + \Delta t$ , where  $\Delta t$  is a small increment so that the concentration of the tracer is constant,  $C(t)$ , is virtually constant.  $\nu$  is the volumetrical flow. We can then set up the relationship shown in (2.4)

$$\Delta N = C(t)\nu\Delta t \quad (2.4)$$

If (2.4) is divided by the total amount of tracer,  $N_0$ , the equation becomes:

$$\frac{\Delta N}{N_0} = \frac{\nu C(t)}{N_0} \Delta t \quad (2.5)$$



The residence time distribution ( $E(t)$ ) for a pulse injection is then defined as:

$$E(t) = \frac{\nu C(t)}{N_0} \quad (2.6)$$

As a consequence of (2.5) and (2.6) we have the relationship:

$$\frac{\Delta N}{N_0} = E(t)\Delta t \quad (2.7)$$

$E(t)$  can also be expressed as the relationship between the momentary concentration of tracer exiting the reactor divided by the total concentration of tracer injected. The momentary concentration is hard to measure directly, so in stead sensors measure the conductivity ( $G(t)$ ) of the solution entering and exiting the reactor. The change in conductivity is proportional to the change in concentration of tracer, so the RTD can be calculated using the conductivity data directly, as shown in equation (2.8):

$$E(t) = \frac{C(t)}{\int_0^\infty C(t) dt} = \frac{G(t)}{\int_0^\infty G(t) dt} \quad (2.8)$$

## 2.4 Space time ( $\tau$ ) and mean residence time ( $t_m$ )

The average residential time,  $\tau$ , is the relationship between the reactor volume ( $V$ ) and the volumetrical flow ( $\nu$ ) so that:

$$\tau = \frac{V}{\nu} \quad (2.9)$$

So long as the reactor has a volumetrical flow that is constant with no stagnant zones or dispersion, the mean residence time ( $t_m$ ) is equal to the nominal space time ( $\tau$ ) in all cases. However, if these criteria are not fulfilled it must be expected that the experimental value of  $t_m$  will deviate from  $\tau$ , in many cases quite drastically. Due to the fact that in a real reactor the flow will travel in non ideal flow patterns, there will for instance be ineffective contact between the molecules and stagnant zones. Ineffective contact leads to lower conversion than what would have been the case in an ideal reactor, and stagnant zones hold back molecules and prolong their residence time. Because molecules can spend a very different amount of time in the reactor it is useful to calculate a mean value for their residence time,  $t_m$ .

In general, the first moment of the RTD function gives the mean residence time:

$$t_m = \frac{\int_0^\infty tE(t) dt}{\int_0^\infty E(t) dt} = \int_0^\infty tE(t) dt \quad (2.10)$$

## 2.5 Additional exercise

The following task was given:

Pretend the following: A 2nd order  $A \rightarrow B$  reaction takes place in the reactor. Feed concentration  $C_{A0} = 1.5 \text{ mol L}^{-1}$ , reaction rate constant  $k = 0.01 \text{ L mol}^{-1} \text{ s}^{-1}$ . Assume segregated flow (batch reactor math) and calculate the average concentration of A,  $C_A$ , leaving the reactor.

### 2.5.1 Solving the exercise

Setting up the differential equation for a second order reaction.

$$\frac{dC_A}{dt} = -kC_A^2 \quad (2.11)$$

Solving the differential equation.

$$\frac{dC_A}{C_A^2} = -kdt \quad (2.12)$$

Solving using the boundary conditions given in the task.

$$-\frac{1}{C_A} \Big|_{C_{A0}}^{C_A} = -kt \Big|_0^t \quad (2.13)$$

Rearranging:

$$\begin{aligned} -\frac{1}{C_A} + \frac{1}{C_{A0}} &= -kt \\ \frac{C_A}{C_{A0}} + C_A C_{A0} kt &= C_{A0} \\ C_A(1 + C_{A0} kt) &= C_{A0} \\ C_A &= \frac{C_{A0}}{1 + C_{A0} kt} \end{aligned}$$

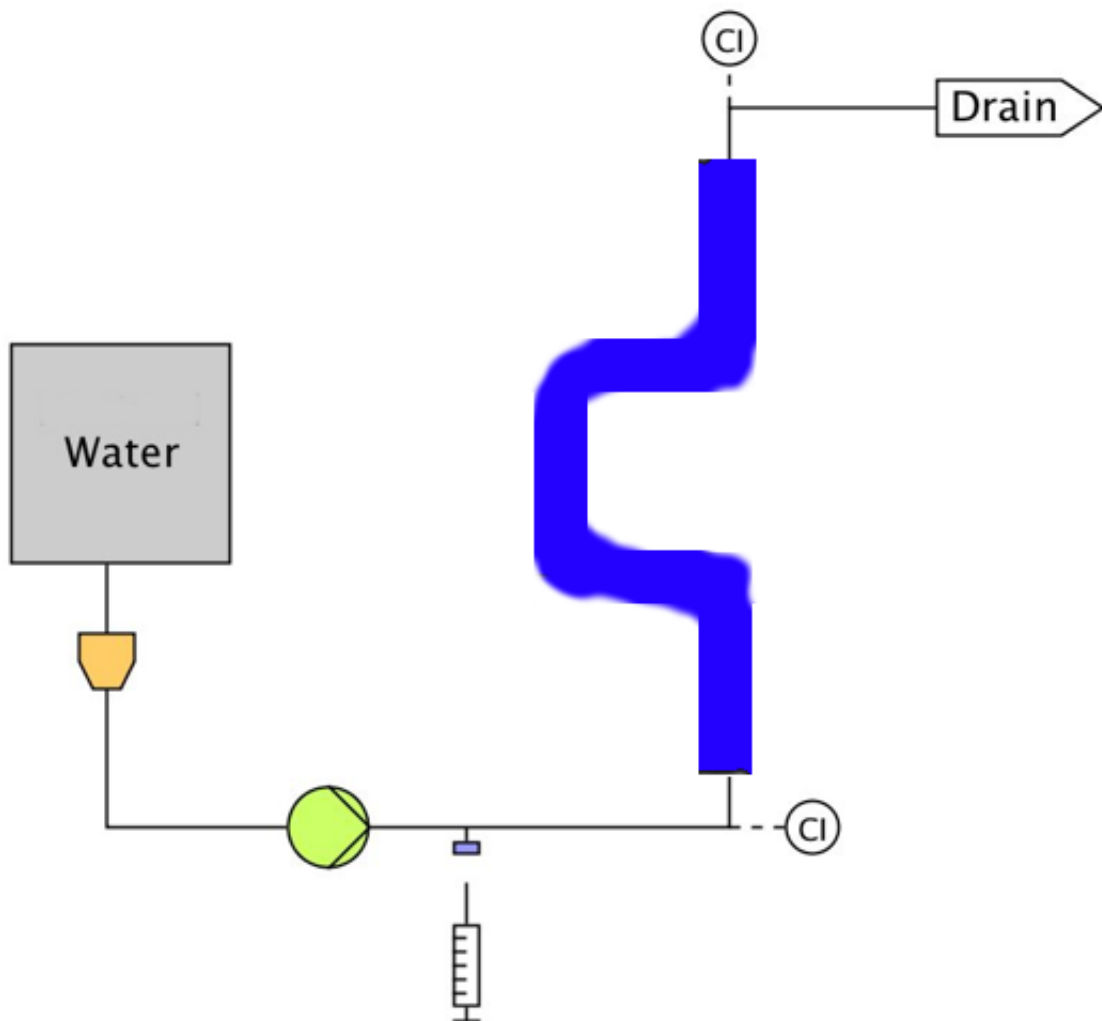
Inserting known values:

$$C_A = \frac{1.5}{1 + 1.5 \cdot 0.01 \cdot t} \quad (2.14)$$

### 3 Experimental

#### 3.1 Experimental Setup

The experimental setup is shown in figure (3). Tap water was pumped into a curved packed tubular reactor, and a tracer was injected into the feed. The concentration of tracer was measured in the fluid going both in and out of the reactor by two conductivity sensors. The concentration and time data was logged by a computer.



**Figure 3:** The figure is showing the experimental setup, including the pump and the injectionsite, as well as the reactor and the sensors for logging concentration.

## 3.2 Experimental procedure

Data for the calculation of the residence time distribution of the reactor was collected by injecting tracer following the pulse method. The experiment was conducted several times with pump speeds of 50%, 60% and 75% of maximum and concentrations of the tracer of  $50 \text{ gL}^{-1}$ ,  $100 \text{ gL}^{-1}$  and  $150 \text{ gL}^{-1}$ . The experiment was executed in accordance with the following procedure:

1. The filter (orange in figure (3)) was checked and the tank was filled with distilled water.
2. The pump must be started.
3. When the conductivity sensors showed that the flow was constant the tracer solution was injected onto the feed flow using a syringe.
4. The conductance data was logged on the computer for each part of the experiment was analysed using Matlab.

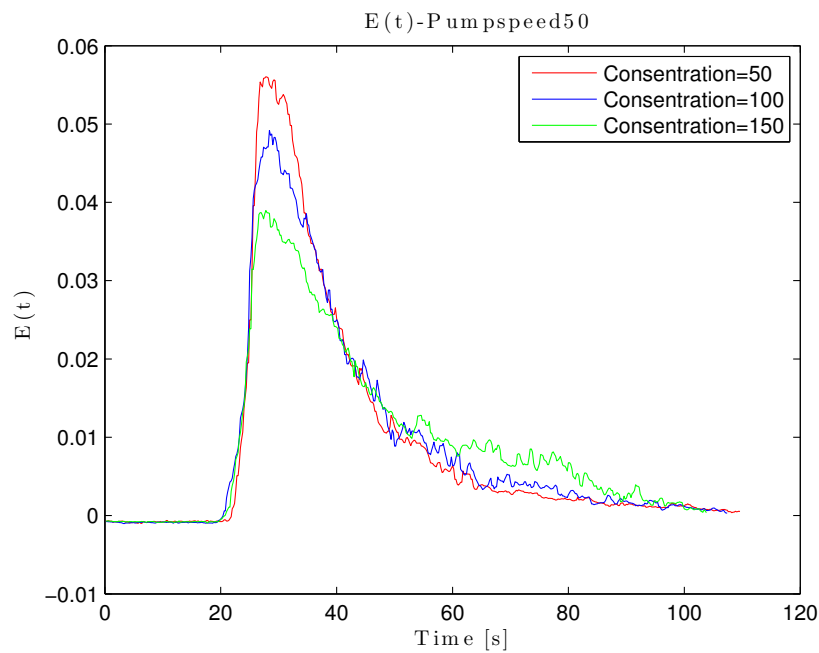
### 3.2.1 Others measurements

In order to calculate the average residence time, the volumetric flow was measured by simply disconnecting the tubes out of the reactor and measuring the volume for a given period of time and then simply using equation (C.1). The reactor volume was calculated by measuring time the fluid spend from sensor 1 to sensor 2, by emptying the reactor and look at the fluid front when the reactor was filled again. Then the reactor volume was calculated from equation (C.2).

## 4 Results

### 4.1 Residence time distribution for pumpspeed 50%

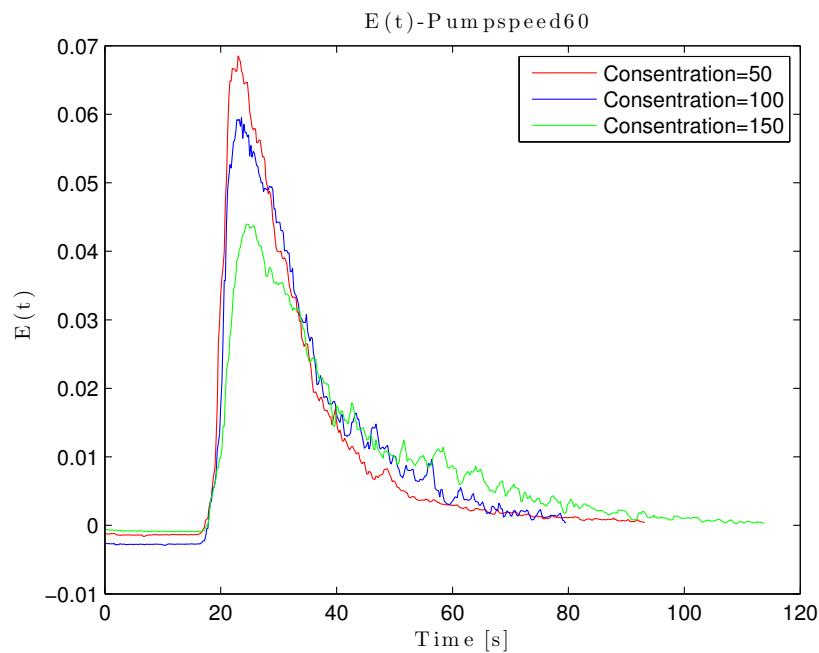
Figure 4 shows the residence time distribution for the three concentrations when the pump was set to 50%.



**Figure 4:** Results for the experiment when the speed of the pump was set to 50%

## 4.2 Residence time distribution for pumpspeed 60%

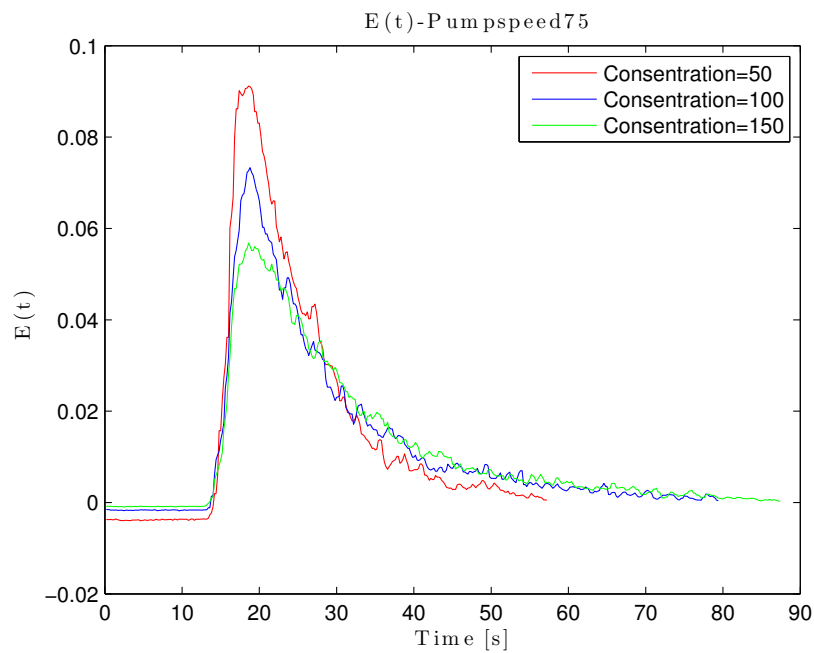
Figure 5 shows the residence time distribution,  $E(t)$  for the three concentrations when the pump was set to 60%.



**Figure 5:** Results for the experiment when the speed of the pump was set to 60%

### 4.3 Residence time distribution for pumpspeed 75%

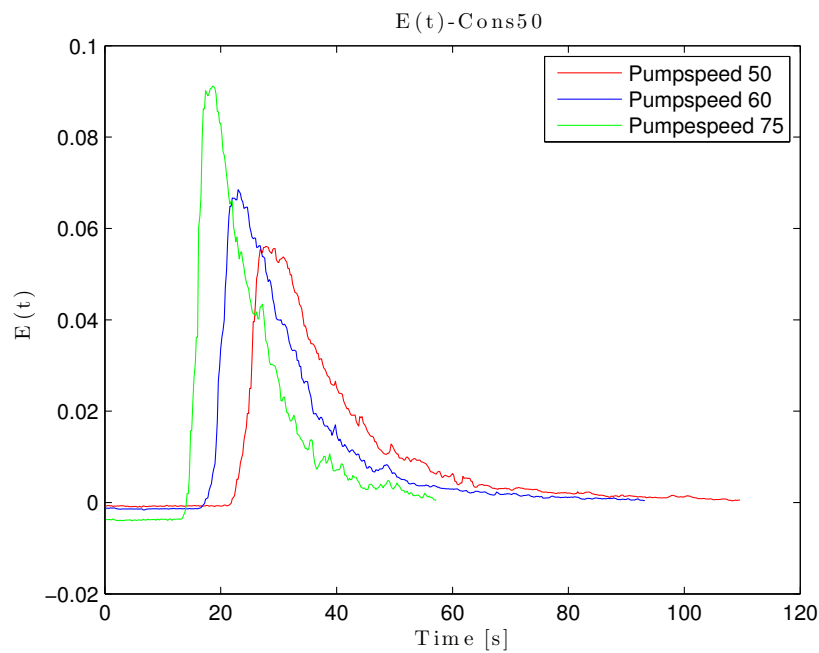
Figure 6 shows the residence time distribution,  $E(t)$  for the three concentrations when the pump was set to 75%.



**Figure 6:** Results for the experiment when the speed of the pump was set to 75%

#### 4.4 Comparison of different pumpspeeds

In figure 7, the concentration is equal ( $50 \text{ gL}^{-1}$  in all three cases, but with different pump speeds. This shows the impact of flow rate both in time and in the residence time distribution.

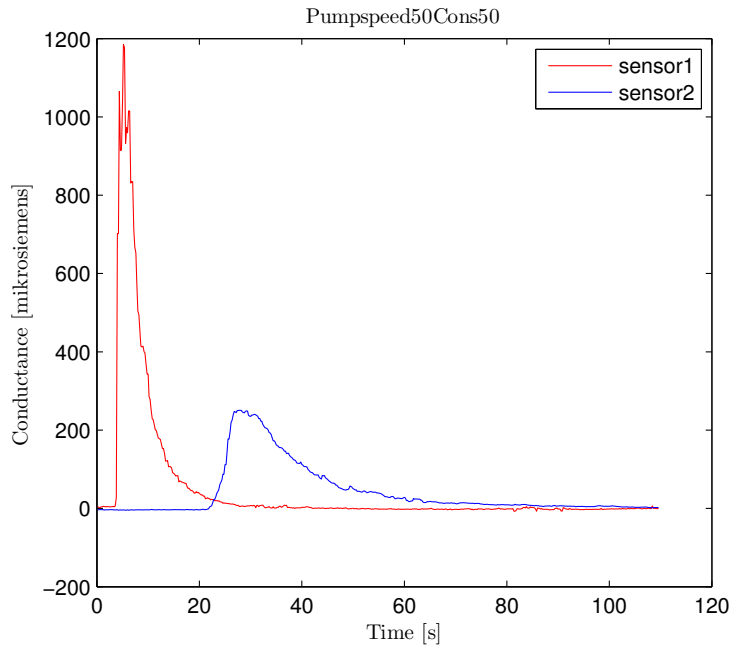


**Figure 7:** Residence time distribution for three different pump speeds when the concentration of the tracer was  $50 \text{ gL}^{-1}$

#### 4.5 Sensor- inlet and outlet

Figure 8 is showing the sensor input and output, for the case when the pump speed was set to 50% and the concentration of the tracer was  $50 \text{ gL}^{-1}$ . The baseline has been set to zero, so the actual conductance is the conductance on the plot + the conductance of pure water.





**Figure 8:** Results for the experiment when the speed of the pump is set to 50% and the concentration is  $50 \text{ g L}^{-1}$ .

Other plots with reactor inlet and outlet conductances is included in Appendix B

## 4.6 Nominal space time ( $\tau$ )

As mentioned in section 2.2.1, in an ideal reactor all elements entering through the feed stream will spend the same amount of time inside the reactor, given by equation (2.9). Presuming that the reactor in this experiment is ideal, the theoretical average residence times of the different pump speeds are given in table 1.

**Table 1:** Nominal space time of the different pump speeds calculated with equation (2.9).

Pumpspeed	50%	60%	75%
$\tau$	28,0 s	23,7 s	18,9 s

## 4.7 Mean residence time

Using the MATLAB-script in appendix A.4, table 2 was generated.

**Table 2:** The tables shows the mean residence time,  $t_m$  in seconds for a random particle in the reactor at different speeds (% of maximum speed) and inlet concentrations of the tracer (in  $\text{g L}^{-1}$ )

Cons	Pumpspeed	$t_m$
50	50	40.36
100	50	41.75
150	50	45.86
50	60	32.66
100	60	34.77
150	60	40.78
50	75	25.37
100	75	29.01
150	75	30.81

## 4.8 Additional exercise

Using equation (2.14) and the residence time distribution found in the experiment, the average concentration out of the reactor is given by (4.1):

$$\bar{C}_A = \int_0^{\infty} C(t)E(t) dt \quad (4.1)$$

Using the MATLAB script in appendix A.5, table 3 was generated to find the outlet concentration in  $\text{mol L}^{-1}$  Cons is the concentration of the tracer in  $\text{g mol}^{-1}$  in the reactor inlet, and Pumpspeed is the speed of the pump in % of maximum speed. Setting up the concentration from the experiment is unnecessary, since the inlet concentration was given in the task. However, different concentrations give different RTDs used to calculate the average outlet concentration of species  $A$ .

## 5 Discussion

As mentioned in section 2.2.1 the residence time distribution for a Plug Flow Reactor (PFR) corresponds with the Dirac delta function (Appendix D). However, the data from the lab obviously deviate quite distinctively from the theoretical prediction, so it is necessary to discuss what might have caused this non ideal behaviour.

**Table 3:** The tables shows the concentration at the reactor outlet in mol L<sup>-1</sup>

Cons	Pumpspeed	$C_A$
50	50	0.00003038
100	50	0.00001819
150	50	0.00002179
50	60	0.00002725
100	60	0.00002678
150	60	0.00001495
50	75	0.00004066
100	75	0.00002737
150	75	0.00001854

## 5.1 Causes of non-ideality

If the reactor in this experiment had been ideal the outlet data would have been identical to the inlet data but for a delay in time equal to  $\tau$ . It becomes clear from figure 8 that this is not the case, as the outlet data curve spans over a longer period of time and have a lower peak than the inlet data. The same trend can also be observed in the plots in appendix B. What causes this behaviour?

The deviation from the ideal case in this experiment can result from several factors. An ideal reactor would not contain any stagnant zones or zones of uneven turbulence, but in a real reactor such places will most likely occur. The bent shapes of the reactor in this experiment will certainly have created plentiful of both kinds. Stagnant zones will also occur in places of sudden enlargement or contractions, as shown in figure 2.8-4 in [3]. That this was in fact the case was confirmed by observation of both stagnant and turbulent zones highlighted by the dye moving through the reactor at different pump speeds.

Considering the  $E(t)$ -curves in figure 7 it becomes clear that the data lie closer to ideality when the pump speed is high. During the demonstrations with dye it became apparent that fewer stagnant zones formed at high pump speed; With the pump speed at 75% there was hardly any stagnant zones at all. It seems that even though the flow was still laminar, it moved too fast for any significant stagnant zones to form. This will have contributed to bringing the behaviour of the reactor closer to ideality.

There might also have been inaccuracies in the measured values, first and foremost due to bubbles of air flowing through the reactor disturbing the observed value of the conductance of the solution.

## 5.2 Pump speed vs. mean residence time

The mean residence time,  $t_m$ , varies with both the concentration and the pump speed as can be seen in table 2. When comparing mean residence time with the pump speed, it is easy to spot the trend that mean residence time decreases at higher pump speed for all concentrations. When comparing table 2 with figure 7 this trend is illustrated as the peak of the mean residence time is pushed closer to zero as the pump speed increases. This is as expected because the higher pump speed the faster volumetric flow, and therefore the time an indefinitely small fluid element spends in the reactor decreases.

## 5.3 Concentration vs. mean residence time

The mean residence time,  $t_m$ , also varies with the concentration. From table 2 it can be observed that the mean residence time increases with higher concentration. This correlates with figure 4 where the higher concentration yields a lower peak for the RTD as the fluid elements are spread out over a larger period of time. Note that the differences in concentration does not affect the mean residence as much as the pump speed. For example: An increase from 50% to 60% in pump speed decreases the  $t_m$  by 8 seconds when the concentration is  $50 \text{ g L}^{-1}$ . An increase in concentration from  $50 \text{ g L}^{-1}$  to  $100 \text{ g L}^{-1}$  only increases the  $t_m$  with 1.4 seconds when the pump speed is 50% of maximum.

## 5.4 Nominal space time versus mean residence time

Comparing the figures in table 1 to those of table 2 it becomes clear that particles spend a longer time in the reactor than what they theoretically would have done in an ideal reactor. These figures strongly support the trends indicated by the plots of the  $E(t)$  (figures 4, 5 and 6) and the raw data (figure 8). For instance, the higher the concentration of the tracer at a given pump speed, the longer the residence time. This is logical because a larger number of particles will lead to a larger number of particles trapped in stagnant zones etc., which will lead the residence time distribution curve to be prolonged over time. This behaviour can be observed in figure 4,5 and 6.

## 5.5 Comparison of outlet concentration in theoretical reaction

For the additional exercise, the average concentration was calculated from the MATLAB-script in appendix A.5 and the outlet concentrations is found in 3. The outlet concentrations (typically  $10^{-5} \text{ mol L}^{-1}$ ) is very low compared to the inlet concentration ( $1.5 \text{ mol L}^{-1}$ ). This indicates that most of reactant *A* is converted to product *B* in the reactor. Note that the average concentration is very low, but for the last period of time, the typical concentration of *A* would have been very small in a reactor without a reaction as well. When comparing the outlet concentrations with each other, it is difficult to spot a clear trend. It decreases and increases for both different pump speeds and different concentrations. This was rather unexpected so it can not ruled out that there is an error somewhere in the calculation. What could be expected was a decrease in outlet concentration when the pump speed was high, due to shorter mean residence time.

## 6 Conclusion

In this experiment the properties of a packed-tube reactor have been explored with respect to residence time distribution. The reactor in question has been shown to behave more like an ideal reactor the higher the flow rate through it, as this made the residence time distribution narrower and more similar to the Dirac delta function. The biggest sources of error are the bubbles of air that occasionally entered the feed stream and human error in handling of instruments.

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Trondheim, October 4, 2013

## References

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- [3] Geankoplis, Christie John *Transport Processes and Separation Process Principles (includes unit operations)*, 4. ed; Prentice Hall, 2003
- [4] New Mexico State University, <http://www.slideshare.net/narendrakumard/rtd-8786036>, visited 18.9.2013

## List of Symbols

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Symbol	Unit	Description
$A$	$\text{m}^2$	Reactor cross-section area
$C(t)$	$\text{g mol}^{-1}$	Concentration of tracer leaving reactor at time $t$
$C_A$	mol	Concentration of component A
$C_{A0}$	mol	Start concentration of component A
$D$	m	Reactor diameter
$E(t)$	probability density	Probability density distribution of residence time for fluid elements entering the reactor
$F(t)$		Cumulative distribution on tracer
$G(t)$	$\mu\text{S}$	Conductivity at time $t$
$k$	dimensionless	Reaction rate constant
$N_0$	mol	Total amount of tracer
$N_{Re}$	dimensionless	Reynolds number
$\Delta N$	mol	Amount of tracer leaving reactor in a time interval from $t = t$ to $t = t + \Delta t$
$r$	m	Reactor cross-section radius
$t$	s	Time
$t_m$	s	Mean residence time
$V$	mL	Reactor volume
$v$	$\text{m s}^{-1}$	Flow velocity
$\delta(t)$	dimensionless	Dirac delta function
$\mu$	$\text{kg m}^{-1} \text{s}^{-1}$	Dynamic viscosity
$\nu$	$\text{mL s}^{-1}$	Volumetrical flow
$\rho$	$\text{g mL}^{-1}$	Density
$\tau$	s	Nominal space time

---

## A Calculations and MATLAB-script

MATLAB was used in this experiment for all data analysis. Note that the following code was separated into two functions *residence.m* and *rtd.m* and a script *plots.m*.

### A.1 Adjustments of raw data

The raw data was imported, separated for the two sensors, the base line was adjusted to zero, and finally unnecessary data was cut off. The last part was done by assuming that the concentration of tracer was equal to zero when the value of the conductance was 0.5%.

---

```
function [newsensor1 newsensor2 time] = residence(inputdata, var, kons)
%This function takes inn a file (inputdata), a running variable (var) and a
% vector of concentration (kons). The output is an array of values from
% sensor 1 (newsensor1), array from sensor2 (newsensor2) and a time array
% with the same length as the arrys from the sensors

dataset = load(inputdata);
sensor1 = dataset(:,1);
sensor2 = dataset(:,3);
pumpspeed = dataset(1,2);
time = dataset(:,5);

% Hastighet for strØmningen
% Volum sprØyte: 1ml
concentration = kons(var);
m = 58.5; %molarvekt
c = concentration/m;
vs = 1; % Volum sprØyte i mL
N0 = c*vs;

if pumpspeed == 50
    v = 3.306; %mL/s
elseif pumpspeed == 60
    v = 3.909; %mL/s
elseif pumpspeed == 75
    v = 4.895; %mL/s
else
    disp('Feil i pumpehastighet')
```



```
    return
end %if

%Lag ny referanselinje for sensorene
refline1 = sensor1(end);
refline2 = sensor2(end);

newsensor1 = sensor1 - refline1;
newsensor2 = sensor2 - refline2;

len2 = length(newsensor2);
% maksimum
[max2 indx2] = max(newsensor2);
%Fjern un0dvendige plott fra sensor1

for j=indx2:len2
    if newsensor2(j) < 0.005*max2
        newsensor2(j:end) = [];
        newsensor1(j:end) = [];
        time(j:end) = [];
        break
    end
end
end
end
```

---

## A.2 Calculations of the residence time distribution

By using equation (2.8) the residence time distribution was created with the following MATLAB-script.

---

```
function rtd = rtd(sensor,time)
% E(t) plott
% Integral
integral = trapz(time,sensor);
for i=1:length(sensor)
    rtd(i) = sensor(i)/integral;
end

end
```

---

### A.3 Plotting

Plotting was done with MATLAB's built-in *plot*-function.

---

```
for i=1:9
    inputfile = [num2str(i) '.txt'];
    [sensor1{i} sensor2{i} time{i}] = residence(inputfile, i, kons);
end
% Integral of G(t)
for i = 1:9

    et{i} = rtd(sensor2{i},time{i});

end
plottyn = input('Vil du plotte driten? y/n: ', 's');
% Plot of both sensor 1 and 2 in the same plot
if plottyn == 'y'
    hold on
    figname = ['Pumpspeed' num2str(pump(i)) 'Cons' num2str(kons(i))];
    hFig = figure('Name',figname,'Visible','off');
    set(hFig,'NumberTitle','off');
    plot(time{i},sensor1{i},'r',time{i},sensor2{i},'b')
    legend('sensor1','sensor2')
    xlabel('Time [s]','Interpreter','LaTeX')
    ylabel('Conductance [mikrosiemens]','Interpreter','LaTeX');
    title(figname,'Interpreter','LaTeX')
    hold off
    export_fig(figname, '-pdf', '-transparent')

    for j = 1:3:9
        %Plots comparison of concentrations
        figname = ['E(t)-Pumpspeed' num2str(pump(j))];
        konsFig = figure('Name',figname,'Visible','off');
        set(konsFig,'NumberTitle','off');
        plot(time{j},et{j},'r',time{j+1},et{j+1},'b',time{j+2},et{j+2},'g')
        legend('Concentration=50','Concentration=100','Concentration=150')
        xlabel('Time [s]','Interpreter','LaTeX')
        ylabel('E(t)','Interpreter','LaTeX');
        title(figname,'Interpreter','LaTeX')
        export_fig(figname, '-pdf', '-transparent')
    end
end
```

```
% Plots comparison of different pumpspeeds
filename = ['E(t)-Cons' num2str(kons(j))];
konsFig = figure('Name',filename,'Visible','off');
set(konsFig,'NumberTitle','off');
plot(time{1},et{1},'r',time{4},et{4},'b',time{7},et{7},'g')
legend('Pumpspeed 50','Pumpspeed 60','Pumpspeed 75')
xlabel('Time [s]','Interpreter','LaTeX')
ylabel('E(t)','Interpreter','LaTeX');
title(filename,'Interpreter','LaTeX')
export_fig(filename, '-pdf', '-transparent')
```

```
end
```

---

## A.4 Mean residence time

To find the mean residence time,  $t_m$ , the residence time distribution must be integrated by using (2.10). This has to be done with numeric integration, by using MATLAB's *trapz*-function. The following code writes the mean residence time for every injection to a file: *t\_m.txt*

---

```
for i=1:9
    tm(i) = (trapz(time{i},time{i}.*sensor2{i})/trapz(time{i},sensor2{i}));
end
fileID = fopen('tm.txt','w');
fprintf(fileID,'%s %s %s %s \r\n','Cons&','Pumpspeed&','$t_m$', '\\');
fprintf(fileID,'%s \r\n',midrule);
for i=1:length(tm)
    fprintf(fileID,'% -3.f & % -8.f & %12.2f %s \r\n',kons(i),pump(i),tm(i), '\\')
end
fclose(fileID);
```

---

## A.5 Outlet concentration

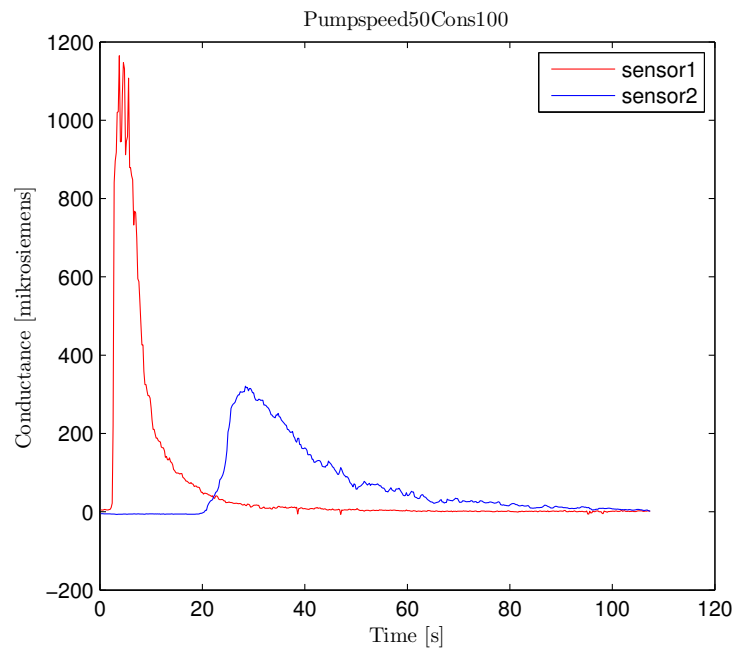
---

```
fileID =fopen('ca.txt','w');
fprintf(fileID,'%s %s %s %s \r\n ','Cons &','Pumpspeed&','$C_A$', '\\');
fprintf(fileID,'%s \r\n',midrule);
for i=1:length(et)
    caout = trapz(time{i},(1.5/(1+0.015.*time{i})).*et{i});
```

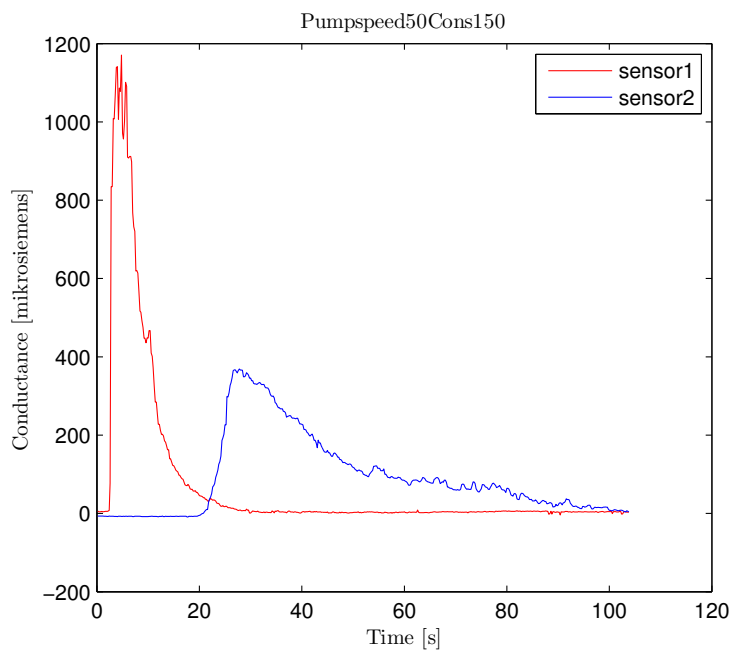
```
    fprintf(fileID, '%-3.f & %-8.f & %12.2f %s \r\n', kons(i), pump(i), caout, '\\')  
end  
fclose(fileID);
```

---

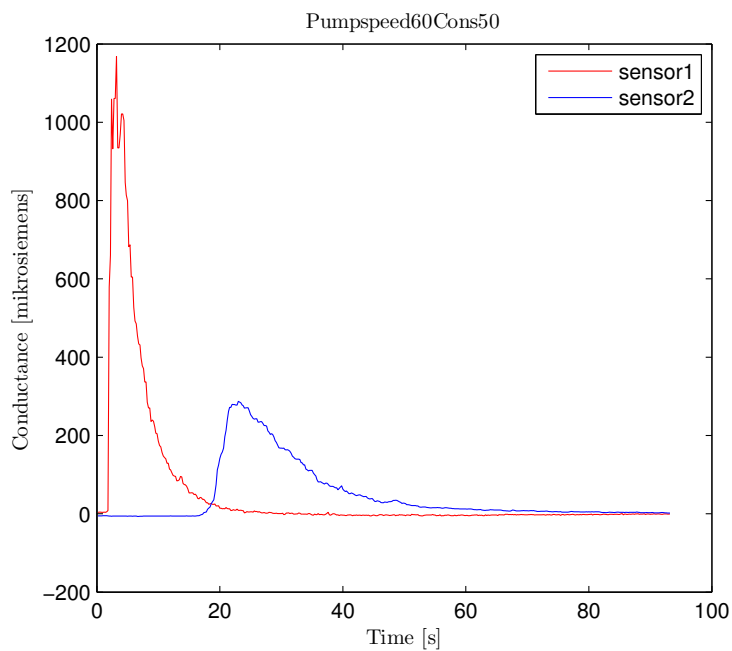
## B Plots



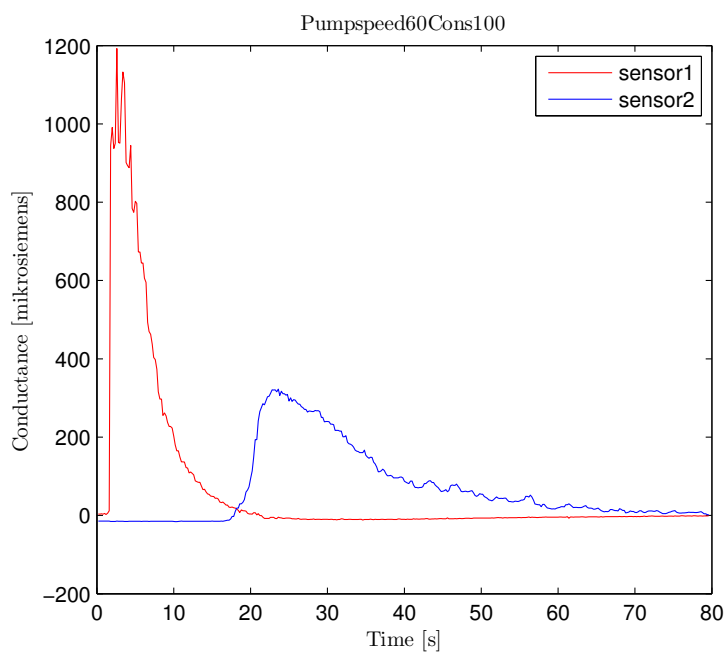
**Figure 9:** Results for the experiment when the speed of the pump is set to 50% and the concentration is  $100 \text{ gL}^{-1}$ .



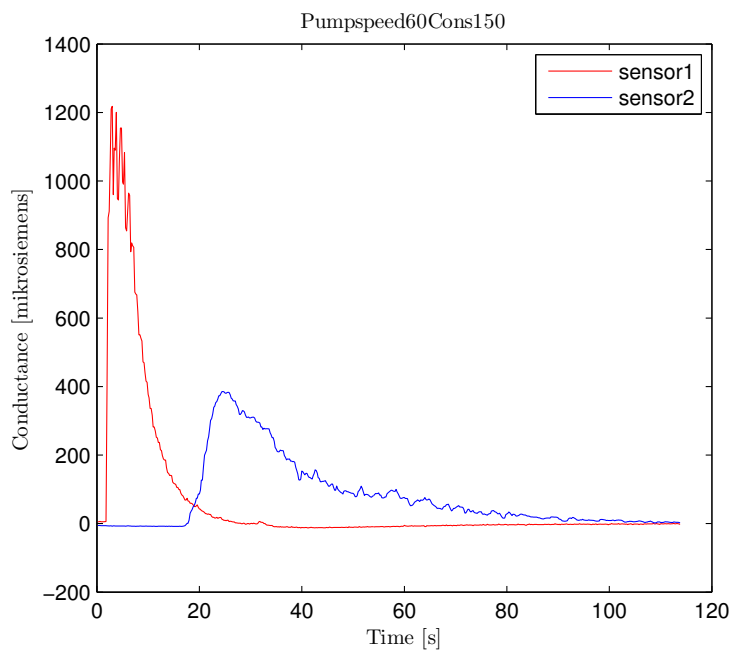
**Figure 10:** Results for the experiment when the speed of the pump is set to 50% and the concentration is  $150 \text{ g L}^{-1}$ .



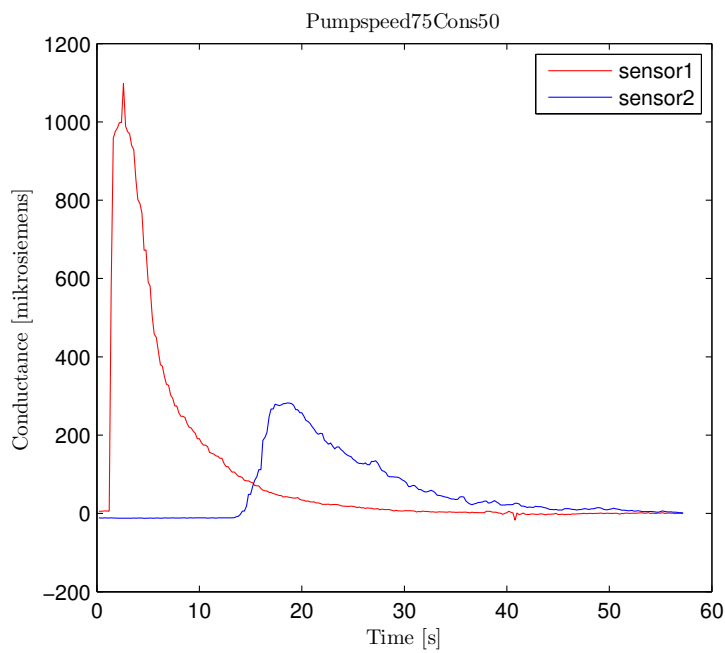
**Figure 11:** Results for the experiment when the speed of the pump is set to 60% and the concentration is  $50 \text{ g L}^{-1}$ .



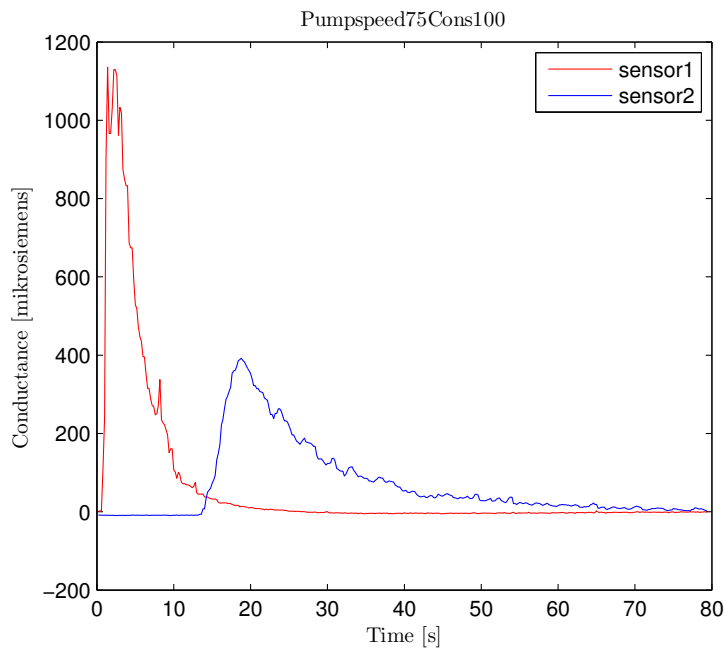
**Figure 12:** Results for the experiment when the speed of the pump is set to 60% and the concentration is  $100 \text{ gL}^{-1}$ .



**Figure 13:** Results for the experiment when the speed of the pump is set to 60% and the concentration is  $150 \text{ gL}^{-1}$ .

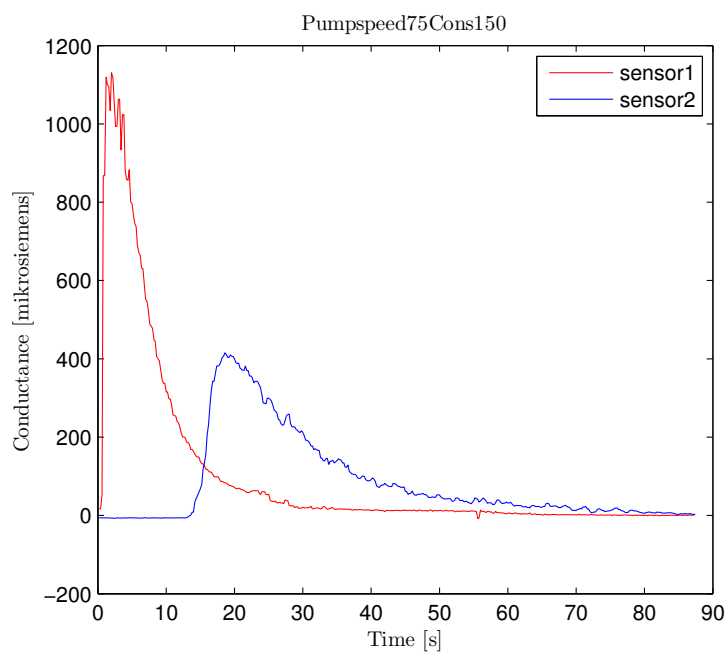


**Figure 14:** Results for the experiment when the speed of the pump is set to 75% and the concentration is  $50 \text{ g L}^{-1}$ .



**Figure 15:** Results for the experiment when the speed of the pump is set to 75% and the concentration is  $100 \text{ g L}^{-1}$ .





**Figure 16:** Results for the experiment when the speed of the pump is set to 75% and the concentration is  $150 \text{ gL}^{-1}$ .

## C Fluid mechanics

In order to measure the volumetric flow, equation (C.1) was used.

$$\nu = \frac{V}{t} \quad (\text{C.1})$$

To calculate the reactor volume equation (C.2) was used.

$$V_{\text{reactor}} = t_{\text{fluidfront,sensor1} \rightarrow \text{sensor2}} \cdot \nu \quad (\text{C.2})$$

### C.1 Assumption of laminar flow

The assumption of laminar flow was taken after calculating the Reynold's number,  $N_{Re}$ , for the flow through the reactor, which is calculated from equation 2.5-1 in [3],

$$N_{Re} = \frac{D \cdot v \cdot \rho}{\mu} \quad (\text{C.3})$$

where  $D$  is the diameter of the pipe,  $v$  is the flow rate,  $\mu$  the viscosity of the fluid, and  $\rho$  is the density of the fluid. The flow rate is calculated by (C.4)

$$v = \frac{\nu}{A} \quad (\text{C.4})$$

where  $\nu$  is the volumetric flow and  $A$  is cross-section area. Equation (C.3) was solved for the radius,  $r$ , of the pipe, giving equation (C.5)

$$\frac{N_{Re}\mu\pi s}{2\rho v} = \frac{1}{r} \quad (\text{C.5})$$

A flow is turbulent if the value of Reynold's number is over 4000 (section 2.5C in [3]). By inserting known values in equation (C.5):

$$\left(\frac{28 \cdot \pi \cdot 4000 \cdot 10^{-3}}{2 \cdot 92.57 \cdot 10^{-6} \cdot 1000}\right)^{-1} > r = 5.26 \cdot 10^{-4} \text{m} \quad (\text{C.6})$$

In order to be turbulent, the radius of the pipes in the reactor has to be smaller than 0.5 mm. The pipes were significantly larger, ergo the flow was laminar. Note that even though the flow itself is laminar, it way has zones of turbulence and stagnation.

## D The Dirac Delta Function, $\delta(x)$

For an ideal Plug Flow Reactor (PFR) the residence time distribution follows the Dirac delta function given by equation (D.1) where  $\delta$  is the Dirac delta function,  $t$  is the time and  $\tau$  is the nominal space time.

$$E(t) = \delta(t - \tau) \tag{D.1}$$

The Dirac delta function has the following properties:

$$\delta(x) = \begin{cases} 0 & \text{when } x \neq 0 \\ \infty & \text{when } x = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$