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Fuzzy Sets and Systems 83 (1996) 135–141

FUZZY sets and systems

# Uncertainty in fault tree analysis: A fuzzy approach

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Received November 1995

**Abstract**

In fault tree analysis, the uncertainties in the failure probability and/or failure rate of system components or basic events can be propagated to find the uncertainty in the overall system failure probability. The conventional approach is Monte-Carlo simulation by assuming a probability distribution for the failure probability. In addition, a new methodology based on fuzzy set theory is also being used in the fault tree analysis for quantifying the basic event uncertainty and for propagating it. However, identification of the components which contribute maximum to the system failure probability is also important in fault tree analysis. Similarly, ranking the components based on their contribution of uncertainty to the uncertainty of the system failure probability is also very important. This paper presents a comparative study of probabilistic and fuzzy methodologies for top event uncertainty evaluation. Further, it explains a new approach to rank the system components or basic events depending on (1) their contribution to the top event failure probability and (2) their uncertainty contribution to the uncertainty of the top event based on fuzzy set theory.

*Keywords:* Uncertainty analysis; Fault tree analysis; Fuzzy set theory; Importance measures; Monte-Carlo simulation

## 1. Introduction

Fault tree analysis (FTA) is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequences and combinations of faults and failure events. The conventional FTA based on probabilistic approach has been used extensively in the past. However, it is often very difficult to estimate precise failure rates or failure probabilities of individual components or failure events. This happens particularly in systems like nuclear power plants where available data are insufficient for statistical inferences or the data show a large variation [2]. Therefore, in the absence of accurate data, it may be necessary to work with rough estimates of probabilities. However, to incorporate the variation in the estimated values, the failure rates or the failure probabilities are treated as random variables with known probability distributions [7]. This requires, of course, that data be available from which these probability distributions can reasonably be deduced. Fuzzy methods might be the only resort when little quantitative information is available regarding fluctuations in the parameters [4, 5].

In the conventional uncertainty analysis, the point estimates of the primary events are replaced by probability distributions. Hence, one could derive a probability distribution for the probability of occurrence of the top event in the fault tree.

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*SSDI* 0165-0114(95)00386-X

However, an analytical method may be difficult and a simulation may require enormous computer time. In fuzzy approach the algebraic operations are easy and straightforward.

Fuzzy set theory (FST) developed by Zadeh [11] way back in 1965 has been applied to realiability and fault tree analysis [8, 10]. In this paper, we have considered FST for the uncertainty analysis in FTA. Instead of assuming the input parameter as a random variable it is considered as a fuzzy number and the uncertainty to the top event is propagated. In earlier works on the application of fuzzy set theory in fault tree analysis [4, 8], the failure probabilities of components are assumed as fuzzy probabilities and the extension principle is used for algebraic operations. However, these approaches cannot be applied to a fault tree with repeated events and they are computationally intensive too. Soman and Misra [9] provided a simple method for fuzzy fault tree analysis based on the $\alpha$-cut method, also known as resolution identity. In this paper, the $\alpha$-cut method is used for the top event failure probability calculation, and the approach is elaborated in Section 3. The comparison of the results of both the approaches is given in the concluding section.

In FTA, the concept of importance is used to evaluate how far a basic event contributes to the top event. An importance analysis is useful for the design modifications of the system. Up to date, a number of applications of importance have been presented, most of which are based on probabilistic concepts. Pan and Tai [6] developed a model for computing the importance measure of basic components using variance importance measure. The Monte-Carlo simulation, which is generally used in the determination of variance importance measure, introduces its own uncertainty into the model; also it takes more computer time. If the system components are large in number, the whole procedure has to be repeated that many times. Even though we expect the same measure of uncertainty for the components which have the same probability distribution parameters, the results are slightly different because of the use of Monte-Carlo simulation.

However, these methods are not suitable for the fuzzy approach. Fuzzy importance which can be used in fuzzy fault tree analysis is introduced in [1] which is equivalent to structural importance. Liang and Wang [3] proposed another importance measure known as fuzzy importance index (FII) and the calculation of FII is based on a ranking method of triangular fuzzy numbers with maximising and minimising sets. A simple method is proposed in this paper to evaluate an importance measure called fuzzy importance measure (FIM) and it is based on the Euclidean distance approach.

As it is important to identify the critical components, it is also very essential to identify the components which have the maximum contribution of uncertainty to the uncertainty of the top event. A new method is proposed in this paper for uncertainty importance, which is called fuzzy uncertainty importance measure (FUIM). FUIM plays an important role in the reduction of uncertainty, for it is used to identify those sources of uncertainty having greatest impact on the uncertainty of the top event. These measures are further explained in Section 4. A numerical example is provided and the results are compared with Pan and Tai's variance importance measure.

## 2. Probabilistic approach to uncertainty analysis

In the case of a system comprising a large number of components, failure may occur due to various failure combinations involving one or more components. This relationship between component and system failure is represented in a fault tree. The component data uncertainties are propagated in the fault tree to obtain the uncertainty in the system failure probability. The present probabilistic approach to uncertainty analysis consists of treating the failure rates as a random variable represented by a specified probability distribution. A lognormal distribution is generally considered [7], which is represented by a median and an error factor, when sufficient data are available for a component. The range propagation to the system level is carried out using Monte-Carlo simulation. Thus, apart from the uncertainty in data and models etc., further uncertainty is introduced by the simulation process.

In fault tree analysis, the system failure probability can be expressed using minimal cut-sets of the components. The unreliability function of a system can be written as the sum of partial products of the unreliability of the components. Thus the system failure probability is

$$Q = f(q_1, q_2, \dots, q_i, \dots, q_n), \tag{1}$$

where $q_i$ is the unreliability or failure probability of component $i$ and $n$ is the total number of components.

Fault tree analysis program (FTAP) module of the software PSAPACK has been used for generating the minimal cut-sets and top event failure probability point estimate. Another software for uncertainty analysis, similar to SAMPLE [7], developed by us has been used to obtain the 90% confidence interval of the top event failure probability using Monte-Carlo simulation.

## 3. Fuzzy approach to uncertainty analysis

The probabilistic approach to uncertainty analysis basically depends upon the assumption of a probability distribution of failure probability as explained earlier which can be obtained only when a sufficient amount of failure data is available. In addition, the distributions are propagated using simulation methods to obtain the top event failure probability distribution. To overcome some of the difficulties, the use of fuzzy set theory [3, 8] is being considered of late. In FST, the input parameter is treated as a fuzzy number and the variability is characterised by the membership function which can be obtained based on available information or the expert's opinion. The membership function of each fuzzy set is usually assumed to be a triangular or trapezoidal function and is treated as a possibility distribution.

When the unreliability of each component has a point estimate, the top event unreliability will also be a point estimate. In this paper, the component failure probabilities are considered as triangular fuzzy sets to incorporate the uncertainties in the parameter. The membership function of a triangular fuzzy set is defined as

$$\mu_X(x) = \begin{cases} \max[0, 1 - |(x - x^{(1)})/(x^{(2)} - x^{(1)})|], & x^{(1)} \leqslant x \leqslant x^{(2)}, \\ 1, & x = x^{(2)}, \\ \max[0, 1 - |(x^{(3)} - x)/(x^{(3)} - x^{(2)})|], & x^{(2)} \leqslant x \leqslant x^{(3)}, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

with

$$\mu_X(x^{(2)}) = 1, \tag{3}$$

$$\mu_X(x^{(1)}) = \mu_X(x^{(3)}) = 0, \tag{4}$$

and $[x^{(1)}, x^{(3)}]$ are [lower, upper] bounds of triangular fuzzy sets. For demonstration, the lower and upper bounds may be obtained from the point median value and the error factor (EF) of the failure probability [3]. The lower bound, middle value and the upper bound are defined as

$$x^{(1)} = \frac{q_p}{\mathrm{EF}}, \tag{5}$$

$$x^{(2)} = q_p, \tag{6}$$

$$x^{(3)} = q_p \, \mathrm{EF}, \tag{7}$$

where $q_p$ is the point median value of the failure probability. The fuzzy evaluation of the failure probability of the top event in a fault tree (i.e. Eq. (1)) is carried out using the $\alpha$-cut method. The top event can be represented by an $N \times 2$ array, where $N$ is the number of $\alpha$-cuts.

## 4. Importance measures

The identification of critical components is essential as far as the safety analysis of any system is concerned. Many measures are available in probabilistic approach like risk achievement worth, Birnbaum importance, Fussel-Vesely importance, etc. Pan and Tai [6] have developed a methodology for variance importance measure using Monte-Carlo simulation. In fuzzy methodology, two different importance measures are introduced and they are (1) fuzzy importance measure and (2) fuzzy uncertainty importance measure, which are further explained below.

### 4.1. Fuzzy importance measure

The evaluation of the contribution of different basic events is essential to identify the critical components in the system. The top event failure probability by making the component '$i$' fully unavailable (i.e. $q_i = 1$) is

$$Q_{q_i=1} = f(q_1, q_2, \ldots, q_{i-1}, 1, q_{i+1}, \ldots, q_n). \quad (8)$$

Similarly when the component '$i$' is fully available,

$$Q_{q_i=0} = f(q_1, q_2, \ldots, q_{i-1}, 0, q_{i+1}, \ldots, q_n). \quad (9)$$

Thus the total contribution of component '$i$' to the system failure probability is the difference between $Q_{q_i=1}$ and $Q_{q_i=0}$ and is called Birnbaum importance in conventional approach. Pan and Tai evaluated the variance importance measure by averaging the square of this importance using Monte-Carlo simulation.

However, in fuzzy fault tree analysis both $Q_{q_i=1}$ and $Q_{q_i=0}$ are fuzzy numbers and neither point estimate nor Monte-Carlo simulation can be used. The authors propose a simple method to identify the critical components based on the fuzzy importance measure (FIM), which is defined as

$$\mathrm{FIM}_i = \mathrm{ED}[Q_{q_i=1},\ Q_{q_i=0}], \quad (10)$$

where $\mathrm{ED}[A, B]$ is the Euclidean distance between two fuzzy sets $A$ and $B$ and is defined as

$$\mathrm{ED}[A, B] = \sum_{\alpha_i = 1, 2, \ldots, N} ((a^{\mathrm{L}} - b^{\mathrm{L}})^2 + (a^{\mathrm{U}} - b^{\mathrm{U}})^2)^{0.5}_{\alpha_i}, \quad (11)$$

where $a^{\mathrm{L}}$ and $a^{\mathrm{U}}$ are the lower and upper values of fuzzy set $A$ at each $\alpha$-level.

### 4.2. Fuzzy uncertainty importance measure

FIM can be used to identify the critical components. However, it is also important to know the components whose uncertainty of failure probability contribute significantly to the uncertainty of the failure probability of the system. This helps in deciding the components for which more information should be collected so that the uncertainty in the calculated system failure probability can be lowered. An importance measure known as fuzzy uncertainty importance measure is proposed to identify the components which contribute maximum uncertainty to the uncertainty of the top event and is defined as

$$\mathrm{FUIM}_i = \mathrm{ED}[Q, Q_i], \quad (12)$$

where $Q$ = top event failure probability (Eq. (1)), $Q_i$ = top event failure probability when error factor for component '$i$' is unity (i.e. $\mathrm{EF}_i = 1$), i.e. the parameter of the basic event has a point value or crisp value.

## 5. Discussions and conclusions

In order to further illustrate the methodology of this paper, let us consider a simplified fault tree for the reactor protective system [7] as shown in Fig. 1. The input data are given in Table 1 with the corresponding error factors. The results of the Monte-Carlo simulation after 1200 trials are as follows:

median point value $= 3.426 \times 10^{-5}$,

median value (50%) $= 6.082 \times 10^{-5}$,

low value (5%) $= 1.321 \times 10^{-5}$,

high value (95%) $= 3.346 \times 10^{-4}$.

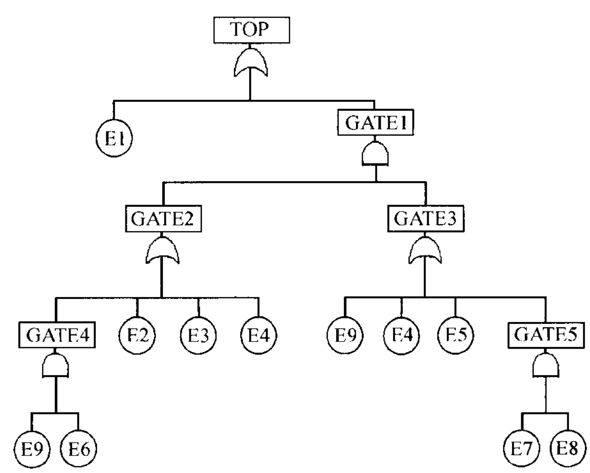


Fig. 1. Reduced fault tree for the reactor protective system [WASH-1400].

Table 1
Failure probability and ranking for different components

| Event no. | Failure probability (median) | Error factor | Lower bound | Upper bound | FIM (rank) | FUIM (rank) |
|---|---|---|---|---|---|---|
| 1 | 1.7E−5 | 10 | 1.7E−6 | 1.7E−4 | 4.69 (1) | 3.01E−4 (2) |
| 2 | 1.0E−3 | 3 | 3.3E−4 | 3.0E−3 | 2.97E−2 (3) | 4.53E−5 (4) |
| 3 | 3.6E−4 | 3 | 1.2E−4 | 1.1E−3 | 2.97E−2 (3) | 1.63E−5 (5) |
| 4 | 1.0E−3 | 3 | 3.3E−4 | 3.0E−3 | 2.97E−2 (3) | 4.53E−5 (4) |
| 5 | 3.6E−4 | 3 | 1.2E−4 | 1.1E−3 | 2.97E−2 (3) | 1.63E−5 (5) |
| 6 | 6.1E−3 | 4 | 1.5E−3 | 2.4E−2 | 2.0E−2 (4) | 2.88E−4 (3) |
| 7 | 6.1E−3 | 4 | 1.5E−3 | 2.4E−2 | 2.0E−2 (4) | 2.88E−4 (3) |
| 8 | 9.7E−4 | 10 | 9.7E−5 | 9.7E−3 | 8.5E−2 (2) | 5.54E−4 (1) |
| 9 | 9.7E−4 | 10 | 9.7E−5 | 9.7E−3 | 8.5E−2 (2) | 5.54E−4 (1) |

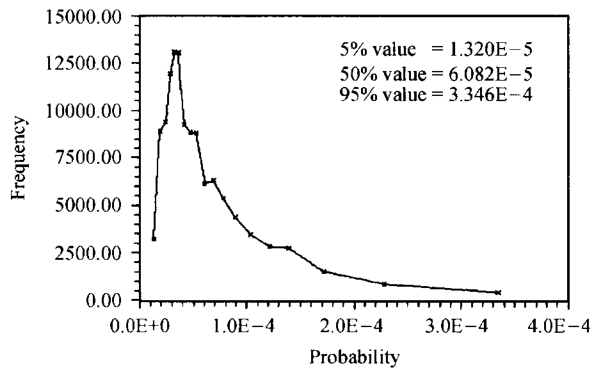


Fig. 2. Frequency distribution for top event failure probability.

Fig. 2 gives the frequency distribution of the top event obtained by this approach. Table 2 provides the function values for the different confidence limits.

The membership function for each basic component is evaluated using Eqs. (5), (6), (7) and (2) and the lower and upper values are given in Table 1. The membership function for the top event is evaluated using the $\alpha$-cut method and the fuzzy top event failure probability is given below:

lower value $= 2.30 \times 10^{-6}$,

middle value $= 3.43 \times 10^{-5}$,

upper value $= 8.33 \times 10^{-4}$.

Table 2
Function values for different confidence limits (Monte-Carlo simulation)

| Confidence (%) | Function value |
|---|---|
| 00.50 | 5.866E−06 |
| 01.00 | 7.128E−06 |
| 02.50 | 9.614E−06 |
| 05.00 | 1.321E−05 |
| 10.00 | 1.882E−05 |
| 20.00 | 2.833E−05 |
| 25.00 | 3.214E−05 |
| 30.00 | 3.596E−05 |
| 40.00 | 4.700E−05 |
| 50.00 | 6.082E−05 |
| 60.00 | 7.800E−05 |
| 70.00 | 1.037E−04 |
| 75.00 | 1.212E−04 |
| 80.00 | 1.393E−04 |
| 90.00 | 2.278E−04 |
| 95.00 | 3.346E−04 |
| 97.50 | 4.716E−04 |
| 99.00 | 6.871E−04 |
| 99.50 | 9.362E−04 |

Fig. 3 gives the membership function for the fuzzy top event failure probability and Table 3 gives the lower and upper bound values ([$N \times 2$] array) of the top event failure probability for the different $\alpha$ values.

From the results of the present analysis, based on Monte-Carlo simulation, it is seen that the 90%

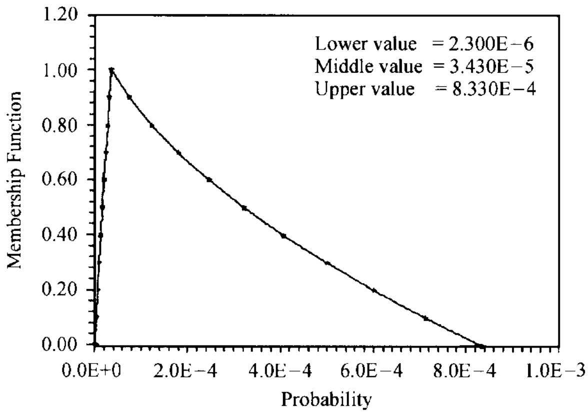


Fig. 3. Membership function for top event failure probability.

Table 3
Lower and upper bound values for top event failure probability for different α-levels

| α-level | Lower bound | Upper bound |
|---|---|---|
| 0.00 | 2.30E−06 | 8.33E−04 |
| 0.10 | 4.49E−06 | 7.13E−04 |
| 0.20 | 6.91E−06 | 6.01E−04 |
| 0.30 | 9.55E−06 | 4.99E−04 |
| 0.40 | 1.24E−05 | 4.06E−04 |
| 0.50 | 1.55E−05 | 3.21E−04 |
| 0.60 | 1.88E−05 | 2.46E−04 |
| 0.70 | 2.23E−05 | 1.79E−04 |
| 0.80 | 2.61E−05 | 1.22E−04 |
| 0.90 | 3.01E−05 | 7.36E−05 |
| 1.00 | 3.43E−05 | 3.43E−05 |

confidence limit for the top event failure probability is $1.32 \times 10^{-5}$ to $3.346 \times 10^{-4}$. However, the values can lie anywhere between 0 and $\infty$ with different probabilities. In the case of fuzzy representation, the total possible range is $2.30 \times 10^{-6}$ to $8.33 \times 10^{-4}$, with a high possibility (0.9) range of $3.01 \times 10^{-5}$ to $7.36 \times 10^{-5}$. Thus, it can be seen that the range for the high possibility is small in fuzzy representation. The computer time used for the probabilistic approach is very high compared to the fuzzy set approach. By assuming a probability distribution at system level for further propagation we are introducing uncertainty once more in the probability method, and in the fuzzy approach uncertainty is introduced only at component level and the analytical method is used to propagate it further. However, the fuzzy set approach is still at research level while the probability method has a well-established procedure in fault tree analysis.

Both FIM and FUIM have been calculated based on Eqs. (10) and (12) for all basic components. The results are summarized in Table 1. The top event fuzzy failure probabilities, with component 6 fully available (i.e. $q_6 = 0$) and fully unavailable (i.e. $q_6 = 1$), are given in Fig. 4 for FIM

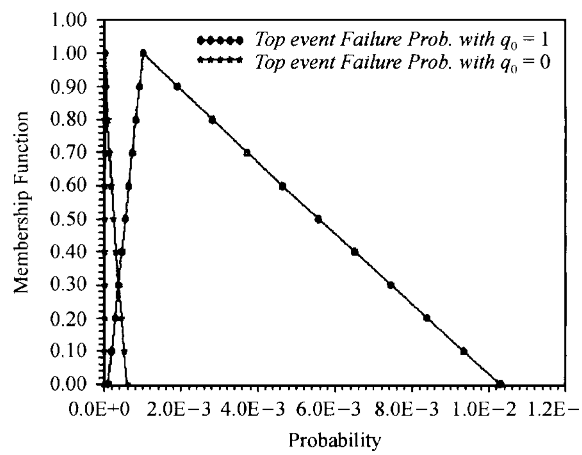


Fig. 4. FIM for component 6 (with failure prob. 1 and 0).

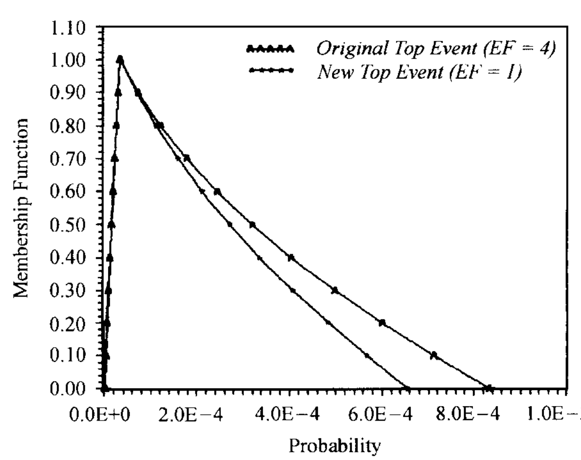


Fig. 5. FUIM for component 6 (with error factor 4 and 1).

calculations. Similarly FUIM calculation of component 6 is shown in Fig. 5.

The ranking based on FIM is 1. (1), 2. (8, 9), 3. (2, 3, 4, 5) and 4. (6, 7) and is the same as that of Pan and Tai [6]. Even though Monte-Carlo simulation [6] gives slightly different variance important measures for the events 2, 3, 4 and 5, the same rank was given for all these events. In this approach, the fuzzy importance measures are equal for the same rank. However, the ranking for FUIM is 1. (8, 9), 2. (1), 3. (6, 7), 4. (2, 4) and 5. (3, 5), which is different from FIM as expected. FIM can be used to find out the critical component which may be useful for design modifications of the system. The results of FUIM can be utilised to provide insight on the design of data and information gathering strategies that focus on the reduction of the total uncertainty.

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