Helicopter project
TTK4115 Linear System Theory

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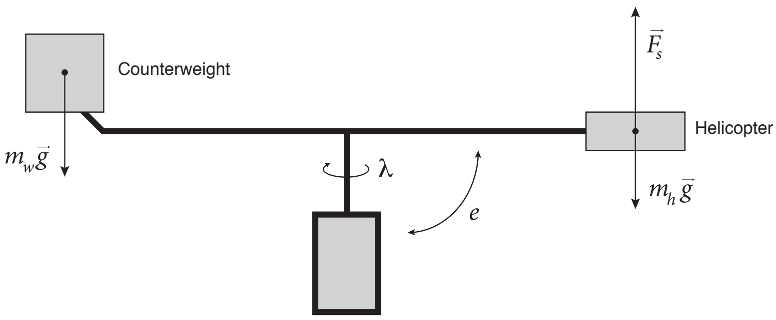
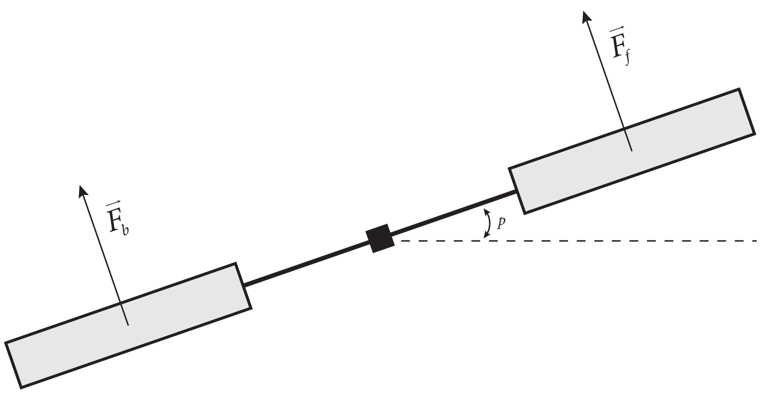
1 Mathematical modelling

1.1 Part I

The first part of the assignment was to develop the equations describing the helicopters motion – i.e. the differential equations for the pitch angle p, the travel angle λ and the elevation angle e.



(a) Helicopter side



(b) Helicopter front

Figure 1: Helicopter model

We use fig. 1a, 1b and Newtons 2. law for rotation to obtain the differential equations.

$$\Sigma\tau = J_p\ddot{p} = (F_f - F_b)l_h = K_f l_h V_d$$
$$\Updownarrow$$
$$\ddot{p} = \frac{K_f l_h}{J_p} V_d = K_1 V_d \tag{1}$$

Where

$$K_1 = \frac{K_f l_h}{J_p} \tag{2}$$

$$\Sigma\tau = J_t\ddot{\lambda} = -K_f l_a \sin(p) - K_a l_a |\dot{\lambda}|\dot{\lambda}$$
$$\Updownarrow$$
$$\ddot{\lambda} = -\frac{K_f l_a}{J_t}\sin(p) - \frac{K_a l_a}{J_t}|\dot{\lambda}|\dot{\lambda} = -K_2\sin(p) - k_l|\dot{\lambda}|\dot{\lambda} \tag{3}$$

Where

$$K_2 = \frac{K_f l_h}{J_p},$$
$$k_l$$

$$\Sigma\tau = J_e\ddot{e} \ = (F_f + F_b) l_a \cos(p) = K_f l_a \cos(p) V_s$$
$$\Updownarrow$$
$$\ddot{e} = \frac{K_f l_a}{J_e}\cos(p) V_s = K_3 \cos(p) V_s \tag{4}$$

Where

$$K_3 = \frac{K_f l_a}{J_e} \tag{5}$$

$$V_s = V_f + V_b \tag{6}$$
$$V_d = V_f - V_b \tag{7}$$

$$V_f = \frac{1}{2}(V_s + V_d) \tag{8}$$
$$V_b = \frac{1}{2}(V_s - V_d) \tag{9}$$

$$\ddot{p} = K_1 V_d \tag{10}$$
$$\ddot{\lambda} = -K_2\sin(p) - k_l|\dot{\lambda}|\dot{\lambda} \tag{11}$$
$$\ddot{e} = K_3 V_s - K_4 \tag{12}$$

Where

$$K_1 = \frac{l_h K_f}{J_p} =$$
$$K_2 = \frac{l_a K_f}{J_t} =$$
$$K_3 = \frac{l_a K_f}{J_e} =$$
$$K_4 = \frac{l_a m_g g}{J_e} = 42$$

2 Part II – Mono-variable control

In this part of the assignment, we wanted to implement mono-variable control for the elevation e, pitch p, and travel rate λ.

2.1 Problem 1

We want to use a PD controller to control the pitch angle p.

$$V_d = K_{pp}(p_c - p) - K_{pd}\dot{p} \tag{13}$$
$$\tag{14}$$

We apply the Laplace transform to (??) and (1) (with zero initial conditions):

$$\begin{aligned} \frac{s^2 p}{K_1} &= K_{pp}(p_c - p) - sK_{pd}p \\ &\Updownarrow \\ p(s^2 + sK_1K_{pd} + K_1K_{pp}) &= p_c K_1 K_{pp} \\ &\Updownarrow \\ \frac{p}{p_c}(s) &= \frac{K_1 K_{pp}}{s^2 + sK_1K_{pd} + K_1K_{pp}} \end{aligned} \tag{15}$$

As a starting point we set the regulator parameters such that the regulator is as fast as possible without oscillations – i.e. we choose the parameters such that $\zeta = 1$. We choose $K_{pp} = 1$ and find K_{pd}.

$$\begin{aligned} \zeta &= \frac{\sqrt{K_1}K_{pd}}{2\sqrt{K_{pp}}} \\ &\Updownarrow \\ 1 &= \frac{\sqrt{K_1}K_{pd}}{2\sqrt{1}} \\ &\Updownarrow \\ K_{pd} &= \frac{2}{\sqrt{K_1}} \end{aligned}$$

lol

3 Appendix

p	–	Pitch angle
λ	–	Travel angle
e	–	Elevation angle
V_f	–	Voltage, front motor
V_b	–	Voltage, rear motor
V_d	–	Voltage difference, $V_f - V_b$
V_s	–	Voltage sum, $V_f + V_b$
K_{pp}	–	Controller gain
K_{pd}	–	Controller gain
K_{rp}	–	Controller gain
p_c	–	Pitch reference
$\dot{\lambda}_c$	–	Travel rate reference
e_c	–	Elevation reference
l_a	–	Distance from axis of elevation to helicopter body [m]
l_h	–	Distance from pitch axis to motor [m]
K_f	–	Motor force constant [N/V]
J_e	–	Moment of inertia about elevation axis [kg m^2]
J_t	–	Moment of inertia about travel axis [kg m^2]
J_p	–	Moment of inertia about pitch axis [kg m^2]
m_h	–	Helicopter body mass [kg]
m_w	–	Counterweight mass [kg]
m_g	–	Net mass of helicopter and counterweight [kg]
K_p	–	Force needed to lift the helicopter body from the table top (g$\cdot m_g$) [N]

Table 1: List of variables and constants