NTNU
Department of Engineering Cybernetics

TTK4115 LINEAR SYSTEM THEORY

Helicopter Lab

Report

October 7, 2011

By

KenanTrnka
and
TorjusSveierOttemo

Preface

Contents

Chapter 1

Calculations Temp

We are to find the transfer function to

$$V_d = K_{pp}(p_c - p) - K_{pd}\dot{p} \tag{1.1}$$

From previous derivations, we have that

$$\ddot{p} = K_1 V_d \leftrightarrow V_d = \frac{\ddot{p}}{K_1} \tag{1.2}$$

Inserting (X.X) into (Y.Y) yields

$$\frac{\ddot{p}}{K_1} = K_{pp}(p_c - p) - K_{pd}\dot{p} \tag{1.3}$$

Replacing derivatives with the appropriate Laplace transforms and then solving to find $\frac{p}{p_c}(s)$

$$\frac{s^2 p}{K_1} = K_{pp}p_c - K_{pp}p - K_{pd}ps \tag{1.4}$$

$$p(\frac{s^2}{K_1} + K_{pd}s + K_{pp}) = K_{pp}p_c \tag{1.5}$$

$$\frac{p}{p_c}(s) = \frac{K_1 K_{pp}}{s^2 + K_1 K_{pd}s + K_1 K_{pp}} \tag{1.6}$$

Deriving the transfer function $\frac{r}{r_c}(s)$ is fairly simple. From previous derivations we have, under the assumption p and r are small, the linear equation

$$\dot{r} = -K_2 p \leftrightarrow p = -\frac{\dot{r}}{K_2} \tag{1.7}$$

The simple P controller we will implement are given by this equation

$$p_c = K_{rp}(r_c - r),\ K_{rp} < 0 \tag{1.8}$$

Inserting (X.X) into (Y.Y) and assuming perfect control ($p = p_c$) yields

$$-\frac{\dot{r}}{K_2} = K_{rp}(r_c - r) \tag{1.9}$$

Replacing the derivative with the appropriate Laplace transform and then solving to find $\frac{r}{r_c}(s)$

$$sr = -K_2 K_{rp}(r_c - r),\ \rho = K_2 K_{rp} \tag{1.10}$$

$$r(s - \rho) = -\rho r_c \tag{1.11}$$

It should be emphasized that this transfer function are based on some rough approximations, and great care should be taken when using it to analyze the system.

$$\frac{r}{r_c}(s) = \frac{\rho}{s - \rho} \tag{1.12}$$

Part III

The first thing we do is to initialize the constants derived earlier by executing the following code:

```
Kf = m_g*9.81/3.85
K1 = Kf*l_h/J_p
K2 = Kf*3.85*l_a/J_e
K3 = l_a*Kf/J_e
K4 = m_g*9.81*l_a/J_e
```

We then have three equations to include in the system:

$$\ddot{e} = K_3V_f + K_3V_s - K_4 \tag{1.13}$$

$$\dot{p} = \dot{p} \tag{1.14}$$

$$\ddot{p} = K_1V_f - K_1V_b \tag{1.15}$$

Which translates to the following system:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} K_3 & K_3 \\ 0 & 0 \\ K_1 & -K_1 \end{bmatrix} \boldsymbol{u} \tag{1.16}$$

where $\boldsymbol{x} = \begin{bmatrix} \dot{e} \\ p \\ \dot{p} \end{bmatrix}$ and $\boldsymbol{u} = \begin{bmatrix} V_f \\ V_b \end{bmatrix}$

The constant K_4 is ignored since it doesn't influence neither the states nor the inputs. We are interested in checking the controllability of the system, and hence we use MATLAB to calculate the controllability matrix with the following code (the calculated values for K_1 and K_3 will replace the variables when the script is executed):

```
A = [0,0,0;0,0,1;0,0,0]
B = [K_3, K_3;0,0;K_1,-K_1]
Co = ctrb(A,B)
us = length(A)-rank(Co)
```

The last variable, us (uncontrollable states), evaluates to 0. This means the system is controllable.

We are now interested in using the LQR (Linear Quadratic Regulator) method. We set $u = Pr - Kx$, where P and K er gain (weight) matrices for the reference and states respectively.

To find P we need to do som algebra. We start by defining $y = Cx$ and $\dot{x} = Ax + Bu$. The matrix C is:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{1.17}$$

We replace u with $Pr - Kx$ which gives us (after a little shuffling)

$$\dot{x} - (A - BK)x = BPr \tag{1.18}$$

We now replace x with y by multiplying with the C matrix:

$$\dot{y} - (A - BK)y = CBPr \tag{1.19}$$

And then we Laplace transform the equation:

$$Ys - (A - BK)Y = CBPr \tag{1.20}$$

Finally, we set up the transfer function, dividing with s since r is an impulse

$$\frac{Y}{r}(s) = \frac{CBP}{s(s - (A - BK))} \tag{1.21}$$

We now use the final value theorem to find the value when $t \to \infty$:

$$\lim_{s \to 0} s\frac{Y}{r}(s) = s\frac{CBP}{s(s - (A - BK))} = \frac{CBP}{BK - A} = 1 \tag{1.22}$$

Where the expression evaluates to 1 because after an infinite amount of time, $r = y$. We now use the result to solve for P:

$$CBP = BK - A \leftrightarrow P = C^{-1}B^{-1}(BK - A) \tag{1.23}$$

And this is the final answer. We used the following script to calculate these matrices:

```
A = [0,0,0;
     0,0,1;
     0,0,0]
B = [K3,K3;
     0,0;
     K1,-K1]
Q = [2.5*10,0,0;
     0,6*10,0;
     0,0,4*10]
R = [4*10,0;
     0,4*10]
K = lqr(A,B,Q,R)
C = [1,0,0;
     0,1,0]
P = inv(C*inv(-A+B*K)*B) %(inv(C)*inv(B))*(B*K-A)
```

We found some values of Q and R that gave an OK response in the system, which can be seen in the above code.

Chapter 2

Introduction

2.1 ?!

Chapter 3

Theory

3.1 ?!

Appendix A

Equations

Bibliography

[1] Oxford University Press *Linear System Theory and Design*
by Chi-Tsong Chen