11 Mathematical Models for Performing Human Reliability and Error Analysis in Engineering Maintenance

11.1 INTRODUCTION

Mathematical modeling is a widely used approach to perform various types of analysis in engineering systems. In this case, the components of a system are denoted by idealized elements assumed to have representative characteristics of real-life components and whose behavior can be described by equations. However, the degree of realism of mathematical models depends on the type of assumptions imposed on them.

Over the years, a large number of mathematical models have been developed to study human reliability and error in engineering systems. Most of these models were developed using stochastic processes including the Markov approach [1, 2]. Although the usefulness of such models can vary from one situation to another, some of the human reliability and error models are being used quite successfully to represent various types of real-life environments in the industrial sector [3]. Thus, some of these models can also be used to tackle human reliability and error problems in the area of engineering maintenance.

This chapter presents the mathematical models considered quite useful to perform various types of human reliability and error-related analysis in engineering maintenance.

11.2 MODELS FOR PREDICTING MAINTENANCE PERSONNEL RELIABILITY IN NORMAL AND FLUCTUATING ENVIRONMENTS

Maintenance personnel perform various types of time-continuous tasks including monitoring, tracking, and operating. Environments under which such tasks are performed can be either normal or fluctuating. In performing such tasks, maintenance personnel can make various types of errors including critical and noncritical errors. Therefore, this section presents three mathematical models to predict maintenance worker performance reliability and to perform maintenance error-related analysis under the above-described conditions.

11.2.1 Model I

This model is concerned with predicting the maintenance worker performance reliability under normal conditions—more specifically, the probability of performing a time-continuous task correctly by a maintenance worker. An expression to predict the maintenance worker performance reliability is developed below [1, 2, 4, 5].

The probability of human error in a maintenance task in the finite time interval Δt with event D given is expressed by

$$P(C/D) = z(t)\,\Delta t \tag{11.1}$$

where C is an event that human error will occur in time interval $[t, t + \Delta t]$, D is an errorless performance event of duration t, and $z(t)$ is the human error rate at time t.

The joint probability of the errorless performance is given by

$$P(\bar{C}/D) = P(D) - P(C/D)P(D) \tag{11.2}$$

where $P(D)$ is the occurrence probability of event D and $\bar{C}$ is the event that human error will not occur in time interval $[t, t + \Delta t]$.

Equation (11.2) denotes an errorless performance probability over time intervals $[0, t]$ and $[t, t + \Delta t]$ and is rewritten as

$$R_h(t) - R_h(t)P(C/D) = R_h(t + \Delta t) \tag{11.3}$$

where $R_h(t)$ is the maintenance worker reliability at time t.

By substituting Equation (11.1) into Equation (11.3), we get

$$\lim_{\Delta t \to 0} \frac{R_h(t+\Delta t) - R_h(t)}{\Delta t} = -R_h(t)z(t) \tag{11.4}$$

In the limiting case Equation (11.4) becomes

$$\lim_{\Delta t \to 0} \frac{R_h(t+\Delta t) - R_h(t)}{\Delta t} = \frac{dR_h(t)}{dt} = -R_h(t)z(t) \tag{11.5}$$

At time $t = 0$, $R_h(0) = 1$.

By rearranging Equation (11.5), we get

$$\frac{1}{R_h(t)} \cdot dR_h(t) = -z(t)dt \tag{11.6}$$

Integrating both sides of Equation (11.6) over the time interval [0, *t*], we get

$$\int_{1}^{R_h(t)} \frac{1}{R_h(t)} \cdot dR_h(t) = -\int_{0}^{t} z(t)dt \tag{11.7}$$

After evaluating Equation (11.7) we obtain

$$R_h(t) = -e^{-\int_0^t z(t)dt} \tag{11.8}$$

Equation (11.8) is the general expression to compute maintenance worker performance reliability for any time to human error statistical distribution (e.g., Weibull, normal, and exponential).

By integrating Equation (11.8) over the time interval $[0, \infty]$, we get the following general equation for the mean time to human error [1]:

$$MTTHE = \int_{0}^{\infty} \left[e^{-\int_0^t z(t)dt} \right] dt \tag{11.9}$$

where *MTTHE* is the mean time to human error of a maintenance worker.

Example 11.1

Assume that a maintenance worker is performing a certain task and his or her error rate is 0.001 errors/hour (i.e., times to human error are exponentially distributed). Calculate the maintenance worker's reliability during a 6-hour work period.

Thus, we have [1]

$$z(t) = 0.001 \text{ errors/hour}$$

By substituting the above value and the given value for time *t* into Equation (11.8), we get

$$\begin{aligned} R_h = (6) &= e^{-\int_0^6 (0.001)dt} \\ &= e^{-(0.001)(6)} \\ &= 0.9940 \end{aligned}$$

Thus, the maintenance worker's reliability during the 6-hour work period is 0.9940.

11.2.2 Model II

This model represents a maintenance worker performing time-continuous tasks under fluctuating environment (i.e., normal and stressful) [1, 6]. One example of such an environment is weather changing from normal to stormy and vice versa.

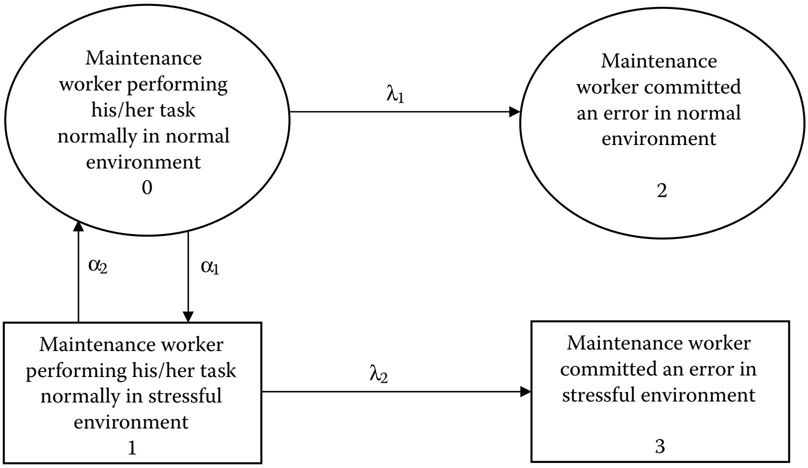

FIGURE 11.1 State space diagram for model II.

As the rate of a maintenance worker's errors from a normal work environment to a stressful environment can vary quite significantly, the model considers two separate maintenance worker error rates (i.e., one for normal environment and the other for stressful environment).

Thus, the model can be used to determine the maintenance worker's reliability and mean time to human error under the fluctuating environment. The model state space diagram is shown in Figure 11.1. The numerals in circles and boxes denote the maintenance worker's states.

The following assumptions are associated with the model:

- Maintenance worker error rates are constant.
- All maintenance worker errors occur independently.
- Environment change rates (i.e., from normal to stressful and vice versa) are constant.

The following symbols are associated with the diagram:

i is the ith state of the maintenance worker; $i = 0$ (maintenance worker performing his or her task normally in a normal environment), $i = 1$ (maintenance worker performing his or her task normally in a stressful environment), $i = 2$ (maintenance worker committed an error in a normal environment), $i = 3$ (maintenance worker committed an error in a stressful environment).

$P_i(t)$ is the probability of the maintenance worker being in state i at time t, for $i = 0, 1, 2, 3$.

λ_1 is the constant error rate of the maintenance worker performing his or her task in a normal environment.

λ_2 is the constant error rate of the maintenance worker performing his or her task in a stressful environment.
α_1 is the constant transition rate from normal environment to stressful environment.
α_2 is the constant transition rate from stressful environment to normal environment.

Using the Markov approach described in Chapter 4, we write down the following set of equations for the diagram shown in Figure 11.1 [6]:

$$\frac{dP_0(t)}{dt} + (\lambda_1 + \alpha_1)\,P_0(t) = \alpha_2\,P_1(t) \tag{11.10}$$

$$\frac{dP_1(t)}{dt} + (\lambda_2 + \alpha_2)\,P_1(t) = \alpha_1\,P_0(t) \tag{11.11}$$

$$\frac{dP_2(t)}{dt} = \lambda_1\,P_0(t) \tag{11.12}$$

$$\frac{dP_3(t)}{dt} = \lambda_2\,P_1(t) \tag{11.13}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = P_2(0) = P_3(0) = 0$.

By solving Equations (11.10)–(11.13), we get the following state probability equations:

$$P_0(t) = \frac{1}{(y_2 - y_1)}[(y_2 + \lambda_2 + \alpha_2)\,e^{y_2 t} - (y_1 + \lambda_2 + \alpha_2)\,e^{y_1 t}] \tag{11.14}$$

where

$$y_1 = \left[-a_1 + \left(a_1^2 - 4a_2\right)^{1/2}\right]/2 \tag{11.15}$$

$$y_2 = \left[-a_1 - \left(a_1^2 - 4a_2\right)^{1/2}\right]/2 \tag{11.16}$$

$$a_1 = \lambda_1 + \lambda_2 + \alpha_2 + \alpha_1 \tag{11.17}$$

$$a_2 = \lambda_1\,(\lambda_2 + \alpha_2) + \alpha_1\,\lambda_2 \tag{11.18}$$

$$P_2(t) = a_4 + a_5 e^{y_2 t} - \alpha_6\,e^{y_1 t} \tag{11.19}$$

where

$$a_3 = \frac{1}{y_2 - y_1} \tag{11.20}$$

$$a_4 = \lambda_1 (\lambda_2 + \alpha_2)/y_1 \, y_2 \tag{11.21}$$

$$a_5 = a_3(\lambda_1 + a_4 y_1) \tag{11.22}$$

$$a_6 = a_3 (\lambda_1 + a_4 y_2) \tag{11.23}$$

$$P_1 (t) = \alpha_1 a_3 \, (e^{y_2 t} - e^{y_1 t}) \tag{11.24}$$

$$P_3(t) = a_7 \left[(1 + a_3) \left(y_1 e^{y_2 t} - y_2 e^{y_1 t} \right) \right] \tag{11.25}$$

where

$$a_7 = \lambda_2 \, \alpha_1 / y_1 \, y_2 \tag{11.26}$$

The maintenance worker's reliability is expressed by

$$R_{mw} (t) = P_0 (t) + P_1 (t) \tag{11.27}$$

where $R_{mw}(t)$ is the maintenance worker's reliability of performing tasks in fluctuating environments.

The maintenance worker's mean time to human error is given by

$$\begin{aligned} MTTHE_{mw} &= \int_0^{\infty} R_{mw}(t) dt \\ &= \frac{\lambda_2 + \alpha_1 + \alpha_2}{\lambda_1 (\lambda_2 + \alpha_2) + \alpha_1 \lambda_2} \end{aligned} \tag{11.28}$$

where $MTTHE_{mw}$ is the mean time to human error of the maintenance worker performing his or her task in a fluctuating environment.

Example 11.2

Assume that a maintenance worker's constant error rates in normal and stressful environments are 0.0001 errors/hour and 0.0005 errors/hour, respectively. The values of the transition rates from normal to stressful environment and vice versa are 0.002 times per hour and 0.003 times per hour, respectively. Calculate the mean time to human error of the maintenance worker.

Substituting the given data values into Equation (11.28) yields

$$MTTHE_{mw} = \frac{0.0005 + 0.002 + 0.003}{0.0001(0.0005 + 0.003) + (0.002)(0.0005)}$$
$$= 4074.1 \text{ hours}$$

Thus, the mean time to human error of the maintenance worker is 4074.1 hours.

11.2.3 Model III

This model represents a maintenance worker performing a time-continuous task subjected to critical and noncritical errors. The model can be used to calculate the maintenance worker reliability at time *t*, the maintenance worker mean time to human error, the probability of the maintenance worker committing a critical error at time *t*, and the probability of the maintenance worker committing a noncritical error at time *t*.

The model state space diagram is shown in Figure 11.2. The numerals in the boxes denote the maintenance worker's states.

The model is subjected to the following assumptions:

- All maintenance worker errors occur independently.
- Maintenance worker critical and noncritical error rates are constant.

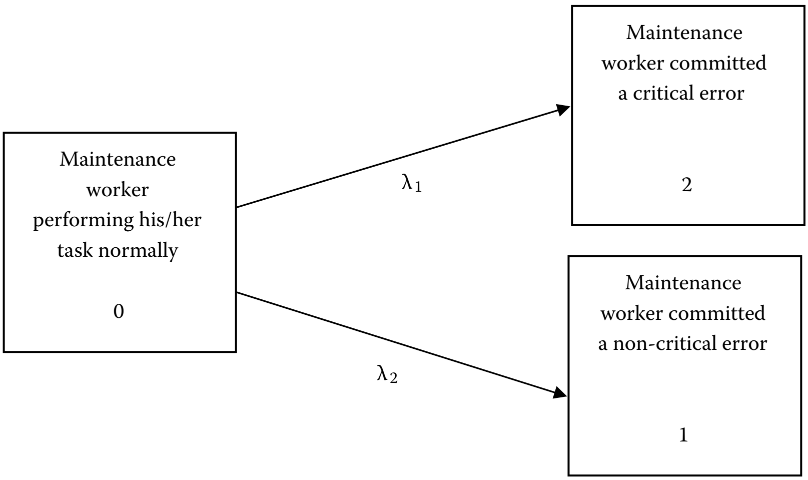

FIGURE 11.2 State space diagram for model III.

The following symbols are associated with the diagram:

i is the ith state of the maintenance worker; $i = 0$ (maintenance worker performing his or her task normally), $i = 1$ (maintenance worker committed a noncritical error), $i = 2$ (maintenance worker committed a critical error).
$P_i\,(t)$ is the probability of the maintenance worker being in state i at time t, for $i = 0, 1, 2$.
λ_1 is the constant critical human error rate of the maintenance worker.
λ_2 is the constant noncritical human error rate of the maintenance worker.

Using the Markov method, we write down the following equations for the diagram [1, 7]:

$$\frac{dP_0\,(t)}{dt} + (\lambda_2 + \lambda_1)\,P_0(t) = 0 \tag{11.29}$$

$$\frac{dP_1\,(t)}{dt} - \lambda_2 P_0(t) = 0 \tag{11.30}$$

$$\frac{dP_2\,(t)}{dt} - \lambda_1 P_0(t) = 0 \tag{11.31}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, and $P_2(0) = 0$.

Solving Equations (11.29)–(11.31), we obtain the following equations:

$$P_0(t) = e^{-(\lambda_2+\lambda_1)t} \tag{11.32}$$

$$P_1(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2}[1 - e^{-(\lambda_2+\lambda_1)t}] \tag{11.33}$$

$$P_2(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2}[1 - e^{-(\lambda_2+\lambda_1)t}] \tag{11.34}$$

The above three equations can be used to obtain the maintenance worker's probabilities of being in state 0, 1, and 2. The maintenance worker reliability is given by

$$\begin{aligned} R_m(t) &= P_0(t) \\ &= e^{-(\lambda_2+\lambda_1)t} \end{aligned} \tag{11.35}$$

where $R_m(t)$ is the maintenance worker's reliability at time t.

The mean time to human error of the maintenance worker is given by [1, 7].

$$\begin{aligned} MTTHE_m &= \int_0^{\infty} R_m(t)dt \\ &= \int_0^{\infty} e^{-(\lambda_2 + \lambda_1)t} dt \\ &= \frac{1}{\lambda_2 + \lambda_1} \end{aligned} \tag{11.36}$$

where $MTTHE_m$ is the mean time to human error of the maintenance worker.

Example 11.3

Assume that a maintenance worker is performing a time-continuous task and his or her constant critical and noncritical error rates are 0.0001 errors/hour and 0.0006 errors/hour, respectively. Calculate the maintenance worker's reliability for a 6-hour mission and mean time to human error.

By substituting the given data values into Equations (11.35) and (11.36), we obtain

$$\begin{aligned} R_m(6) &= e^{-(0.0006+0.0001)(6)} \\ &= 0.9958 \end{aligned}$$

and

$$\begin{aligned} MTTHE_m &= \frac{1}{0.0006 + 0.0001} \\ &= 1428.6 \text{ hours} \end{aligned}$$

Thus, the maintenance worker's reliability and mean time to human error are 0.9958 and 1428.6 hours, respectively.

11.3 MODELS FOR PERFORMING SINGLE SYSTEMS MAINTENANCE ERROR ANALYSIS

Past experiences indicate that systems can fail or degrade due to maintenance errors. Over the years, various mathematical models have been developed to perform reliability and availability analysis of such systems [1, 3, 7]. Two of these models are presented below.

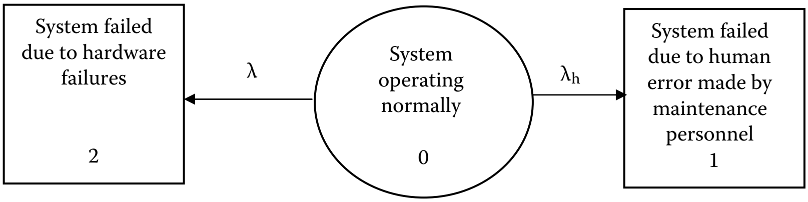

FIGURE 11.3 State space diagram for model I.

11.3.1 Model I

This model represents a system that can fail either due to human errors made by maintenance personnel or due to hardware failures. The model state space diagram is shown in Figure 11.3 where the numerals in the circle and boxes denote system states. It is to be noted that mathematically this model is the same as model III in Section 11.2 above, but its application is different.

The following two assumptions are associated with the model:

- Hardware failures and human errors occur independently.
- Both hardware failure and human error rates are constant.

The following symbols are associated with the diagram:

λ is the constant hardware failure rate of the system.
λ_h is the constant human error rate of the maintenance personnel.
j is the jth state of the system; $j = 0$ (system operating normally), $j = 1$ (system failed due to human error made by maintenance personnel), $j = 2$ (system failed due to hardware failures).
$P_j(t)$ is the probability of the system being in state j at time t, for $j = 0, 1, 2$.

By using the Markov method, we write down the following three equations for the diagram [1, 7]:

$$\frac{dP_0(t)}{dt} + (\lambda_h + \lambda)P_0(t) = 0 \tag{11.37}$$

$$\frac{dP_1(t)}{dt} - \lambda_h P_0(t) = 0 \tag{11.38}$$

$$\frac{dP_2(t)}{dt} - \lambda P_0(t) = 0 \tag{11.39}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, and $P_2(0) = 0$.

By solving Equations (11.37)–(11.39), we get

$$P_0(t) = e^{-(\lambda_h+\lambda)t} \tag{11.40}$$

$$P_1(t) = \frac{\lambda_h}{\lambda_h+\lambda}[1-e^{-(\lambda_h+\lambda)t}] \tag{11.41}$$

$$P_2(t) = \frac{\lambda}{\lambda_h+\lambda}[1-e^{-(\lambda_h+\lambda)t}] \tag{11.42}$$

The system reliability is given by

$$\begin{aligned} R_S(t) &= P_0(t) \\ &= e^{-(\lambda_h+\lambda)t} \end{aligned} \tag{11.43}$$

where $R_S(t)$ is the system reliability at time t.

The system mean time to failure is expressed by

$$\begin{aligned} MTTF_S &= \int_0^\infty R_S(t)dt \\ &= \int_0^\infty e^{-(\lambda_h+\lambda)t}dt \\ &= \frac{1}{\lambda_h+\lambda} \end{aligned} \tag{11.44}$$

where $MTTF_S$ is the system mean time to failure.

Example 11.4

Assume that a system can fail either due to human error made by maintenance personnel or due to hardware failures. The system constant human error and hardware failure rates are 0.0001 errors/hour and 0.0009 failures/hour, respectively.

Calculate the probability that the system will fail due to a human error made by maintenance personnel during a 12-hour mission. By substituting the specified data values into Equation (11.41), we obtain

$$\begin{aligned} P_1(12) &= \frac{0.0001}{(0.0001+0.0009)}[1-e^{-(0.0001+0.0009)(12)}] \\ &= 0.0012 \end{aligned}$$

Thus, the probability that the system will fail due to a human error made by maintenance personnel is 0.0012.

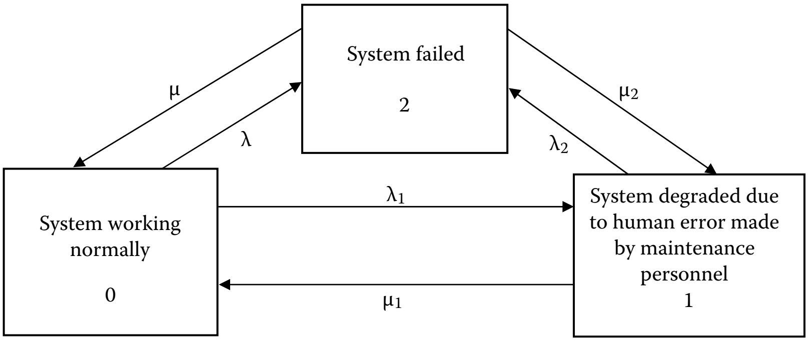

FIGURE 11.4 State space diagram for model II.

11.3.2 Model II

This model represents a system that can only fail due to hardware failures, but human errors made by maintenance personnel can degrade its performance.

The system is repaired from failed and degraded states. The system state space diagram is shown in Figure 11.4. The numerals in boxes denote system states.

The following assumptions are associated with the model:

- The occurrence of human error by maintenance personnel can only result in system degradation, but not failure.
- Human error and hardware failure rates are constant.
- The totally or partially failed system is repaired and preventive maintenance is performed on a regular basis.
- The degraded system can only fail due to hardware failures.
- All system repair rates are constant.
- The repaired system is as good as new.

The following symbols are associated with the diagram:

λ_1 is the constant human error rate due to maintenance personnel.
λ_2 is the system constant failure rate from its degraded state.
λ is the system constant failure rate.
μ is the system constant repair rate.
μ_1 is the constant repair rate from the system degraded state to normal working state.
μ_2 is the constant repair rate from the system failed state to degraded or partially working state.
j is the jth state of the system; $j = 0$ (system operating normally), $j = 1$ (system degraded due to human error made by maintenance personnel), $j = 2$ (system failed).
$P_j(t)$ is the probability that the system is in state j at time t, for $j = 0, 1, 2$.

Using the Markov method and Fig. 11.4, we write down the following equations [1, 7, 8]:

$$\frac{dP_0(t)}{dt}+(\lambda_1+\lambda)P_0(t)=\mu_1 P_1(t)+\mu P_2(t) \tag{11.45}$$

$$\frac{dP_1(t)}{dt}+(\mu_1+\lambda_2)P_1(t)=\mu_2 P_2(t)+P_0(t)\lambda_1 \tag{11.46}$$

$$\frac{dP_2(t)}{dt}+(\mu+\mu_2)P_2(t)=\lambda_2 P_1(t)+\lambda P_0(t) \tag{11.47}$$

At time $t=0$, $P_0(0)=1$, $P_1(0)=0$, and $P_2(0)=0$.

By solving Equations (11.45)–(11.47), we get

$$\begin{aligned} P_0(t)=&\frac{\mu_1\mu+\lambda_2\mu+\mu_1\mu_2}{A_1A_2}\\ &+\left[\mu_1A_1+\mu A_1+\mu_2A_1+\lambda_2A_1+A_1^2+\mu_1\mu+\lambda_2\mu+\mu_1\mu_2\right]e^{A_1t}\\ &+\left\{1-\left(\frac{\mu_1\mu+\lambda_2\mu+\mu_1\mu_2}{A_1A_2}\right)\right.\\ &\left.-\left[\frac{\mu_1A_1+\mu A_1+\mu_2A_1+A_1\lambda_2+A_1^2+\mu_1\mu+\lambda_2\mu+\mu_1\mu_2}{A_1(A_1-A_2)}\right]\right\}e^{A_2t} \end{aligned} \tag{11.48}$$

where

$$A_1, A_2=\frac{\left[-D\pm\sqrt{D^2-4(\mu_1\mu+\lambda_2\mu+\mu_1\mu_2+\mu\lambda_1+\lambda_1\mu_2+\lambda_1\lambda_2+\mu_1\lambda+\lambda\mu_2+\lambda\lambda_2)}\right]}{2}$$

$$D=\lambda_1+\lambda+\lambda_2+\mu_1+\mu+\mu_2$$

$$A_1A_2=\mu_1\mu+\lambda_2\mu+\mu_1\mu_2+\mu\lambda_1+\lambda_1\mu_2+\lambda_1\lambda_2+\mu_1\lambda+\lambda\mu_2+\lambda\lambda_2$$

$$\begin{aligned} P_1(t)=&\frac{\lambda_1\mu+\lambda_1\mu_2+\lambda\mu_2}{A_1A_2}+\left[\frac{A_1\lambda_1+\lambda_1\mu+\lambda_1\mu_2+\lambda\mu_2}{A_1(A_1-A_2)}\right]e^{A_1t}\\ &-\left[\frac{\lambda_1\mu+\lambda_1\mu_2+\lambda\mu_2}{A_1A_2}+\frac{A_1\lambda_1+\lambda_1\mu+\lambda_1\mu_2+\lambda\mu_2}{A_1(A_1-A_2)}\right]e^{A_2t} \end{aligned} \tag{11.49}$$

$$\begin{aligned} P_2(t)=&\frac{\lambda_1\lambda_2+\mu_1\lambda+\lambda\lambda_2}{A_1A_2}+\left[\frac{A_1\lambda+\lambda_1\lambda_2+\lambda\mu_1+\lambda\lambda_2}{A_1(A_1-A_2)}\right]e^{A_1t}\\ &-\left[\frac{\lambda_1\lambda_2+\mu_1\lambda+\lambda\lambda_2}{A_1A_2}+\frac{A_1\lambda+\lambda_1\lambda_2+\mu_1\lambda+\lambda\lambda_2}{A_1(A_1-A_2)}\right]e^{A_2t} \end{aligned} \tag{11.50}$$

The probability of system degradation due to human error by maintenance personnel is given by Equation (11.49). As time t becomes very large, Equation (11.49) reduces to

$$P_1 = \frac{\lambda_1 \mu + \lambda_1 \mu_2 + \lambda \mu_2}{A_1 A_2} \tag{11.51}$$

where P_1 is the steady-state probability of system degradation due to human error by maintenance personnel.

The time-dependent system operational availability is given by

$$AV_S(t) = P_0(t) + P_1(t) \tag{11.52}$$

where $AV_S(t)$ is the system operational availability at time t.

As t becomes very large, Equation (11.52) becomes

$$AV_S = \frac{\mu_1\mu + \lambda_2 \mu + \mu_1\mu_2 + \lambda_1\mu + \lambda_1\mu_2 + \lambda\mu_2}{A_1 A_2} \tag{11.53}$$

where AV_S is the system steady-state operational availability.

Example 11.5

Assume that for a system we have the following data values:

$\lambda = 0.007$ failures per hour
$\lambda_1 = 0.0002$ errors per hour
$\lambda_2 = 0.002$ failures per hour
$\mu = 0.03$ repairs per hour
$\mu_1 = 0.006$ repairs per hour
$\mu_2 = 0.04$ repairs per hour

Calculate the steady-state probability of system degradation due to human error by maintenance personnel.

By inserting the specified data values into Equation (11.51), we obtain

$$P_1 = \frac{(0.0002)(0.03) + (0.0002)(0.04) + (0.007)(0.04)}{(0.006)(0.03) + (0.002)(0.03) + (0.006)(0.04) + (0.03)(0.0002) + (0.0002)(0.04) + (0.0002)(0.002) + (0.006)(0.007) + (0.007)(0.04) + (0.007)(0.002)}$$
$$= 0.3540$$

Thus, the steady-state probability of system degradation due to human error by maintenance personnel is 0.3540.

11.4 MODELS FOR PERFORMING REDUNDANT SYSTEMS MAINTENANCE ERROR ANALYSIS

Past experiences indicate that human error by maintenance personnel can cause not only the failure of single unit systems but also of redundant unit systems. In the published literature, there are many mathematical models that can be used to perform maintenance error analysis of redundant systems [1]. Two of these models are presented below.

11.4.1 Model I

This mathematical model represents a two-identical-units parallel system subjected to periodic preventive maintenance. The system/unit can fail due to hardware failures or maintenance or other errors. The system state space diagram is shown in Figure 11.5. The numerals in circles and boxes denote system states.

The following assumptions are associated with the model:

- All failures and errors occur independently.
- Both units are independent, active, and identical.
- Maintenance or other errors may occur when either both system units are good or when one system unit is good.
- The system is subjected to periodic preventive maintenance.
- Both failure and error rates are constant.
- The total system fails due to maintenance or other errors.

The following symbols are associated with the diagram:

i is the ith state of the system; $i = 0$ (both units operating normally), $i = 1$ (one unit failed due to hardware failure, the other operating normally), $i = 2$ (system failed due to maintenance or other errors), $i = 3$ (system failed due to hardware failures).

$P_i(t)$ is the probability that the system is in state i at time t, for $i = 0, 1, 2, 3$.

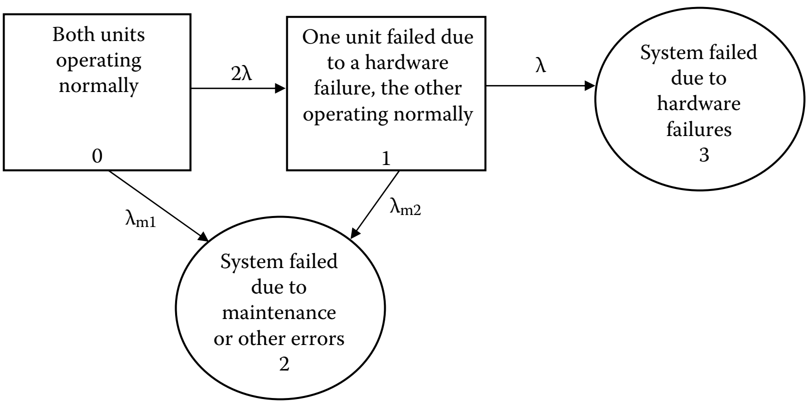

FIGURE 11.5 State space diagram for model I.

λ is the unit constant failure rate.
λ_{m1} is the constant maintenance or other error rate when both units are operating normally.
λ_{m2} is the constant maintenance or other error rate when only one unit is operating normally.

Using the Markov method and Figure 11.5, we get the following equations [1, 8]:

$$\frac{dP_0(t)}{dt} + (2\lambda + \lambda_{m1}) P_0(t) = 0 \tag{11.54}$$

$$\frac{dP_1(t)}{dt} + (\lambda + \lambda_{m2}) P_1(t) = 2\lambda P_0(t) \tag{11.55}$$

$$\frac{dP_2(t)}{dt} = \lambda_{m1} P_0(t) + \lambda_{m2} P_1(t) \tag{11.56}$$

$$\frac{dP_3(t)}{dt} = \lambda P_1(t) \tag{11.57}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, $P_2(0) = 0$, and $P_3(0) = 0$.

By solving Equations (11.54)–(11.57), we obtain

$$P_0(t) = e^{-A_1 t} \tag{11.58}$$

where

$$A_1 = 2\lambda + \lambda_{m1} \tag{11.59}$$

$$P_1(t) = B_1(e^{-A_1 t} - e^{-A_2 t}) \tag{11.60}$$

where

$$A_2 = \lambda + \lambda_{m2} \tag{11.61}$$

$$B_1 = \frac{2\lambda}{A_2 - A_1} \tag{11.62}$$

$$P_2(t) = B_2 - B_3 e^{-A_1 t} - B_4 e^{-A_2 t} \tag{11.63}$$

where

$$B_2 = \frac{2\lambda\lambda_{m2} + \lambda_{m1} A_2}{A_1 A_2} \tag{11.64}$$

$$B_3 = \frac{2\lambda\lambda_{m2} + \lambda_{m1}(A_2 - A_1)}{A_1(A_2 - A_1)} \tag{11.65}$$

$$B_4 = \frac{2\lambda\lambda_{m2}}{A_1(A_1 - A_2)} \tag{11.66}$$

$$P_3(t) = B_5 - B_6 e^{-A_1 t} - B_7 e^{-A_2 t} \tag{11.67}$$

where

$$B_5 = \frac{2\lambda^2}{A_1 A_2} \tag{11.68}$$

$$B_6 = \frac{2\lambda^2}{A_1(A_2 - A_1)} \tag{11.69}$$

$$B_7 = \frac{2\lambda^2}{A_2(A_1 - A_2)} \tag{11.70}$$

The system reliability is given by

$$\begin{aligned} R_S(t) &= P_0(t) + P_1(t) \\ &= e^{-A_1 t} + B_1 (e^{-A_1 t} - e^{-A_2 t}) \end{aligned} \tag{11.71}$$

where $R_S(t)$ is the system reliability at time t.

The system mean time to failure is given by [1, 8]

$$\begin{aligned} MTTF_S &= \int_0^\infty R_S(t)dt \\ &= \int_0^\infty \left[e^{-A_1 t} + B_1 (e^{-A_1 t} - e^{-A_2 t}) \right] dt \\ &= \frac{3\lambda + \lambda_{m2}}{(2\lambda + \lambda_{m1})(2\lambda + \lambda_{m2})} \end{aligned} \tag{11.72}$$

where $MTTF_S$ is the system mean time to failure.

Example 11.6

Assume that a system is composed of two independent and identical units in parallel. The unit constant failure rate and the constant maintenance or other error rate when both units operate normally are 0.02 failures/hour and 0.004 errors/hour, respectively. The constant maintenance or other error rate, when only one unit operates normally, is 0.001 errors/hour.

Calculate the system mean time to failure.

By substituting the given data values into Equation (11.72), we get

$$MTTF_S = \frac{3(0.02)+0.001}{[2\,(0.02)+0.004](0.02+0.001)}$$
$$= 66.01 \text{ hours}$$

Thus, the system mean time to failure is 66.01 hours.

11.4.2 Model II

This model represents a system with two independent and identical units forming a parallel configuration subjected to periodic maintenance and failed unit repair. The system/unit can malfunction due to hardware failures or maintenance or other errors. The system state space diagram is shown in Figure 11.6. The numerals in boxes and circles denote system states.

The model is subjected to the following assumptions:

- Both units are active, independent, and identical.
- All failure, error, and repair rates are constant.
- All failures and errors occur independently.
- The total system fails due to maintenance or other errors.

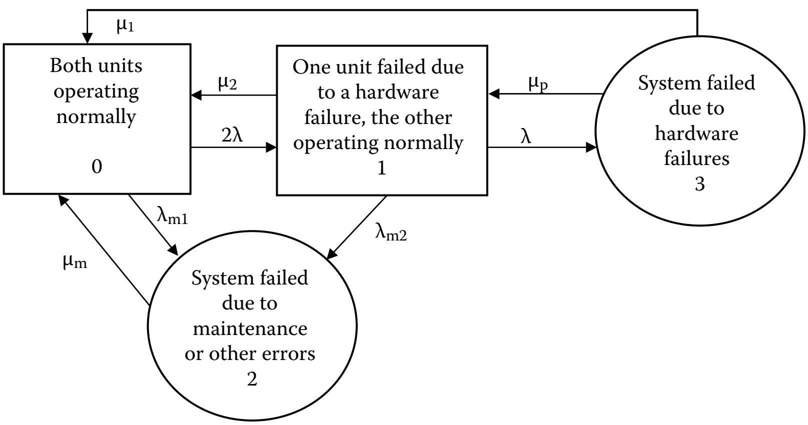

FIGURE 11.6 State space diagram for model II.

- Maintenance or other errors may occur when either both system units are good or when one system unit is good.
- The repaired system or unit is as good as new.

The following symbols are associated with Figure 11.6:

λ is the unit constant failure rate.
λ_{m1} is the constant maintenance or other error rate when both units are operating normally.
λ_{m2} is the constant maintenance or other error rate when only one unit is operating normally.
j is the jth state of the system; $j = 0$ (both units operating normally), $j = 1$ (one unit failed due to a hardware failure, the other operating normally), $j = 2$ (system failed due to maintenance or other errors), $j = 3$ (system failed due to hardware failures).
$P_j(t)$ is the probability that the system is in state j at time t, for $j = 0, 1, 2, 3$.
μ_1 is the system constant repair rate from state 3 to state 0.
μ_2 is the system constant repair rate from state 1 to state 0.
μ_m is the system constant repair rate from state 2 to state 0.
μ_p is the system constant repair rate from state 3 to state 1.

By using the Markov method and Figure 11.6, we write down the following equations [1, 8]:

$$\frac{dP_0(t)}{dt} + (2\lambda + \lambda_{m1})\, P_0(t) = P_1(t)\,\mu_2 + P_3(t)\,\mu_1 + P_2(t)\,\mu_m \tag{11.73}$$

$$\frac{dP_1(t)}{dt} + (\lambda + \lambda_{m2} + \mu_2)\, P_1(t) = P_0(t)\, 2\lambda + P_3(t)\,\mu_p \tag{11.74}$$

$$\frac{dP_2(t)}{dt} + \mu_m\, P_2(t) = P_0(t)\,\lambda_{m1} + P_1(t)\,\mu_{m2} \tag{11.75}$$

$$\frac{dP_3(t)}{dt} + (\mu_p + \mu_1)\, P_3(t) = P_1(t)\,\lambda \tag{11.76}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, $P_2(0) = 0$, and $P_3(0) = 0$.

By solving Equations (11.73)–(11.76), we obtain the following steady-state probability equations [1, 8]:

$$P_0 = \left[1 + D_1 + 2\lambda^2 D + \frac{1}{\mu_m}(\lambda_{m1} + D_1\,\lambda_{m2})\right]^{-1} \tag{11.77}$$

where

$$D = [(\mu_1 + \mu_p)(\lambda + \lambda_{m2} + \mu_2) - \lambda\mu_p]^{-1}$$

$$D_1 = 2\lambda(1 + \lambda\mu_p D)/(\lambda + \lambda_{m2} + \mu_2)$$

$$P_1 = P_0 D_1 \tag{11.78}$$

$$P_2 = P_0(\lambda_{m1} + D_1\lambda_{m2})/\mu_m \tag{11.79}$$

$$P_3 = P_0 2\lambda^2 D \tag{11.80}$$

where P_0, P_1, P_2, and P_3 are the steady-state probabilities of the system being in states 0, 1, 2, and 3, respectively.

The system steady-state availability is given by

$$AV_{SS} = P_0 + P_1 \tag{11.81}$$

where AV_{SS} is the system steady-state availability.

Additional information on this model is available in Refs. [1, 9].

11.5 PROBLEMS

1. A maintenance worker is performing a certain task and his or her error rate is 0.004 errors/hour (i.e., times to human error are exponentially distributed). Calculate the maintenance worker's reliability during an 8-hour work period.
2. Prove Equation (11.28) by using Equation (11.27).
3. Assume that a maintenance worker's constant error rates in normal and stressful environments are 0.0002 errors/hour and 0.0006 errors/hour, respectively. The values of the transition rates from normal to stressful environment and vice versa are 0.004 times per hour and 0.006 times per hour, respectively. Calculate the mean time to human error of the maintenance worker.
4. Prove that the sum of Equations (11.32)–(11.34) is equal to unity and explain why.
5. A system can fail either due to human errors made by maintenance personnel or due to hardware failures. The system constant human error and hardware failure rates are 0.0002 errors/hour and 0.0008 failures/hour, respectively. Calculate the probability that the system will fail due to a human error made by maintenance personnel during a 10-hour mission.
6. Prove Equations (11.40)–(11.42) by using Equations (11.37)–(11.39).
7. Prove Equation (11.51) by using Equation (11.49).
8. Assume that for a system we have the following data values:

$\lambda = 0.008$ failures/hour
$\lambda_1 = 0.0001$ errors/hour
$\lambda_2 = 0.002$ failures/hour
$\mu = 0.02$ repairs/hour
$\mu_1 = 0.004$ repairs/hour
$\mu_2 = 0.03$ repairs/hour
Calculate the steady-state probability of system degradation due to human error by maintenance personnel, by using Equation (11.51).

9. A system is composed of two independent and identical units in parallel. The unit constant failure rate and the constant maintenance or other error rate when both units operate normally are 0.03 failures/hour and 0.005 errors/hour, respectively. The constant maintenance or other error rate when only one unit operates normally is 0.002 errors/hour. Calculate the system mean time to failure.
10. Prove Equations (11.77)–(11.80) by using Equations (11.73)–(11.76).

REFERENCES

1. Dhillon, B.S., *Human Reliability: With Human Factors*, Pergamon Press, New York, 1986.
2. Regulinski, T.L., Askren, W.B., Stochastic Modeling of Human Performance Effectiveness Functions, *Proceedings of the Annual Reliability and Maintainability Symposium*, 1972, pp. 407–416.
3. Dhillon, B.S., *Human Reliability and Error in Transportation Systems*, Springer, London, 2007.
4. Askren, W.B., Regulinski, T.L., Quantifying Human Performance for Reliability Analysis of Systems, *Human Factors*, Vol. 11, 1969, pp. 393–396.
5. Regulinski, T.L., Askren, W.B., Mathematical Modeling of Human Performance Reliability, *Proceedings of the Annual Symposium on Reliability*, 1969, pp. 5–11.
6. Dhillon, B.S., Stochastic Models for Predicting Human Reliability, Microelectronics and Reliability, Vol. 25, 1985, pp. 729–752.
7. Dhillon, B.S., *Design Reliability: Fundamentals and Applications*, CRC Press, Boca Raton, FL, 1999.
8. Dhillon, B.S., *Engineering Maintenance: A Modern Approach*, CRC Press, Boca Raton, FL, 2002.
9. Dhillon, B.S., Rayapati, S.N., Analysis of Redundant Systems with Human Errors, *Proceedings of the Annual Reliability and Maintainability Symposium*, 1985, pp. 315–321.