TFY4225 Nuclear and Radiation physics

1.)
Basic concepts (Lilley Chap.1)

The Nuclei

Notation

The composition of a nucleus is often described using the notation:

$${}^{A}_{Z}\mathrm{X}_{N}$$

X represents the atoms name. A is defined to be the mass number, Z is the atomic number and N is the neutron number.
It is of course sufficient to describe the nuclei by ^{A}X, since X automatically determines the letter Z, which was defined above to be the atom number.

Particle masses

Particle	Index	Mass
Neutron	m_n	$m_n = 1.008665u$
Proton	m_p	$m_p = 1.007276u$
Electron	m_e	$m_e = 0.000549u$

Where u is the atomic mass unit, and 1u$\equiv \frac{1}{12}m({}^{12}C)$

Particle data

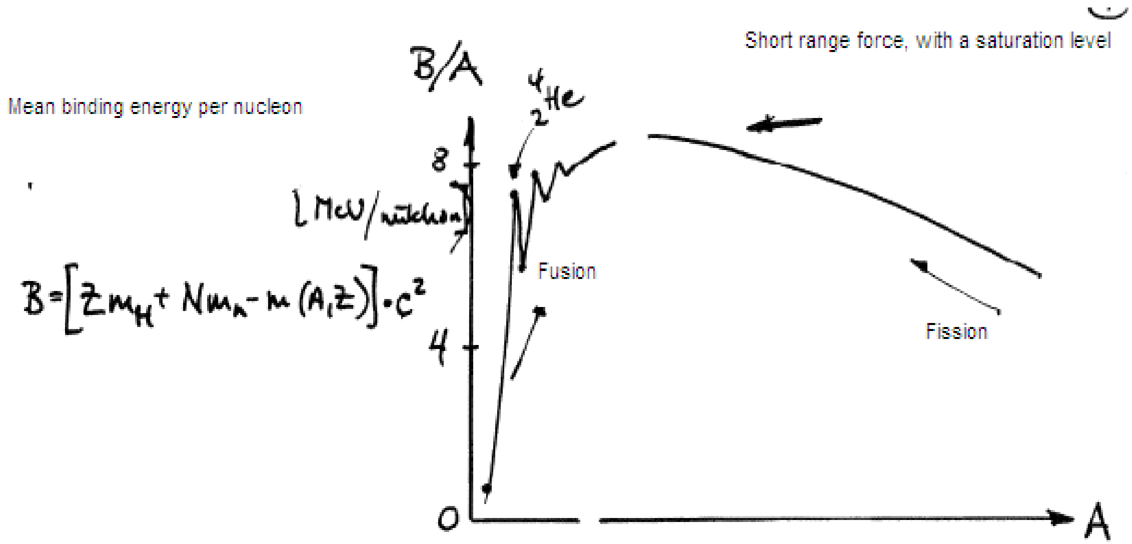
All of the three particles above are spin- $\frac{1}{2}$ fermions with non-zero magnetic moments μ_b. The neutron and the proton belong to the Baryon (composition of three quarks) family and the electron is a lepton.

Atomic mass of nucleus ${}^{A}_{Z}\mathbf{X}$

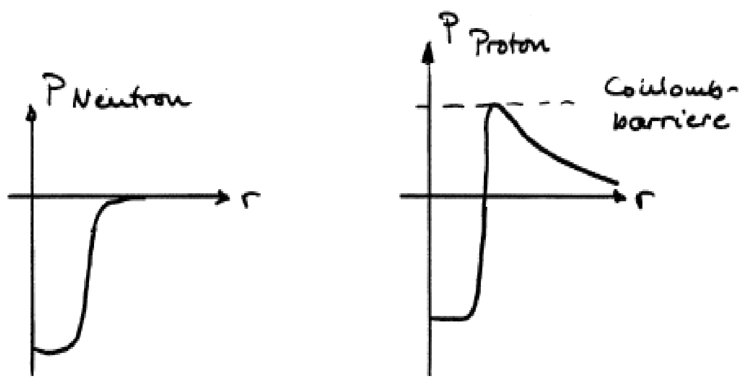
$$m(A, Z) = Zm_H + (A - Z)m_n - \frac{B}{c^2} \tag{1}$$

Where B represents the total binding energy of ${}^{A}_{Z}X$. For this to be valid, one has assumed that the mean binding energy of the electrons in ${}^{A}_{Z}X$ is the same as in ${}^{1}_{1}H$. Mass excess of ${}^{A}_{Z}X$ is defined in atomic mass units(u) to be:

$$\Delta = m(A, Z) - A \tag{2}$$


Short range force, with a saturation level
B/A
Mean binding energy per nucleon
4 2 He
8
[MeV/nucleon]
B=[Zm_H+ Nm_n- m (A,Z)]·c^2
Fusion
4
Fission
0
A

The nuclear potential (Strong force)



The potential within a nucleus can be approximately modelled as an infinite spherical potential well where the potential is zero inside a given radius, and infinity outside it. This can be expressed as:

$$V = \begin{cases} 0, & \text{if r} \leq \text{a} \\ \infty, & \text{if r} > \text{a} \end{cases} \tag{3}$$

Inserting 3 into the Schrødinger equation:

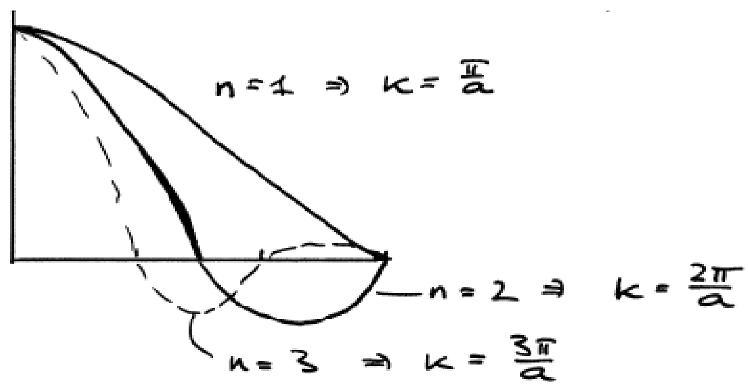
$$H\psi = E\psi \tag{4}$$

Assuming a separable wave function solution of the form $\psi = R(r) \cdot Y_l^m(\phi, \theta)$ where Y_l^m represents the spherical harmonics.

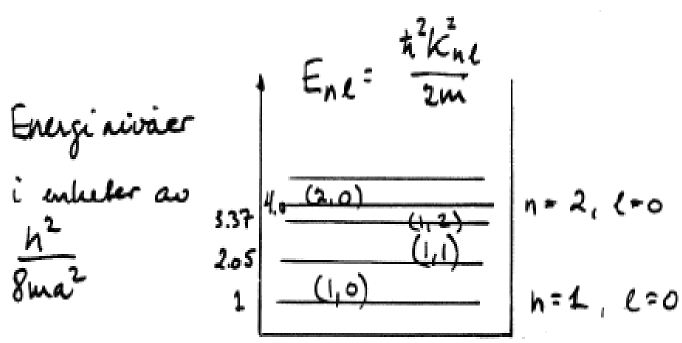
The radial part of the wave function $R(r) = j_l(kr)$, is a spherical Bessel function.

Boundary condition: $j_l(kr) = 0$ for $kr = ka$

$l = 0 : j_0(kr) = \frac{\sin kr}{kr} \rightarrow j_0(ka) = 0$ for $ka = n \cdot \pi$. The wave function has its n'th zero at $r = a$.
$l = 1 : j_1(kr) = \frac{\sin kr}{(kr)^2} - \frac{coskr}{kr}$



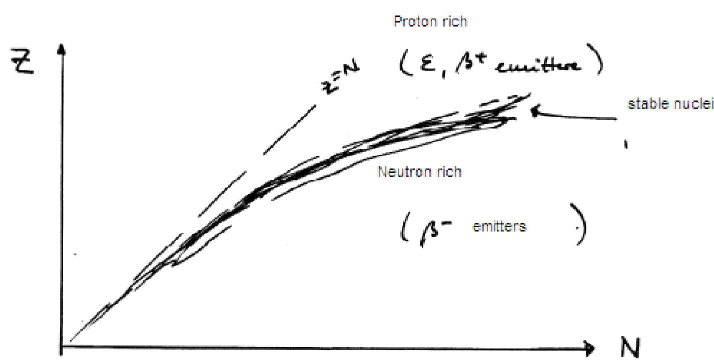
A centrifugal potential arises from the angular motion for $l \neq 0$.⇒ Energy levels $E = E_{nl}$. l is substituted with s,p,d,f for l=0,1,2,3....
For each value of l we have $2l + 1$ values for the quantum number $m_l = 0, \pm 1, \pm 2.... \pm l$



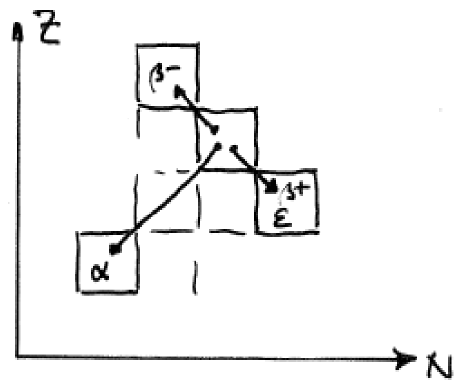
This simple model arranges the energy levels, E_{nl},in the right order up to a nucleus size of A=40.

Stability and existence of nuclei

Chart of nuclides



Radioactivity



Spontaneous radioactive processes:

$$\text{With or without a secondary gamma ray emission.} \quad \begin{cases} \alpha \\ \beta^- \\ \beta^+ \\ electroncapture \end{cases}$$

$\underline{\alpha}$: Induced by strong interactions ${}^A_Z X \rightarrow {}^{A-4}_{Z-2} X' + {}^4_2 \alpha$, $Q_\alpha = c^2(m_P - m_D - m_{He}) = T_{x'} + T_\alpha$

$T_\alpha = \frac{Q_\alpha}{1+\frac{m_\alpha}{m_{x'}}}$

$\underline{\beta^-}$: Induced by weak interactions ${}^A_Z X \rightarrow {}^A_{Z+1} X' + \beta^- + \overline{\nu}$ $Q_{\beta_-} = c^2(m_P - m_D) = T_{x'} + T_{\beta^-} + T_{\overline{\nu}}$

$Q_{\beta^-} = (\Delta_P - \Delta_D), T_{x'} \simeq 0$

$\underline{\beta^+}$: Induced by weak interactions ${}^A_Z X \rightarrow {}^A_{Z-1} X' + \beta^+ + \nu$ $Q_{\beta^+} = c^2(m_P - m_D - 2m_e) = T_{x'} + T_{\beta^+} + T_\nu$

$Q_{\beta^+} = (\Delta_P - \Delta_D - 2m_e)c^2$, $T_{x'} \simeq 0$

$\underline{\varepsilon}$: Induced by weak interactions ${}^A_Z X + e^- \rightarrow {}^A_{Z-1} X' + \nu$ $Q_{EC} = (m_P - m_D)c^2 - E_B = T_\nu$

Electron capture, where an electron is absorbed by the nucleus, is an energetically favorable process which is competing with the β^+ disintegration process. ε is followed by characteristic X-ray radiation.

$\underline{\gamma}$: Induced by E.M interactions ${}^A_Z X^* \rightarrow {}^A_Z X + \gamma$ $Q_\gamma = (m_P - m_D)c^2 = T'_x + h\nu$

$T_{x'} \simeq 0$

γ- and X-ray radiation are both secondary processes, which are characteristic of the final daughter nucleus after a disintegration.

The disintegration constant λ

$$\frac{dN}{dt} = -\lambda \cdot N \Rightarrow N(t) = N(0)e^{-\lambda t} \tag{5}$$

In the equation above, one can see that λ represents a constant transition probability per unit time.
$[\lambda] = s^{-1} = Bq$
A good argument supporting the assumed disintegration model in 5 is based on elementary time-dependent perturbation theory.

Radioactivity. Disintegration kinetics

Statistically defined variables:

Half-life	$T_{\frac{1}{2}}$,	$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$
Mean life-time	τ,	$\tau = \frac{1}{N_0}\int_0^\infty t\lambda N(t)dt = \frac{1}{\lambda}$
Activity	A,	$A = \lambda \cdot N$
Specific activity	SA	$SA = \lambda \cdot n$ (n is the number of atoms per mass unit)
		$n = \frac{N_A}{A}$ where N_A is Avogadro's number,and A is the molar mass of the atom.

1Bq is defined to be the amount of radio-nuclei you need of a specific isotope, to get one disintegration per second.

Disintegration chains

A disintegration chain appears when the daughter nucleus of the previous disintegration is unstable.

$$\text{A} \overset{\lambda_A}{\rightarrow} \text{B} \overset{\lambda_B}{\rightarrow} \text{C}$$

Using equation 5 in several steps, assuming that nucleus C is stable, this reaction becomes:

$$\frac{dN_A}{dt} = -\lambda_A \cdot N_A; \qquad \frac{dN_B}{dt} = \lambda_A \cdot N_A - \lambda_B \cdot N_B; \qquad \frac{dN_C}{dt} = \lambda_B \cdot N_B \tag{6}$$

Example:

C stable $\Rightarrow N_A + N_B + N_C = N_0$, initial values: $N_A(0) = N_0$; $N_B(0) = N_C = 0$
$\Rightarrow$

$N_B = \frac{\lambda_A N_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t})$ (Correction:In this equation NA=NA(0)=N0)

$N_A = N_0 e^{-\lambda_A t}$
$\Rightarrow$
$N_B = \frac{\lambda_A}{\lambda_B} N_0 (1 - e^{-\lambda_B t})$ if $\lambda_A << \lambda_B$

Permanent equilibrium for $t >> 1/\lambda_B$ ($T_A \gg T_B$):

$Q_B = \lambda_B N_B \rightarrow \lambda_A N_A = Q_A$

Transient equilibrium($T_A > T_B$):

$Q_B = \lambda_B N_B \rightarrow \frac{\lambda_A \lambda_B N_0}{\lambda_B - \lambda_A} e^{-\lambda_A t} = Q_A$

$Q_B \rightarrow \frac{\lambda_B}{\lambda_B - \lambda_A} Q_A$ When t $\rightarrow \infty$

No equilibrium ($T_A < T_B$)

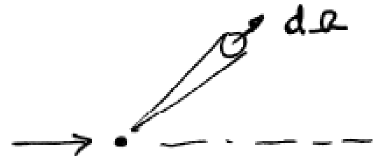
Nuclear reactions

$$\overbrace{a + \underbrace{A}_{Target \quad nucleus}}^{Entrance \quad channel} \rightarrow \overbrace{B + b}^{Exit \quad channel}$$

Energy released: $Q = (m_a + m_A - m_b - m_B)c^2$ $\quad Q \begin{cases} > 0, \text{exoterm, releases energy} \\ < 0, \text{endoterm, absorbs energy} \end{cases}$

Scattering cross-section

Cross-section



Number of particles per second within $d\vec{\Omega}$: $dR = d\sigma \cdot \dot{\Phi}$ per target atom.

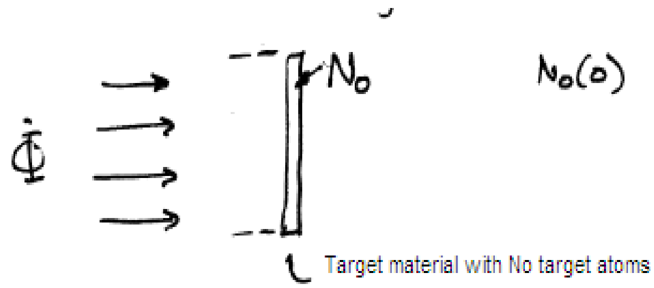
Total cross-section: $\sigma = \int_{\Omega} \frac{d\sigma}{d\Omega} d\Omega$ per target atom

Total rate of particles for a target consisting of N particles: $R = \sigma N \cdot \dot{\Phi}$

Where σ is commonly given in barns(b). 1b=$10^{-28}m^2$

Examples:

Example: Production of isotopes by neutron capture



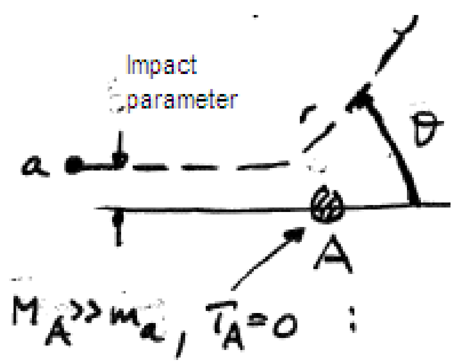
Production rate: $\frac{dN_0(t)}{dt} = -\sigma\dot{\Phi}N_0$

The radioactive nuclei produced have a disintegration constant λ

Rate of change of produced nuclei: $\frac{dN_1(t)}{dt} = \sigma\dot{\Phi}N_0(t) - \lambda N_1(t)$

Instantaneous radioactivity due to the produced nuclei: $A_1 = \lambda \cdot N_1$

Example: Rutherford scattering



Elastic scattering; Central-symmetric Coulomb potential.

Differential cross-section: $\frac{d\sigma}{d\Omega} = \left[\frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 T_a}\right]^2 \frac{1}{\sin^4 \frac{\theta}{2}}$