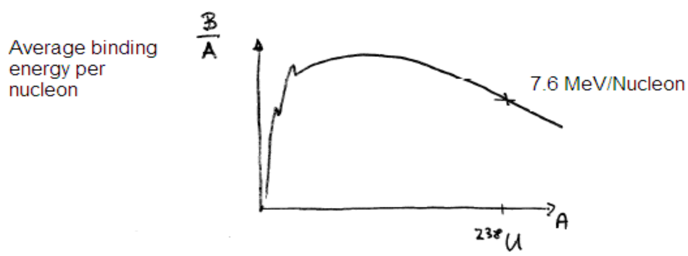
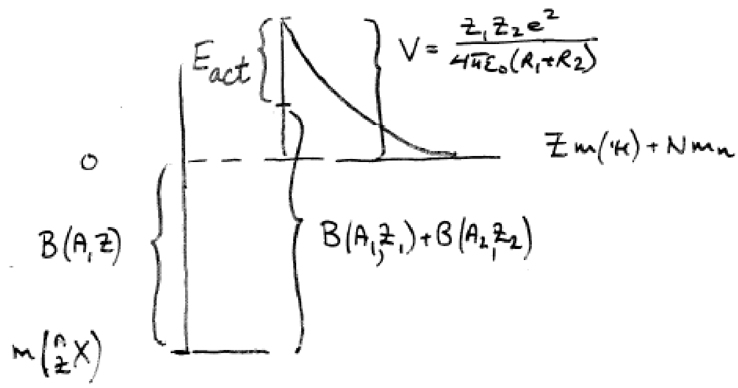
10.)
Fission and fusion

Fission (Lilley Chap.10)

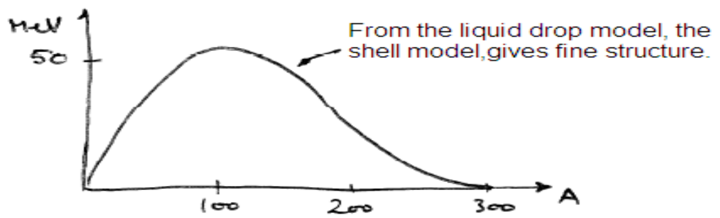
Average binding energy per nucleon:



Nuclear fission: $A \rightarrow A_1 + A_2$



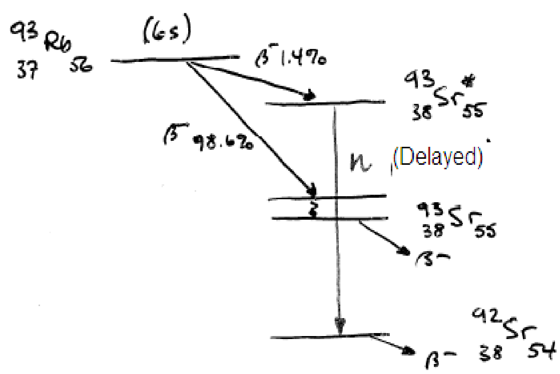
Fission barrier≡ activation energy



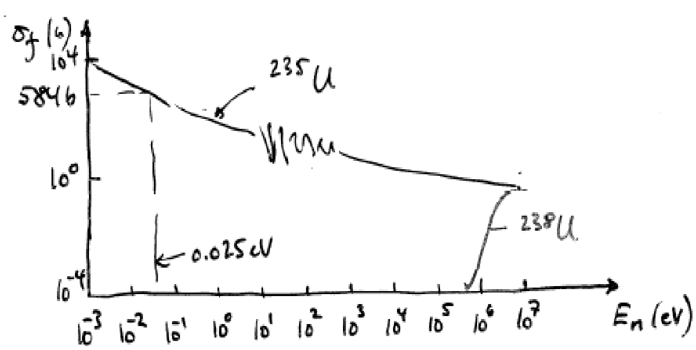
Example (Fission by capture of thermal neutrons)

$$^{235}U + n \rightarrow^{93}_{37} Rb +^{141}_{55} Cs + 2n$$

Where the last term, 2n, represents prompt, fast neutrons. Yield, $\nu = 2.5\frac{n}{fission}$.



Fission cross-section



Thermal neutrons against ^{235}U:

$$\sigma_{fission} = 584b$$

$$\sigma_{rc} = 97b \text{ (radiative capture)}$$

$$\sigma_{sc} = 9b \text{ (elastic scattering)}$$

Neutron capture results in a compound nucleus with an excitation energy E_{ex}.

Reaction:	$^{235}U + n \rightarrow ^{236} U^*$
Excitation energy:	$E_{ex} = \left[m(^{236}U^*) - m(^{236}U)\right]c^2$
For low-energy neutrons (Kinetic energy negligible):	$m(^{236}U^*) = m(^{235}U) + m_n$
$\Rightarrow$	$E_{ex} = 6.5MeV = B_n$

Where B_n represents the binding energy of the captured neutron. Neutron capture in nuclei with odd neutron numbers gives a larger value for E_{ex} than neutron capture in nuclei with even neutron numbers. This is because of pair-contributions to the binding energy. This all results in a large fission cross-section for neutron-induced fission in nuclei with an odd number of neutrons.

Energy distribution

$$^{235}U + n \rightarrow ^{236} U^* \rightarrow ^{93} Rb + ^{141} Cs + 2n, \qquad Q = 181MeV$$
$$\overline{Q} = 200MeV \, \textit{(all possible outcomes)}$$

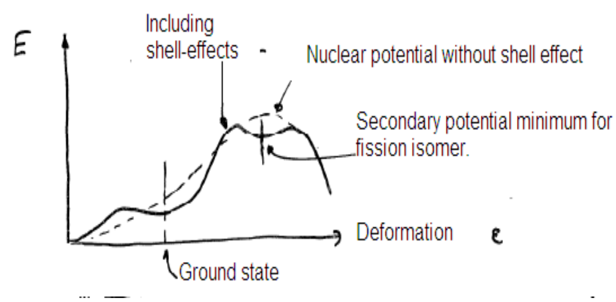
Distribution:

$T_{m_1} + T_{m_2}$	80%	168MeV
T_{2n}		5MeV
Prompt γ		7MeV
Gamma from radiative neutron capture		5MeV
β-disint. of fragments		20MeV
γ-fragments		7MeV
Sum:		212MeV

12 MeV of the β disintegration energy is in the form of neutrino energy, which is not recoverable. Net result is therefore 200 MeV.

Fission and nuclear structure

Deformed nuclei can reach intermediate states (fission isomeric states) with increased deformation which results in a lower fission barrier.



Fission resonance: Transition from one of the ground state's excited levels to one of the fission-isomeric exited states, where energy, spin, and parity coincide with the former state.

Controlled fission reaction

Neutron reproduction factor for an infinite medium: $k_\infty = \eta \cdot \varepsilon \cdot p \cdot f$

Where η represents the yield of fast neutrons for each thermal neutron absorbed in the fission fuel.

$\eta = \nu \frac{\sigma_f}{\sigma_f + \sigma_c}$, $\quad \nu = 2.42 \frac{neutrons}{fission}$ for ^{235}U

$\eta = 1.33$ for $(\underbrace{^{235}U}_{0.72\%} \,\&\, \underbrace{^{238}U}_{99.28\%})$ in naturally occuring U.

ε: Fast fission factor (fast neutron capture$\rightarrow$ fission.

p: Resonance escape probability (i.e. moderation probability)

f: Thermal utilization factor (fraction of thermal neutrons absorbed in the fuel in contrast to the ones absorbed in the moderator or other non-fuel absorbers).

η is determined by the fuel composition. Moreover, ε, p and f are all dependent on both the geometry and the moderator material.

Loss of fast and thermal neutrons l_f and l_t(fractions).

For a finite geometry: $k = k_\infty \cdot (1 - l_f) \cdot (1 - l_t)$

The chain reaction is easier to control due to delayed neutrons after β-disintegration of fission fragments ("Delayed critical").

Nuclear reactor

Fuel:
1.) Naturally occuring ^{235}U (0.72%) or enriched ^{235}U.
2.) ^{239}Pu or ^{233}U from breeder reactors.

Moderator:
1.) Low mass number (effective moderation).
2.) Minimal neutron capture.
3.) Chemical stability
4.) Cheap and accessible.

Moderator materials used: Carbon is ok, but violates 1.), D_2O is good, but violates 4.), H_2O is good, but violates 2.)

Control rods: Cd (Large capture cross section for thermal neutrons)

Reactor types:
1.) Boiling water reactor (has negative power feedback through void fraction)
2.) Pressurized water reactor
3.) Heavy-water reactor
4.) Gas-cooled reactor
5.) Natrium-cooled fast breeder reactor

Breeder reactor

This reactor burns ^{239}Pu. Moreover, it converts ^{238}U to ^{239}Pu and ^{232}Th to ^{233}U:

Conversion of ^{238}U: $$^{238}U + n \rightarrow ^{239}U(23m) \rightarrow ^{239}Np + \beta^- + \overline{\nu}$$

Furthermore, ^{239}Np has a half-life $t_{\frac{1}{2}} = 2.3d$ and decays into $^{239}Pu + \beta^- + \overline{\nu}$.

Conversion of ^{232}Th: $$^{232}Th + n \rightarrow ^{233}Th(22m) \rightarrow ^{233}Pa + \beta^- + \overline{\nu}$$

Furthermore, ^{233}Pa has a half-life $t_{\frac{1}{2}} = 27d$ and decays into $^{233}U + \beta^- + \overline{\nu}$.

Fission products

1.) Can disturb the chain reaction ("reactor poison" due to high neutron capture cross section) for example ^{135}Xe.

2.) Can contain nuclei which are valuable for medical purposes.

3.) Are highly active radioactive waste. (Radioactive waste problems)

Thorium power?

^{232}Th is an abundant, *fertile* nuclide that through conversion to ^{233}U can be used as a component in nuclear reactor fuels, for existing reactors and for new designs (advanced CANDU reactor, molten salt reactor, accelerator-driven systems)

Fusion (Lilley Chap.11)

Advantages relative to a fission reactor for power production:

1.) Easily accessible fuel material (hydrogen, deuterium, tritium).

2.) The reaction products are light and stable nuclei, i.e no problems with highly radioactive waste.

Main problem: To get a reliable reaction going, because the Coulomb-barrier has to be overcome.

Relevant processes:

1.) D-D reaction: $D(d,n)^3He$

$^2H +^2 H \rightarrow^3 He + n$, Q=3.3MeV

$D(d,p)T$

$^2H +^2 H \rightarrow^3 H + p$, Q=4.0MeV

2.) D-T reaction: $D(t,n)^4He$

$^3H +^2 H \rightarrow^4 He + n$, Q=17.6MeV

The D-T reaction is the reaction chosen for further fusion reactor development because:

1.) There is a large output of energy.

2.) The Coulomb barrier is the same as for the D-D reaction.

Coulomb barrier: $V_c = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{Z_a \cdot Z_x}{R_a + R_x}$

D-T reaction: $V_c = 200keV$

Energy: $T_a \simeq 1 - 10keV \ll V_c$ which corresponds to a temperature $10^7 - 10^8 K$

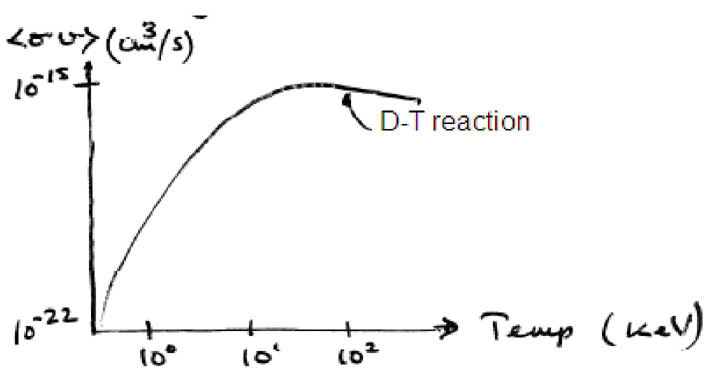
This means that tunneling is required to overcome the barrier.

Fusion cross-section: $\sigma_{fu} \propto \frac{1}{v^2} e^{-2G}$

Reaction rate: $\sigma_{fu} \cdot v$

$p(v) \propto v^2 e^{-\frac{mv^2}{2kT}}$

$\langle \sigma v \rangle = \int \frac{1}{v^2} e^{-2G} \cdot v \cdot e^{-\frac{mv^2}{2kT}} v^2 dv$



Controlled thermal fusion reactor?

Heating the reactor up to about $10^8 K$ (10keV).
Loss due to bremsstrahlung $\ll$ fusion power output at T >4keV.

Fusion energy released per unit volume: $E_f = \frac{1}{4} n^2 < \sigma v > Q\tau$
There are equal densities of D and T, $\frac{n}{2}$. In addition there are free electrons in the plasma, i.e $n_e = n$. Q is the released energy per fusion reaction and is equal to 17.6MeV for D-T. τ is the confinement time, i.e the time the reaction can be maintained by magnetic confinement of plasma. Thermal energy required per unit volume to reach temperature T:

Energy required: $E_{th} = \frac{3}{2} nkT + \frac{3}{2} n_e kT = 3nkT$

Net energy output if $E_f > E_{th}$

$\Rightarrow$ Lawson criterion: $n\tau > \frac{12kT}{<\sigma v> Q} \simeq 10^{20} \frac{s}{m^3}$ for D-T

Fusion reactions in the sun

The sun is a very successful fusion reactor, which maintains nearly constant output power.
Step 1(rate limiting): $^1H +^1 H \rightarrow {}^2H + e^+ + \nu, \quad Q = 1.44MeV$
Low reaction rate due to weak interaction ($p \rightarrow n + e^+ + \nu$) which must take place within the time interval of the collision of the two protons.

Solar temperature: $15 \cdot 10^6 K \simeq 1keV$

Reaction rate: $5 \cdot 10^{-18} s^{-1} \frac{1}{proton} \cdot 10^{56} protons \simeq 10^{38} \frac{reactions}{sec}$

$\Rightarrow$ constant "low" rate.

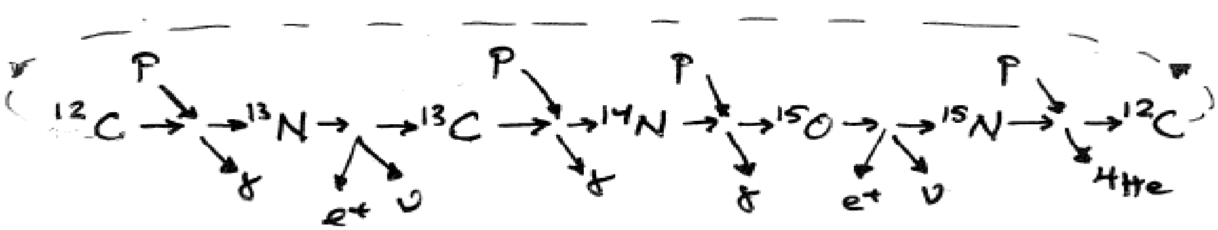
Further reactions follow quickly:

1.) $^2H + {}^1H \rightarrow {}^3He + \gamma$, Q=5.49MeV

2.) $^3He + {}^3He \rightarrow^4 He + 2^1H + \gamma$, Q=12.86MeV

Total result: $4^1H \rightarrow^4 He + 2e^+ + 2\nu$, Q=26.7MeV

The CNO-cycle (in second generation stars)



Same net result: $4^1H \rightarrow^4 He + 2e^+ + 2\nu, \quad Q = 26.7MeV$

Helium burning

Reaction 1: $\alpha + \alpha + \alpha \rightleftharpoons {}^8Be + \alpha \rightleftharpoons^{12} C^* \overset{0.04\%}{\rightarrow} {}^{12}C + \gamma$

Reaction 2: $\alpha +^{12} C \rightarrow^{16} O + \gamma$ etc $\rightarrow {}^{20}Ne,^{24} Mg$

Further burning:

For example: $^{12}C+\,^{12}C \rightarrow \begin{cases} ^{20}Ne+\alpha \\ ^{23}Na+p \\ ^{23}Mg+n \end{cases}$

$$^{16}O +^{16} O \rightarrow^{28} Si + \alpha$$

⇒ Formation of ^{56}Fe, is the last nucleus in this process. Further nucleon synthesis is mainly due to neutron capture and β-disintegration.