2.) Radiation-matter interaction (Lilley Chap.5)

Interaction of charged particles with matter

Coulomb interactions

What characterizes these interactions, is that their origin of existence is due to the long range Coulomb-force.

Type of interaction / Interacts with	Elastic	Inelastic
Electrons		Ionisation
Nuclei	Rutherford Scattering	Brems strahlung

These interaction processes result in a continuous retardation of charged particles, because of the long range Coulomb force.

Heavy charged particles

Energy transfer

Heavy charged particle of mass M, velocity $\vec{V}$, and charge ze interacts with atomic electron of the material.

Assuming the binding energy of the electron, $E_B = 0$ and that initially the electron is found at rest.

Conservation of energy and momentum: $T_M = T'_M + T'_e$

$$\vec{p_M} = \vec{p_M}' + \vec{p_e}'$$

Maximum energy transfer happens when the particles collide head-on. An approximate non relativistic calculation of the maximum energy transfer from the heavy ion to the electron follows below.

Non relativistic calculation: $pc = \sqrt{T(T + 2mc^2)} \simeq c\sqrt{2mT}$

Maximum energy transfer: $T'_{emax} = \frac{4mM}{(m+M)^2} T_M$

For a heavy charged particle $m \ll M \Rightarrow$ $T'_{emax} = 2mV^2$

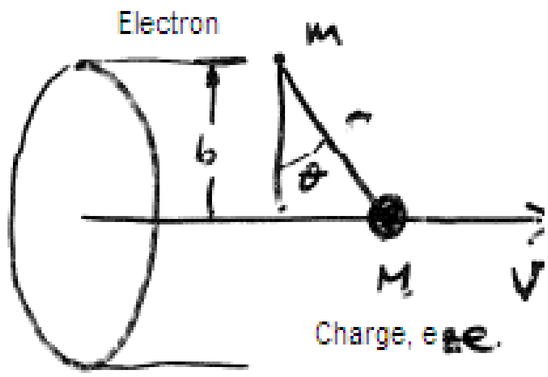
Where V is the initial velocity of the heavy particle, and m is the electron mass. The relativistic expression is a bit more complicated.

Relativistic expression for maximum energy transfer: $T'_{emax} = \frac{2\gamma^2 mV^2}{1 + \frac{2\gamma m}{M} + \frac{m^2}{M^2}}$

Where γ represents the Lorenz factor:

$$\gamma = \frac{1}{\sqrt{1 - (\frac{V}{c})^2}} \tag{1}$$

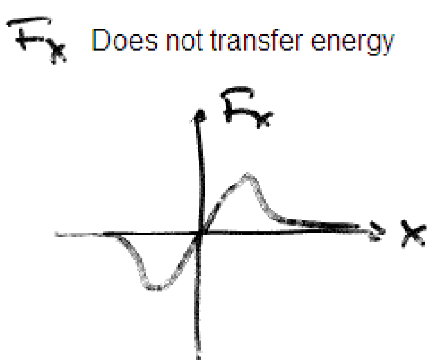
Stopping power for heavy charged particles interacting with electrons.

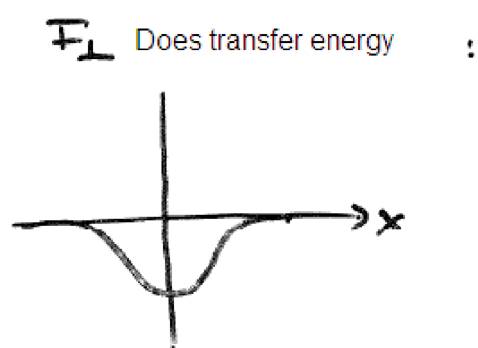


Collision stopping power: $S_c = -\frac{dT}{dx}$

Force acting on the heavy particle: $\vec{F} = \frac{1}{4\pi\varepsilon}\frac{ze^2}{r^2}\hat{e_r}$

S_c is loss of kinetic energy per unit path length in the scattering medium, due to interactions between the heavy charged particle and the electrons.
All the electrons in a cylinder shell with a collision parameter b contribute equally to the stopping power, since the Coulomb force is spherically symmetric.





If the x direction is defined to be along the charged particle's direction as earlier implied, F_x does not transfer energy. However, $F_\perp$ does:

Momentum transfer: $\Delta p_{\perp} = \int |F| \cos\theta dt = \frac{ze^2}{4\pi\varepsilon_0} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta}{b^2} \frac{b}{V} \frac{d\theta}{\cos^2\theta}$

This is found assuming that: V$\simeq$ constant

Energy transferred to the electron: $E = \frac{(\Delta p \perp)^2}{2m_e} = \frac{1}{(4\pi\varepsilon_0)^2} \frac{2z^2e^4}{m_e V^2 b^2}$

The differential cross section for energy transfer between E and $E + dE$, per electron in the stopping medium:

$$d\sigma(E) = \frac{d\sigma(E)}{dE} dE = |2\pi b db| = \frac{2\pi z^2 e^4}{(4\pi\varepsilon_0)^2 m_e V^2} \frac{dE}{E^2} \tag{2}$$

Again returning to the stopping power: $S_c = -\frac{dT}{dx} = -\frac{dE}{dx} = n_v Z \int_{E_{min}}^{E_{max}} \frac{d\sigma}{dE} E dE$

The total contribution to the interaction probability from all of the electrons inside the cylinder shell(d^3V) is worked out below. n_v is the number of atoms per unit volume.

Further on: $n_v Z d^3V = n_v Z \frac{d\sigma(E)}{dE} dE dx$; $n_v = \frac{N_A}{A} \cdot \rho$

The Stopping power: $S_c = \int_{E_{min}}^{E_{max}} n_v Z \frac{2\pi z^2 e^4}{(4\pi\epsilon_0)^2 m_e V^2} \frac{dE}{E^2} E$

The total stopping power then comes out to be:

$$S_c = \frac{2\pi z^2 r_0^2 m_e c^2}{\beta^2} n_v Z \Big[\ln \frac{E_{max}}{E_{min}} \Big]; \quad r_0 = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \tag{3}$$

Going back to the non relativistic case: $E_{max} = \frac{4mM}{(m+M)^2} T_M$

For heavy particles($M \gg m$) $\Rightarrow$ $E_{max} = 2m_e V^2$

$E_{min} = \frac{I^2}{2m_e V^2}$ (I=mean exitation energy)

Mass stopping-power (non relatisvistic): $\frac{S_e}{\rho} = \frac{2\pi z^2 r_0^2}{\beta^2} m_e c^2 N_a \left[\frac{Z}{A}\right] 2 \ln \left[\frac{Q_{max}}{I}\right]$,

$(M \gg m), Q_{max} \equiv E_{max}$

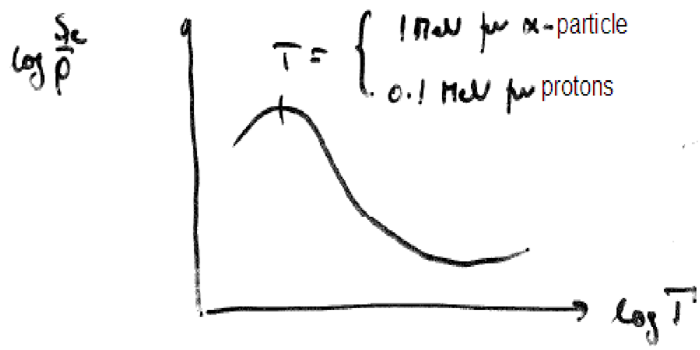
Relativistic expression with corrections:

$$\frac{S_c}{\rho} = N_A \frac{Z}{A} \cdot \frac{z^2 e^4}{4\pi\epsilon_0^2 m_e V^2} \Big[\ln \frac{Q_{max}}{I} - \ln(l - \beta^2) - \beta^2 - \frac{c(\beta^2)}{Z} - \frac{1}{2}\delta \Big] \tag{4}$$

Where A represents the molar mass of the stopping material, V is the particle velocity.

The two last terms in the expression are added as a shell correction and a density effect, respectively.

The last term is a correction which appears because there is also a field set up from other atoms in the stopping material.
Note that this expression is independent of the mass of the incoming particle.

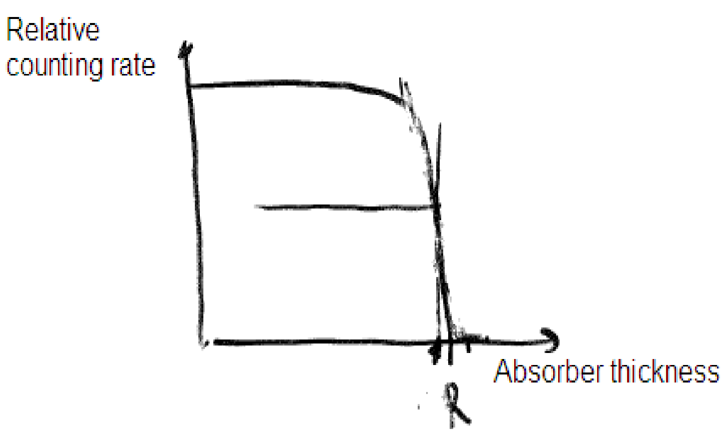


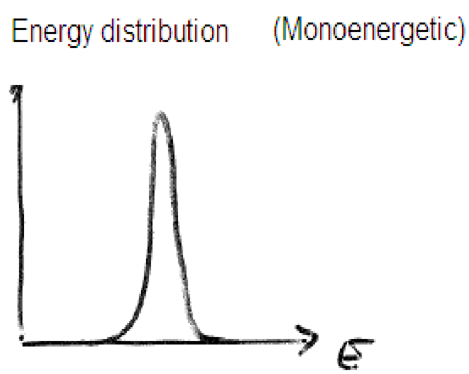
Stopping-power for composite materials: $n_v Z \ln I \Rightarrow \sum_i n_{vi} Z_i \ln I_i$

Range

Range for heavy charged particles

Mono energetic particles, for example α particles:





Particle range in a stopping-material: $R(T) = \int_T^0 \frac{dT}{-\frac{dT}{dx}}$

$-\frac{dT}{dx} = z^2 G(\beta)$

$dT = g(\beta) \cdot M d\beta$

Particle range in a stopping-material: $R(\beta) = \frac{M}{z^2} \int_\beta^0 h(\beta) d\beta = \frac{M}{z^2} f(\beta)$

This is a useful formula for comparing range of particles having identical initial velocity.

Linear energy transfer(LET): $LET = \left[-\frac{dT}{dx}\right]_c$

NOTE! The range is defined to be the distance along the particle track, not the penetration depth. Generally, we have $R > x_0$ where x_0 is the penetration depth. Nevertheless, for heavy charged particles: $R \simeq x_0$. This means that a heavy charged particle, fired at a target medium, will travel along a path that hardly deviates from it's original direction, until it is retarded down to zero velocity.

β-particles

Stopping-power for β-particles (z=1)

$$\frac{S_c}{\rho} = N_A \frac{Z}{A} \frac{e^4}{4\pi\varepsilon_0 m_e c^2 \beta^2} \left[\ln \frac{m_e c^2 \tau \sqrt{\tau + 2}}{\sqrt{2} I} + F^{\pm}(\beta)\right] \tag{5}$$

τ represents the β-particle's kinetic energy: $\tau = \frac{T}{m_e c^2}$

For electrons: $F^{-}(\beta) = \frac{1-\beta^2}{2}\left[1 + \frac{\tau^2}{8} - (2\tau + 1)\ln 2\right]$

For positrons: $F^{+}(\beta) = \ln 2 - \frac{\beta^2}{24}\left[23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right]$

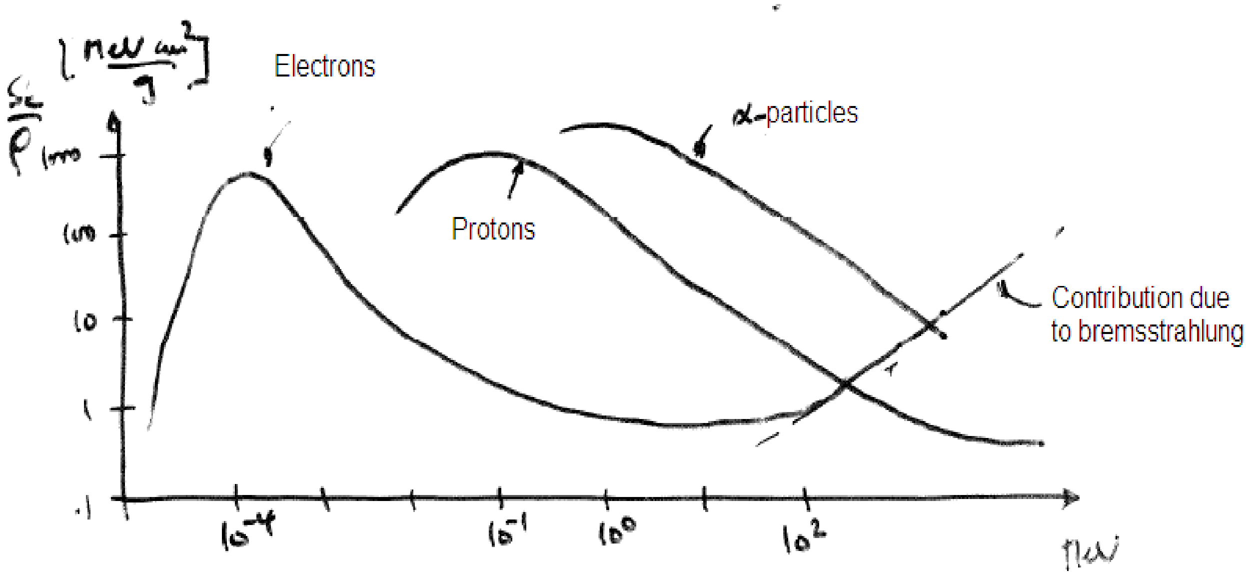
Differences between β, and heavy charged particles' interactions with matter:

1 β-particles can loose all their energy in one collision with an atomic electron.

2 β^--particles are identical with the object they interact with (electrons).

(We assume that the electron with the lowest energy is the one that belonged to the material.)

3 Relativistic formulas are required (for $T_e > 10keV$).



Bremsstrahlung contribution to the stopping power

$$\frac{-\left[\frac{dE}{dx}\right]_{rad}}{-\left[\frac{dE}{dx}\right]_{col}} \simeq \frac{ZE}{800} = \underbrace{2.5 \cdot 10^{-4} ZE}_{E \text{ is total energy in } MeV} \tag{6}$$

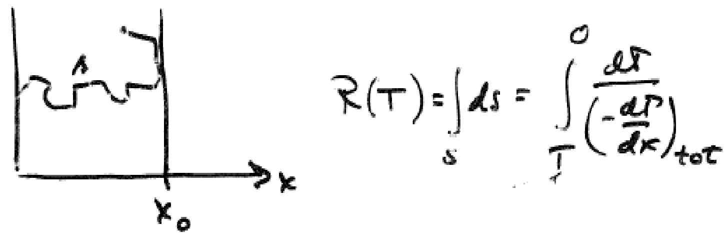
Effective bremsstrahlung contribution:

$$Y(T_0) = \frac{1}{T_0}\int_0^{T_0} y(T)dT \simeq \frac{6 \cdot 10^{-4} Z \overbrace{T}^{MeV}}{1 + 6 \cdot 10^{-4} ZT}; \quad y(T) \equiv \frac{-\left[\frac{dT}{dx}\right]_{rad}}{-\left[\frac{dT}{dx}\right]_{tot}} \tag{7}$$

This is the fraction of the incoming particle's kinetic energy, which is converted into bremsstrahlung during the entire retardation process.

Range for β-particles

Usually, electrons have a continuous energy spectrum up to E_{max}, and the range is defined relative to this energy E_{max}. The electron range is always greater than the penetration depth. NOTE that in this case it is very important to use the total stopping power in the calculations, since the bremsstrahlung contribution is highly significant.



$$R(T) = \int_s ds = \int_T^0 \frac{dT}{-\left[\frac{dT}{dx}\right]_{tot}} \tag{8}$$

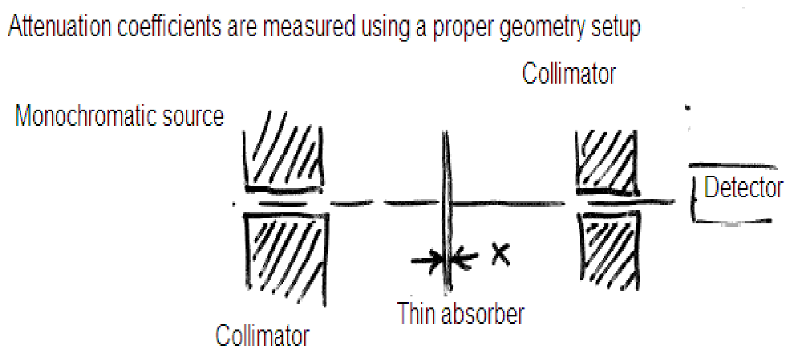
Photons

Photon interactions

Type of interaction: / Interacts with:	Elastic scattering (Coherent)	Inelastic scattering (Incoherent)	Absorption
Atomic electrons	$\sigma_{Coh.sc} \equiv \sigma_R$ Rayleigh	$\sigma_{Incoh.sc} \equiv \sigma_{CT}$ Compton	σ_{pe} Photo-electric effect
Nuclei/Nucleons	Elastic nuclear scattering	Nuclear resonance scattering	Photo-nuclear reactions
Electric field from charged particles			σ_{pp} Pair production

Attenuation coefficients

When measuring attenuation coefficients, one always measure in a "good(proper) geometry" setup.

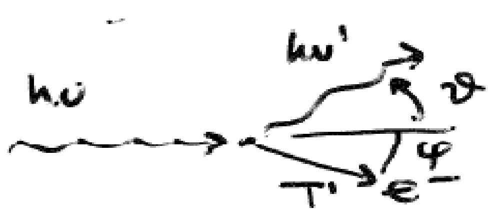


Detected intensity with/without absorber	$\frac{I}{I_0} = e^{-\mu_l \cdot x}$
<u>Linear attenuation coeff</u>:	$\mu_l = \lim_{x \to 0} \frac{1}{x} \ln \frac{I_0}{I} = -\frac{1}{I} \frac{dI}{dx}$
<u>Atomic attenuation coeff</u>:	$\sigma^a = \frac{\mu_l}{n_V}$
<u>Mass attenuation coeff</u>:	$\frac{\mu_l}{\rho} = \sigma^a \frac{N_A}{A}$

The atomic attenuation coefficient is often called the atomic scattering cross-section. This is measured in barn. n_v is the number of atoms per unit volume.
The atomic cross-sections for the different atoms in composite materials are additive.

Photon - atomic electron interaction

Compton scattering:



Assuming that the electron is free and initially at rest:

Conservation of energy: $h\nu + m_e c^2 = h\nu' + \gamma m_e c^2$

Conservation of momentum: $\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e'\cos\phi$

Relativistic electron after interaction: $(p_e'c)^2 = T'(T' + 2m_e c^2)$

Neglecting the electronic binding energy(as earlier implied): $T' = h(\nu - \nu')$

Change in wavelength: $\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - cos\theta)$

Compton wavelength: $\lambda_c = \frac{h}{m_e c}$

Scattered photon's energy:$h\nu' = \frac{h\nu}{1+\alpha(1-\cos\theta)}$,$\alpha = \frac{h\nu}{m_e c^2}$

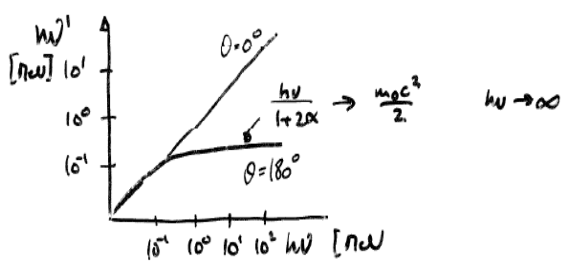
Scattering angles: $\cot\phi = (1+\alpha)\tan\frac{\theta}{2}$

Minimum scattering: $\theta \simeq 0 \Rightarrow \phi = \frac{\pi}{2}$; $h\nu' \simeq h\nu$; $T_e' \simeq 0$

Maximum scattering: $\theta = \pi \Rightarrow \phi = 0$; $h\nu' \rightarrow \frac{h\nu}{1+2\alpha}$; $T_e' = h\nu\frac{2\alpha}{1+2\alpha}$

Fraction of energy scattered: $\frac{h\nu'}{h\nu}$

Fraction of energy transferred to the Compton electron: $(1 - \frac{h\nu'}{h\nu})$



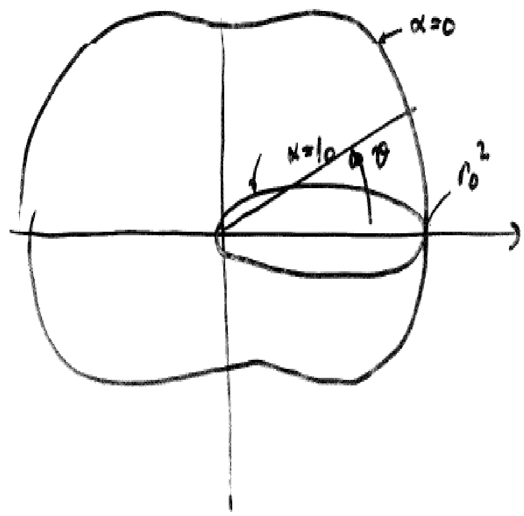
Klein-Nishina cross-section (per electron)

$$\frac{\sigma_{e,KN}}{d\Omega} = \frac{r_0^2}{2}\Big[\frac{1+\cos^2\theta}{[1+\alpha(1-\cos\theta)]^2} + \frac{\alpha^2(1-cos\theta)^2}{[1+\alpha(1-\cos\theta)]^3}\Big] \tag{9}$$

Where r_0 is the classical electron radius as defined before.

Alternatively:

$$\frac{d\sigma_{e,KN}}{d\Omega} = \frac{r_0^2}{2}\Big[\frac{\nu'}{\nu}\Big]^2\Big[\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2(\theta)\Big] \tag{10}$$



For low energies, $\alpha \to 0$:

$$\frac{d\sigma_{KN}}{d\Omega} \to \frac{r_0^2}{2}[1+\cos^2\theta]$$

This cross-section describes scattering of photons by a free electron target, consistent with classical electro-magnetic theory. This is also called the Thomson cross section. This scattering process results in coherent scattering($h\nu' = h\nu$). In reality one has to introduce a scattering form-factor $\mathcal{F}$, for this formula to agree with experimental data.

Cross section for coherent scattering (Low energy description)

$$\frac{d\sigma_{koh.sc}}{d\Omega} = \frac{r_0^2}{2}(1+\cos^2\theta)\Big[F(h\nu,\theta,Z)\Big]^2 \tag{11}$$

Cross section for incoherent scattering

$$\frac{d\sigma_{is}}{d\Omega} = \frac{d\sigma_{KN}}{d\Omega}S(h\nu,\theta,Z) \tag{12}$$

S is here a structure-factor (fraction of incoherent scattering). This factor describes the probability for the target atom to get excited, or ionized after interacting with the incoming photon.
Incoherent scattering $\equiv$ compton scattering:

Total compton scattering cross-section $\sigma_{CT} = \sigma_{CA} + \sigma_{CS}$

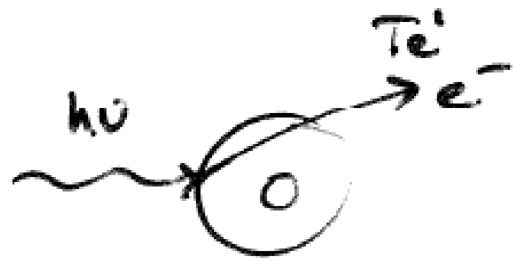
Cross-section describing energy transfer to scattered photon: $\sigma_{CS} = \frac{h\nu'}{h\nu}\sigma_{CT}$

Cross-section describing energy transfer to compton electron: $\sigma_{CA} = \left[1 - \frac{h\nu'}{h\nu}\right]\sigma_{CT}$

Photo-electric effect

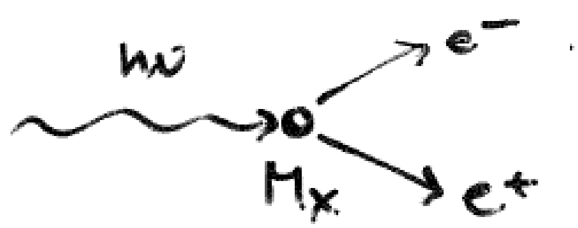
This is not possible for a free electron (There is no solution to the compton equations for $h\nu' = 0$).

Kinetic energy for the electron: $T_e' = h\nu - E_B$



Photon - Coulomb field interaction

Pair production



Threshold energy: $h\nu \geq 2m_oc^2\left[1 + \frac{m_0c^2}{M_xc^2}\right]$

Photon - nuclear Coulomb field interaction($M_x \gg m_0$): $h\nu \geq 2m_0c^2$

Triplet production

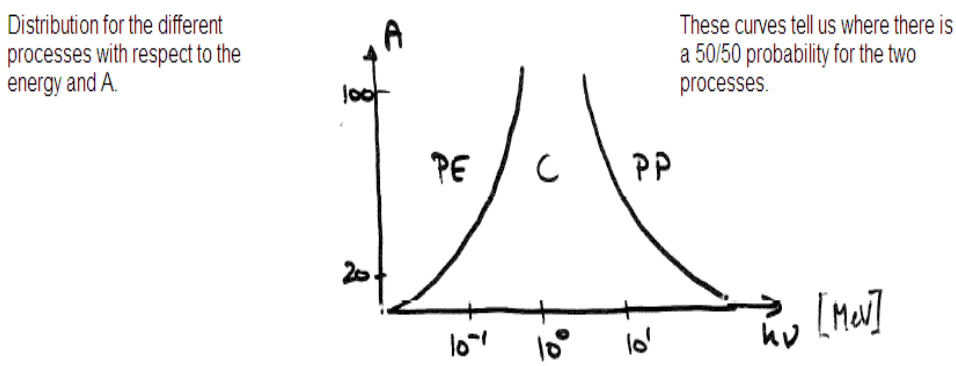
Photon-electronic Coulomb field interaction:($M_x = m_0$): $\quad h\nu \geq 4m_0c^2$

In this case, there is no way telling which two of the electrons are the produced ones, and which one is the original target. That is why the process is called :"triplet production".

β^+ annihilation

β^+ annihilation is usually a result of positronium($\beta^+ \& e^-$) being formed after the β^+ particle has lost its kinetic energy. Positronium has lifetime, $\tau \simeq 10^{-10}s$. Alternatively, the β^+ annihilation can occur "in flight".

Total interaction cross-section for photons



Total attenuation coeff: $\quad \mu = \mu_R + \mu_{PE} + \mu_{CT} + \mu_{PP}$

Mass-energy transfer coefficient, ($\frac{\mu_{tr}}{\rho}$) represents the fraction of the incoming photon's energy, which is transferred to charged particles (secondary electrons), thus increasing their kinetic energy.

$$\frac{\mu_{tr}}{\rho} = \frac{\mu_{PE}}{\rho}\left[1 - \frac{\delta}{h\nu}\right] + \frac{\mu_{CT}}{\rho}\left[1 - \frac{h\nu'}{h\nu}\right] + \frac{\mu_{PP}}{\rho}\left[1 - \frac{2m_0c^2}{h\nu}\right] \tag{13}$$

δ represents the mean energy emitted by characteristic X-ray radiation. $\delta = E_B \cdot$Probability for a de-excitation by X-ray radiation, as opposed to Auger electron emission.

Mass-energy absorption coefficient:

$$\frac{\mu_{en}}{\rho} = \left[\frac{\mu_{tr}}{\rho}\right][1-g] \tag{14}$$

g is the fraction of the secondary electrons' energy, which is emitted as bremsstrahlung. (This energy is not locally deposited in the stopping media)

Z-dependence of the photon cross sections

Generally:	$\sigma^a = Z \cdot \sigma^e$
σ^e is one of the electron cross-sections, for example σ_{KN}	
Linear attenuation coeff:	$\frac{\mu_l}{\rho} = \sigma^a \frac{N_A}{A} = \sigma^e \frac{Z}{A} N_A$
For most materials, Z$\simeq$ 0.45A for $A > 1$:	$\frac{\mu_l}{\rho} \simeq 0.45 N_A \sigma^e$

This means that $\frac{\mu_l}{\rho} \simeq$ constant(close to Z-independency) within the Compton range.

Photo-electric effect:	$\sigma^a_{PE} \propto \frac{Z^4}{(h\nu)^3}$
Compton:	$\sigma^a_{CT} \propto Z \rightarrow \sigma^e_{CT} \simeq$ constant
Pair production:	$\sigma^a_{PP} \propto Z^2$

Neutrons

Classification of neutrons

Thermal neutrons:	$E \simeq 0.025eV$
Epithermal neutrons:	$E \simeq 1eV$
Slow neutrons:	$E \simeq 1keV$
Fast neutrons:	$100keV - 10MeV$

Neutron sources

(α, n)-sources consist of an α-emitter and ^{9}Be:$\Rightarrow$ $\quad {}^4_2He +^9_4 Be \rightarrow^{12}_6 C + n$

For example, a mixture of ^{226}Ra and ^9Be $\Rightarrow$ constant neutron emission rate (not mono-energetic, due to energy loss of the α-particles in the sample).

(γ, n)-sources give nearly mono-energetic neutrons.: $\quad \gamma +^9_4 Be \rightarrow^8_4 Be + n$

The γ-photon's threshold energy for this process to work: $\quad h\nu \geq E_b$

Where E_b is the binding energy of the neutron.

Spontaneous fission, for instance: $\quad ^{252}Cf$

Nuclear reactions: Choosing a specific T_a and exit angle θ $\Rightarrow$ Selective mono-energetic neutron flux.

Example:

$^3H + d \rightarrow^4 He + n$ $\quad$ Q=17.6MeV

$^9Be +^4 He \rightarrow^{12} C + n$ $\quad$ Q=5.7MeV

Reactor as a source: Large flux of neutrons for activation analysis.

Absorption and moderation of neutrons

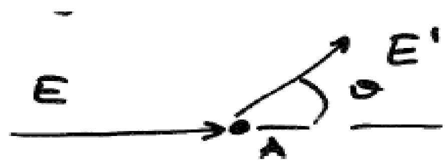
There are several possible reactions for fast neutrons: (n,p), (n,α), (n, 2n) Usually, these reactions have very strong resonances.

Without the resonances: $\quad \sigma \propto \frac{1}{v}$

Attenuation of mono-energetic neutrons: $\quad I = I_0 e^{-\sigma_t n x} = I_0 e^{-\Sigma x}$

Where Σ represents the "macroscopic cross-section".(But really is a linear attenuation coefficient)

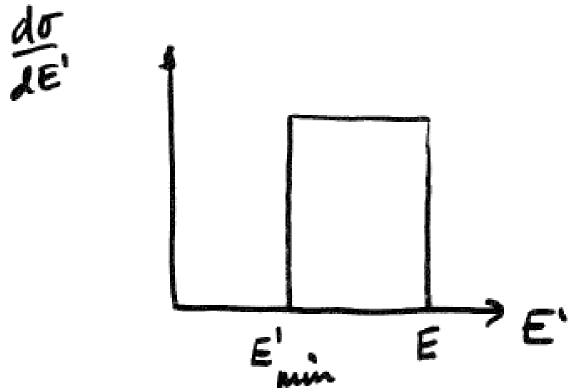
Energy distribution after scattering of mono-energetic neutrons



Scattering is isotropic in the CM frame.

$$\frac{E'}{E} = \frac{A^2 + 2A\cos\theta + 1}{(A+1)^2}$$

$$\left(\frac{E'}{E}\right)_{min} = \left[\frac{A-1}{A+1}\right]^2, \quad for \quad \theta = \pi \tag{15}$$

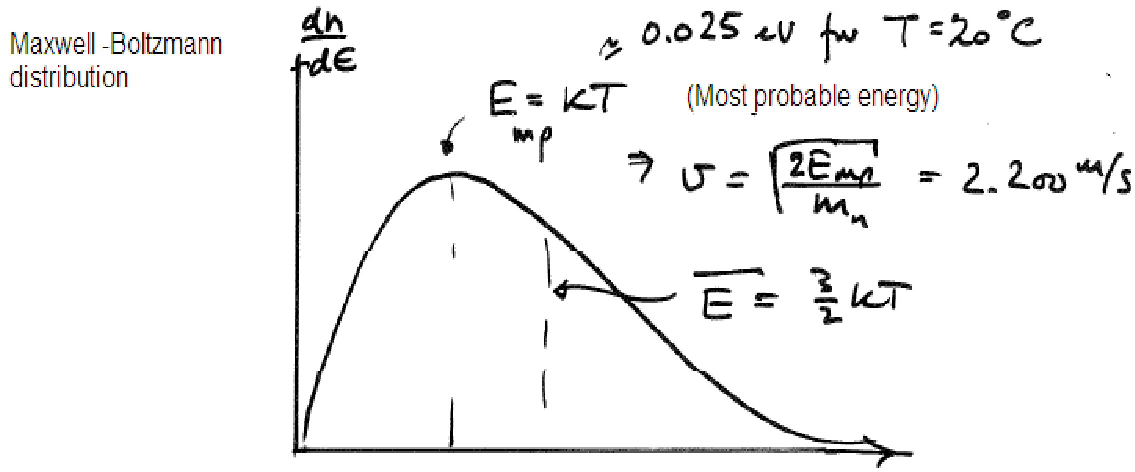


Logarithmic decrement: $\xi = \frac{1}{4\pi}\int \ln\frac{E}{E'} \cdot d\Omega = 1 + \frac{(A-1)^2}{2A}\ln\frac{A-1}{A+1}$

Median energy after n interactions: E'_n

This energy is defined as: $\ln E'_n \equiv \overline{\ln E_n} = \ln E_0 - n\xi$

Example:Thermal moderation of neutrons



Thermalizing 2 MeV neutrons in different moderators:

Moderator	ξ	n
1^H	1.0	18
2^H	0.725	25
^{12}C	0.158	115
^{238}U	0.008	2200