4.)

Nuclear structure (Lilley Chap. 2)

Models

Nuclear force

This is a short range attractive force, but repulsive for even shorter distances$\Rightarrow$ There is a certain optimal distance between nuclear particles.

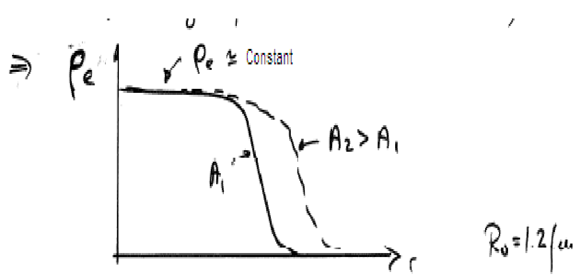
Liquid drop model

The nucleus is considered as a spherical liquid drop with constant internal density.

Evidence for the existence of the liquid drop model:

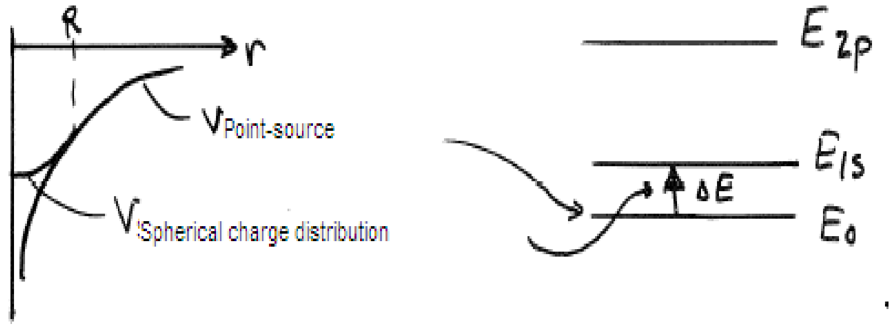
The internal charge distribution:

a.) Electron scattering experiments imply the charge density function below:



Number of nucleons per unit volume is approximately constant $\Rightarrow \quad \rho = \frac{A}{\frac{4}{3}\pi R^3}$

b.) The nuclear charge distribution affects the energy levels of the S-orbital electrons.



c.) The potential energy difference between mirror nuclei:

Example:

$^{13}_{7}N_6 \xrightarrow{\beta^+} {}^{13}_{6}C_7$, Measure E_{max} for β^+

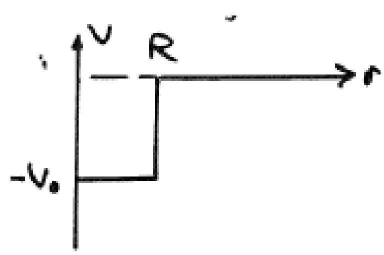
$$\Delta E_C = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{R}\underbrace{\left[Z^2-(Z-1)^2\right]}_{(2Z-1)=A} \Rightarrow \Delta E_c = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{R_0}A^{\frac{2}{3}}$$

The internal mass distribution:

a.) Neutron scattering (elastic)

This is the same calculation as used for electron scattering, remembering to exchange the electron's electro-magnetic potential with the neutron's potential

$\Rightarrow$ Scattering data give the Fourier transform of the mass distribution.



b.) Deviation from the expected angular dependency of Rutherford scattering for $r > R$.

c.) Calculating the tunneling probability for α-disintegration.

d.) Measuring the difference between E_k-energies for atoms

with $\underbrace{\pi - mesons}_{Strong\ force+Coulomb}$ and $\underbrace{muons}_{Coulomb\ only}$ instead of electrons.

These four points, from a.) through d.), result in a conclusion: $\rho_m \simeq \rho_e$, $R = R_0 A^{\frac{1}{3}}$, $R_0 = 1.2$fm

Measuring atomic masses

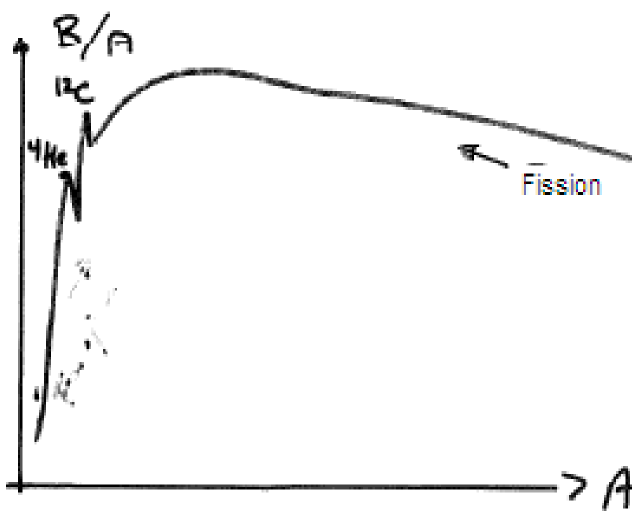
Mass excess, $\Delta = m - A$ is $\begin{cases} \geq 0, & \text{if } A < 12 \\ \leq 0, & \text{if } A > 12 \end{cases}$

Binding energy

Binding energy: $B = \left[Zm(^1H) + Nm_n - m(^A_Z X) \right] c^2$, where $c^2 = 931.5 \frac{MeV}{u}$

Neutron separation energy: $S_n = \left[m_n + m(^{A-1}_{Z} X_{N-1}) - m(^A_Z X_N) \right] c^2$

Proton separation energy: $S_p = \left[m_p + m(^{A-1}_{Z-1} X_N) - m(^A_Z X_N) \right] c^2$



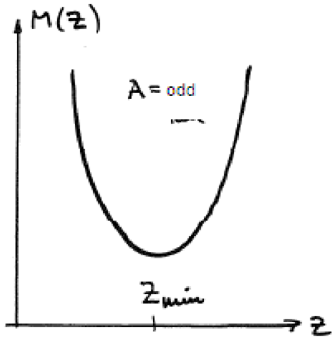
Binding energy: $B = a_v \cdot A - a_s A^{\frac{2}{3}} - a_c \cdot Z(Z-1) A^{-\frac{1}{3}}$ (Liquid drop model)

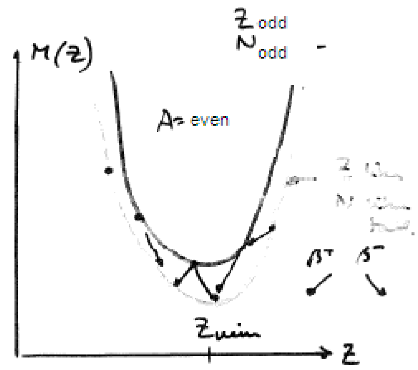
$-a_{sym} \cdot \frac{(A-2Z)^2}{A} + \delta_{pair}$ (Shell effects)

Where $\delta = \begin{cases} +a_p A^{-\frac{3}{4}}, & \text{if Z \& N are even numbers} \\ 0, & \text{if A is an odd number} \\ -a_p A^{-\frac{3}{4}}, & \text{if Z \& N are odd numbers} \end{cases}$

Semi-empirical mass formula: $M(Z, A) = Zm(^1H) + Nm_n - \frac{B(Z,A)}{c^2}$

M(A,Z) is sketched below for fixed values of A:





Minimum mass: $\frac{\partial M}{\partial Z} = 0 \Rightarrow Z = Z_{min} = \frac{[m_n - m(^1H)] + a_c A^{-\frac{1}{3}} + 4a_{sym}}{2a_c A^{-\frac{1}{3}} + 8a_{sym} A^{-1}}$

The nuclear shell model

This model is the nuclear analogy to the electron shell model.
Experimental data show that the ionisation energy decreases and the atomic radius increases rapidly for the first electron outside a full shell. I.e for Li, Na, K etc. The same occurs for nucleons in the nucleus.

Experimental data that justify the theory of a nuclear shell structure

a.) There is a rapid fall in 2-neutron and 2-proton separation energy when passing the magic nucleon numbers; 8, 20, 28, 50, 82, 126

b.) α-energy reaches maximum for radio-nuclei where the daughter nucleus has a structure corresponding to magic numbers.

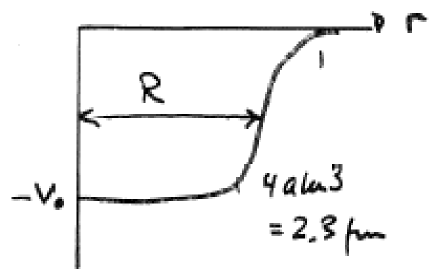
c.) The neutron scattering cross-section for nuclei with N=magic numbers is extraordinarily small.

d.) There is a huge increase in the nuclear radius when the number of neutrons exceed magic numbers.

A realistic potential for the shell model (Woods-Saxon potential):

$$V = \frac{-V_0}{1+e^{\frac{r-R}{a}}} \tag{1}$$

Where $V_0 \simeq 50MeV$, $R = R_0 A^{\frac{1}{3}}$, $R_0 = 1.2fm$



Spin-Orbit coupling

Energy difference: $\Delta E = -(\vec{l}\cdot\vec{s})\; V_{so}, \quad V_{so} > 0$

Total angular momentum: $\vec{j} = \vec{l} + \vec{s}$

From this, it follows that $< \vec{l}\cdot\vec{s} >= \frac{1}{2} < [\vec{j}^2 - \vec{l}^2 - \vec{s}^2] >= \frac{1}{2}[j(j+1) - l(l+1) - s(s+1)]\hbar^2$

Energy splitting: $\delta E = V_{so}[< \vec{l}\cdot\vec{s} >_{j=l-\frac{1}{2}} - < \vec{l}\cdot\vec{s} >_{j=l+\frac{1}{2}}] = \frac{\hbar^2}{2} V_{so}(2l+1)$

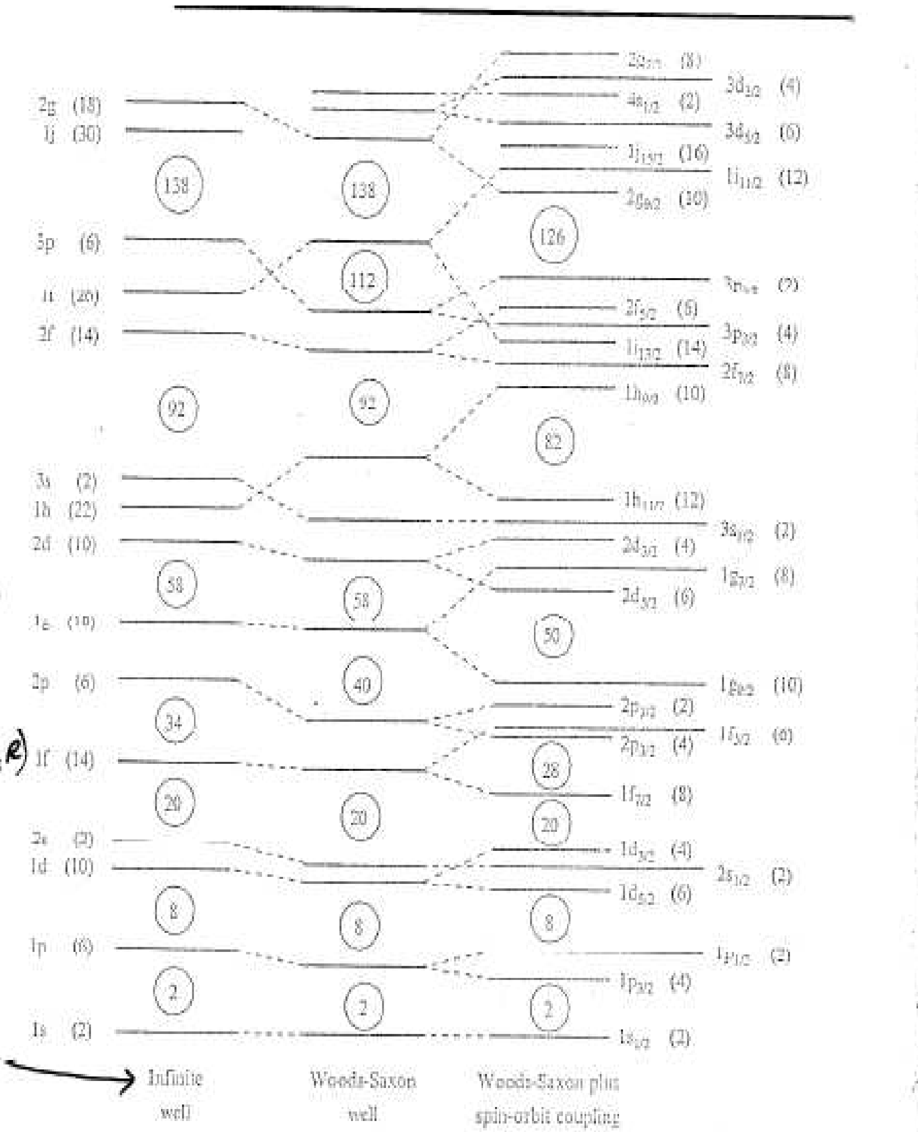
$j_l(k_{nl}R)=0$

$r=R$ gives

n'th zero

for $j_l(kr)$ $r\in(0,R)$

$\rightarrow E_{nl}=\dfrac{\hbar^2 k_{nl}^2}{2m}$



# of identical nucleons per energy level: (2j+1)

Remember that the Pauli principle applies only for identical Fermions (protons and neutrons are counted independently).

$$\text{Parity:}\quad (-1)^l \Rightarrow \begin{cases} \pi^+ & \text{for s, d, g..} \\ \pi^- & \text{for p, f, h..} \end{cases}$$

This shell model with spin-orbit coupling gives the right spin and parity. Further on, it predicts reasonable energy levels, and introduces the magical numbers corresponding to filled shells.

Angular momentum and spin

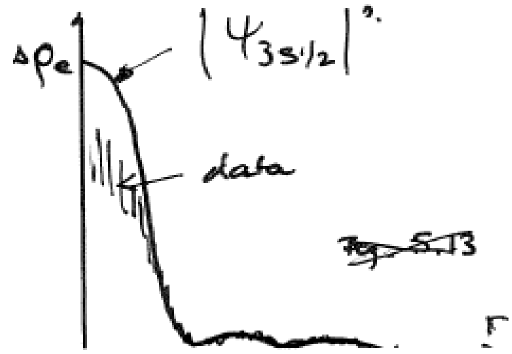
For each nucleon:	$\vec{j} = \vec{l} + \vec{s}$
For the nucleus:	$\vec{I} = \sum \vec{j}_i$
	$\vec{I}^2 = \hbar^2 I(I+1)$
	$I_z = m\hbar$
For nuclei with one valence-nucleon:	$\vec{I} = \vec{j}_{vn}$
For nuclei with two valence-nucleons:	$\vec{I} = \vec{j}_1 + \vec{j}_2$
For nuclei with even numbers of A:	$I \in$ integer
For nuclei with odd numbers of A:	$I \in$ half integer
For even-even nuclei(Z&A even):	I=0 in the ground state

Valence nucleons

Excited states: The valence nucleon jumps to a higher energy state in the shell model by absorbing excitation energy. This model agrees with experimental data for nuclei with one valence nucleon.

Experimental data which justify the orbital model for nucleons

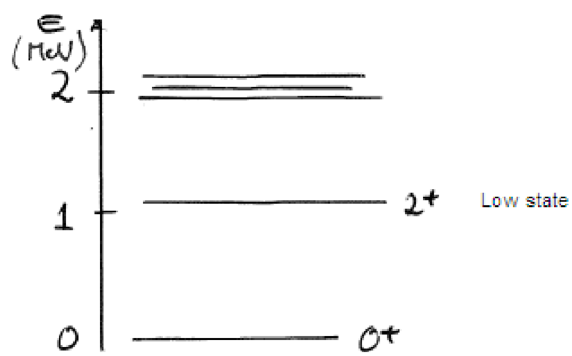
Electron-scattering experiments to find the charge-distribution difference between $^{206}_{82}Pb_{124}$ and $^{205}_{81}Tl_{124}$. The difference, $\Delta\rho_e$, takes place because Pb has one extra proton in a $3S_{\frac{1}{2}}$-state.
$\Rightarrow \Delta\rho_e$ corresponds to a $3s_{\frac{1}{2}}$-orbital.



Protons and neutrons are found as proton- and neutron-pairs in the shell structure. To excite a nucleon, one has to break a pair bond (typically 2MeV binding energy). Energy and spin is then found from the two odd nucleons. Coupling of the two angular momenta $\vec{j}_1 + \vec{j}_2$ gives values from $|j_1 + j_2|$ to $|j_1 - j_2|$.

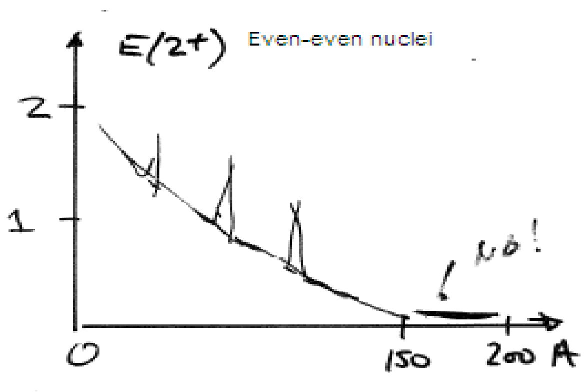
Collective structure contributions in even-even nuclei

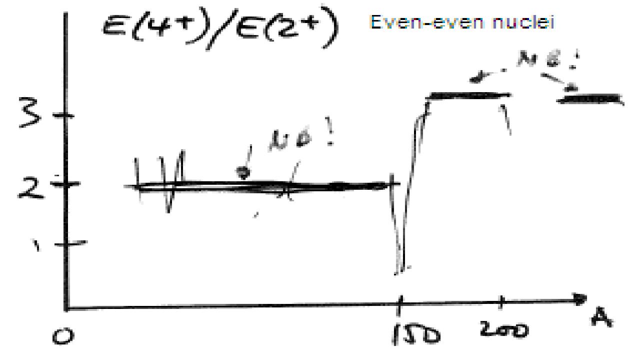
Experimentally:



All even-even nuclei have a low 2^+ excited state with excitation energy around half the energy required to separate a pair of nucleons, indicating another type of excited state than single nucleon excitation.

Experimental data:



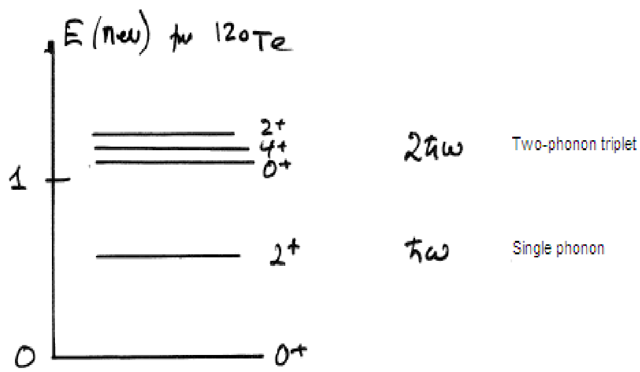


Nuclear vibrations(for $A < 150$)

The nuclear surface:

$$R(t) = R_{av} + \sum_{\lambda \geq 1} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}(\theta, \phi) \tag{2}$$

A nuclear quadrupole-moment corresponds to $Y_{20}(l = 2)$



Exited phonon states with equidistant energy levels $\Rightarrow E = n \cdot \hbar\omega$

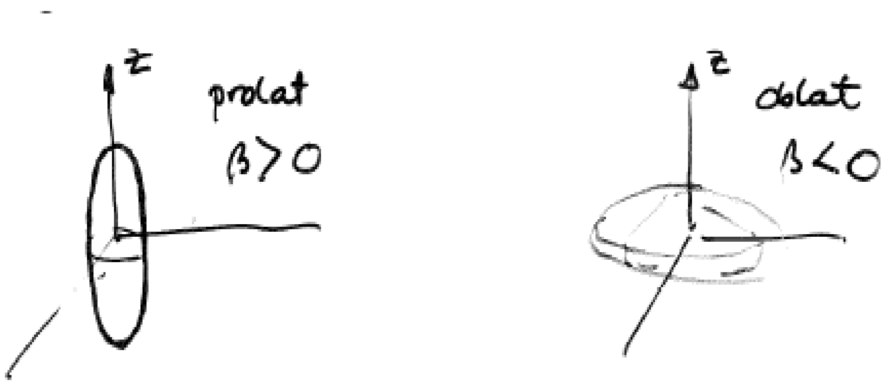
If the 4^+ state is due to a two-phonon excitation and 2^+ corresponds to a one-phonon excitation, one can easily draw the conclusion that $E(4^+)/E(2^+) = 2$. Experimental data for $A < 150$ confirms this model.

Rotating deformed nuclei $(150 < A < 190, A > 220)$

$$R(\theta,\phi) = R_0\big[1+\beta Y_{20}(\theta,\phi)\big] \tag{3}$$

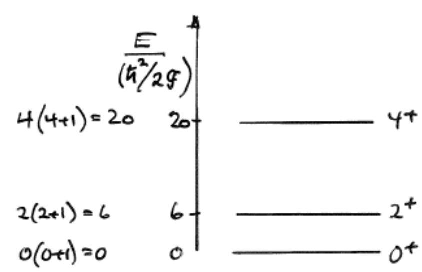
Deformation parameter: $\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}} \simeq \frac{\Delta R}{R_{av}}$

Intrinsic quadrupole moment, Q ,in the nucleus' rest frame: $Q_0 = \frac{3}{\sqrt{5\pi}} \cdot R_{av}^2 Z\beta(1+0.16\beta)$



A rotating $\underbrace{\textit{prolate}}_{Q_0>0}$ ellipsoid rotates perpendicular to the symmetry-axis $\Rightarrow Q < 0$.

Rotational states



$$E = \frac{\hbar^2}{2\Upsilon} I(I+1) \tag{4}$$

The ground state for even-even nuclei has a total angular momentum I=0, and superimposed rotational states have even spin due to symmetry. Υ is the effective nuclear mass moment of inertia. Deformed nuclei are found where Z&N take values far from magic numbers.

Super-deformation

The Schrødinger equation for deformed nuclei gives a new set of states. When deforming a nucleus $\simeq$ 2:1 prolate ellipsoid, a new shell structure arises $\Rightarrow$ super-deformed states.