5.)

Nuclear instability (Lilley Chap 3)

γ-radioactivity

Transitions

Isomeric transition (leaves Z and N unchanged) from an exited nuclear state:	${}^A_Z X^* \rightarrow {}^A_Z X + \gamma$
Conservation of energy:	$E_i = E_f + E_\gamma + T_R$
Conservation of momentum:	$0 = \vec{P}_R + \vec{P}_\gamma \Rightarrow P_R = P_\gamma = \frac{1}{c} E_\gamma$
$\Rightarrow$	$E_\gamma = \frac{\Delta E}{1 + \frac{\Delta E}{2 M_x c^2}} \simeq \Delta E (1 - \frac{\Delta E}{2 M_x c^2})$

Where, E_i and E_f represents the excitation energy in the initial and final states, $\Delta E = E_i - E_f$, and T_R is the recoil energy.

From the theory of classical electromagnetic radiation

Parity for multipole-field of order L:	$\pi(EL) = (-1)^L$, $\pi(ML) = (-1)^{L+1}$
Radiated power:	$P(\sigma L) = \frac{2(L+1)c}{\varepsilon_0 L[(2L+1)!!]^2} \left[\frac{\omega}{c}\right]^{2L+2} [m(\sigma L)]^2$

Where $(2L+1)!! \equiv (2L+1)(2L-1)(2L-3)....1$, $\sigma \in E, M$, and $m(\sigma L)$ is the time dependent multipole amplitude.

A quantum mechanical approach

Multipole moment: $M_{fi}(\sigma L) = \int \psi_f^* m(\sigma L) \psi_i d^3r$

Emitted power: $P(\sigma L) = T(\sigma L) \cdot \hbar\omega$

Emission rate: $T(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\hbar\varepsilon_0 L[(2L+1)!!]^2} \left[\frac{\omega}{c}\right]^{2L+1} B(\sigma L)$

Reduced transition probability: $B(\sigma L) = \left|M_{fi}\right|^2$

Single nucleon (SP) model

Multipole operator: $m(EL) \propto er^L Y_{LM}(\theta, \phi)$

$m(ML) \propto r^{L-1} Y_{LM}(\theta, \phi)$

Weisskopf sp-approximations: $B_{sp}(EL) = \frac{e^2}{4\pi} \left[\frac{3R^L}{L+3}\right]^2$

$B_{sp}(ML) = 10 \left[\frac{\hbar}{m_p c R}\right]^2 B_{sp}(EL)$

These approximations lead to: $T(E1) = 10^{14} A^{\frac{2}{3}} E_\gamma^3$

$T(M1) = 3.1 \cdot 10^{13} E_\gamma^3$

If $L \to L+1$: $T(L+1) \to 6 \cdot 10^{-7} A^{\frac{2}{3}} E_\gamma^2 \cdot T(L)$

Note:

1.) The lowest multipole transition has the highest transition probability

2.) For a given order, $T(EL) \simeq 100 \cdot T(ML)$

Selection rules

The photon is a S=1 Boson. The direction of this spin is either parallel or antiparallel to $\vec{p}_\gamma$. This spin cannot be coupled to $\vec{l} = \vec{r} \times \vec{p}_\gamma$ because $\vec{S} \perp \vec{l}$.

Conservation of angular momentum: $\vec{I}_i = \vec{I}_f + \vec{L}$

$$|I_i - I_f| \leq L \leq |I_i + I_f|, \quad L \neq 0$$

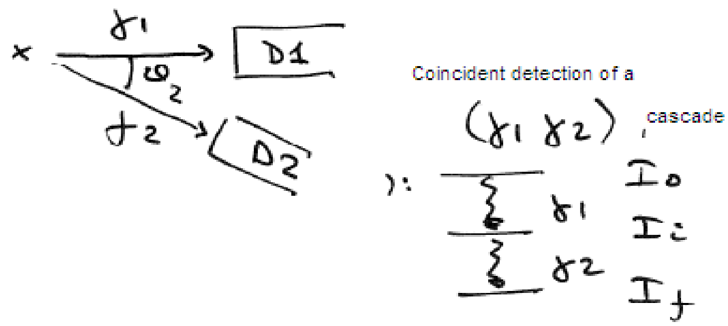
Now, if:

$\Delta\pi = 0$:	Even EL, odd ML $\Rightarrow$	M1, E2, M3....
$\Delta\pi \neq 0$	Odd, EL, even ML $\Rightarrow$	E1, M2, E3....

If I_i or $I_f = 0 \Rightarrow$ A particular value of $L \Rightarrow$ Pure multipole transition.
If $I_i = I_f = 0$ Forbidden transition for γ-transition, but an electron conversion is possible.

Experimental determination of multipole contribution

Generally, $|I_i - I_f| \leq L \leq |I_f + I_i|$ give several possible L-values. This means that L has to be determined experimentally. The easiest way to approach this problem is to find the angular-correlation:



Conversion electrons

The nucleus de-excites by interaction with an atomic electron (mainly S-orbital electrons) $\Rightarrow$ electron emission.

Conservation of energy:	$T_e = \Delta E - E_B$
Binding energy:	$E_B(K) > E_B(L) > E_B(M)....$
Transition probability per unit time:	$\lambda_{tot} = \lambda_\gamma + \lambda_e$
Conversion coeff.:	$\alpha = \frac{\lambda_e}{\lambda_\gamma} \quad \Rightarrow \quad \lambda_t = \lambda_\gamma(1+\alpha)$
	$\alpha = \alpha_K + \alpha_{LI} + \alpha_{LII} + \alpha_{LIII} + \alpha_M......$

Maximum conversion: K-shell electron conversion (n=1) for low-energy, high-polarity transitions ($E \ll 2m_ec^2$) in heavy nuclei ($\propto Z^3$). The difference between $\alpha(EL)$ and $\alpha(ML)$ can be used to determine the change of parity. $\alpha = \infty$ for $0^+ \rightarrow 0^+$ because L=0 is a forbidden γ-emission transition. The competition between conversion electrons and γ-emission is analogous to the process where Auger electrons and characteristic X-ray emission compete when a de-exitation of electronic energy-states takes place. ($K - L_I$ transition is optically forbidden).

β-Disintegration

There are 3 different processes concerning this topic: $\beta^-, \beta^+, \varepsilon$

β^--disintegration

$$^A_ZX \rightarrow ^A_{Z+1} X' + e^- + \overline{\nu}_e$$

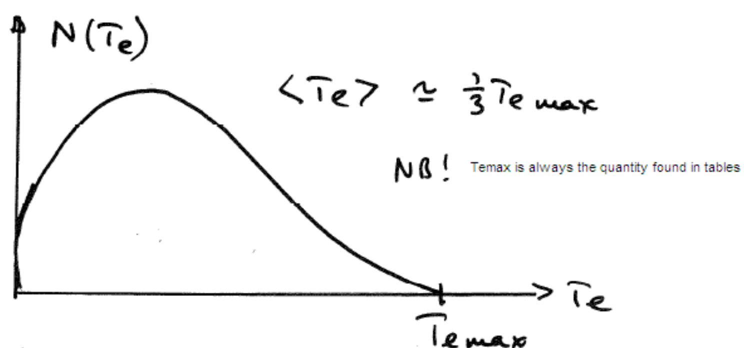
Energy released: $$Q_{\beta^-} = (m_P - m_D)c^2$$

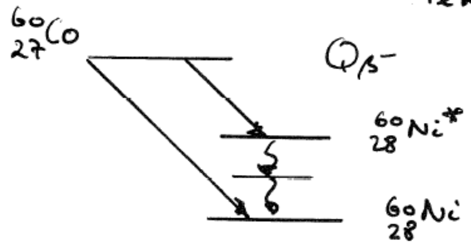
$$Q_{\beta^-} = (\Delta_P - \Delta_D)c^2$$

$$Q_{\beta^-} = T_{X'} + T_e + T_{\overline{\nu}_e}$$

Where $T_{X'}$, the recoil energy, is close to zero and $m_{\overline{\nu}_e} \simeq 0 \Rightarrow T_{\overline{\nu}_e} = E_{\overline{\nu}_e}$. This means that the energy released in the reaction can be written as below.

Energy released: $$Q_{\beta^-} = T_e + T_{\overline{\nu}_e} = T_{e,max} = T_{\overline{\nu}_e,max}$$





β^+-disintegration

$$ {}^A_Z X \rightarrow {}^A_{Z-1} X' + \beta^+ + \nu_e $$

Energy released: $Q_{\beta+} = (m_P - m_D - 2m_e)c^2$

$$Q_{\beta+} = (\Delta_P - \Delta_D - 2m_e)c^2$$

$$Q_{\beta+} = \underbrace{T_{X'}}_{\simeq 0} + T_{\beta_+} + \underbrace{T_{\nu_e}}_{\simeq E_{\nu_e}} \text{ because } m_{\nu_e} \simeq 0$$

$$Q_{\beta^-} = T_{\beta^+,max} = T_{\nu_e,max}$$

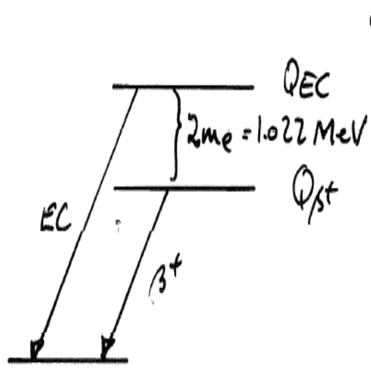
Electron capture (ε or EC)

$$ {}^A_Z X + e^- \rightarrow {}^A_{Z-1} X' + \nu_e $$

Released energy: $Q_{EC} = c^2(m_P - m_D) - E_B$

$$Q_{EC} = T'_X + T_{\nu_e}$$

The recoil energy, T'_X, is very small and can therefore in most cases be neglected. E_B is the binding energy for the captured electron's initial orbital. Since this is a two-body problem, the neutrino is emitted with well-defined energy and is therefore said to be mono-energetic.



Possible Q_{EC} values if $m_P \simeq m_D$: $E_B(K) > E_B(L) \Rightarrow \begin{cases} Q_{EC(K)} < 0 & \text{No transition} \\ Q_{EC(L)} > 0 & \text{Transition possible} \end{cases}$

Fermi theory for β-disintegration

Distinctive traits (in comparison to α-disintegration):

1.) The potential barrier is of no relevance ($m_e \ll m_\alpha$, and therefore $P(tunneling) \simeq 1$).

2.) An electron and an anti neutrino has to be created.

3.) A relativistic approach is necessary.

4.) "3-body problem" for $\beta^\pm$.

Fermi's golden rule: $\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E_f)$

Matrix element: $V_{fi} = g\int \psi_f^* V \psi_i d^3r$

Initial state: $\psi_i = \psi_{iN}$

Final state: $\psi_f = \psi_{fN}\phi_e\phi_{\bar{\nu}_e}$

Where g is a constant which characterizes the strength of the weak interactions.

Number of states: $n = \frac{pL}{h}$, for $x \in [0, L]$ and $p \in [0, p]$

$\Rightarrow$ $d^2n = dn_e dn_{\nu_e} = \frac{(4\pi)^2 V^2 p^2 dpq^2 dq}{h^6}$

Where p is the linear momentum of the electron and q that of the neutrino. For the electron and neutrino states, we use zero order approximations which give allowed transitions.

Electron state: $\phi_e(\vec{r}) = \frac{1}{\sqrt{V}}e^{i\frac{\vec{p}\cdot\vec{r}}{\hbar}} \simeq \frac{1}{\sqrt{V}}\left[1 + i\frac{\vec{p}\cdot\vec{r}}{\hbar} + ...\right] \simeq \frac{1}{\sqrt{V}}$

Neutrino state: $\phi_{\overline{\nu}_e}(\vec{r}) = \frac{1}{\sqrt{V}}e^{i\frac{\vec{q}\cdot\vec{r}}{\hbar}} \simeq \frac{1}{\sqrt{V}}\left[1 + i\frac{\vec{q}\cdot\vec{r}}{\hbar} + ...\right] \simeq \frac{1}{\sqrt{V}}$

Now, by inserting this into Fermi's golden rule one obtains:

Transition probability rate: $d\lambda(p) = \frac{2\pi}{\hbar}\left|g\int \psi^*_{fN}\phi^*_e\phi^*_{\overline{\nu}_e}O_x\psi_i d^3r\right|^2 \frac{(4\pi)^2V^2p^2dpq^2}{h^6}\frac{dq}{dE_f}$

Conservation of energy: $E_f = E_e + E_{\overline{\nu}_e} = E_e + qc$, assuming $M_{\overline{\nu}_e} \equiv 0$

$\Rightarrow$ $\frac{dE_f}{dq} = c$ for fixed E_e

Released energy: $Q = T_e + qc \Rightarrow q = \frac{Q-T_e}{c}$

Transition probability rate: $d\lambda(p) = \frac{2\pi}{\hbar}g^2\left|M_{fi}\right|^2(4\pi)^2\frac{p^2dpq^2}{h^6}\frac{1}{c}$

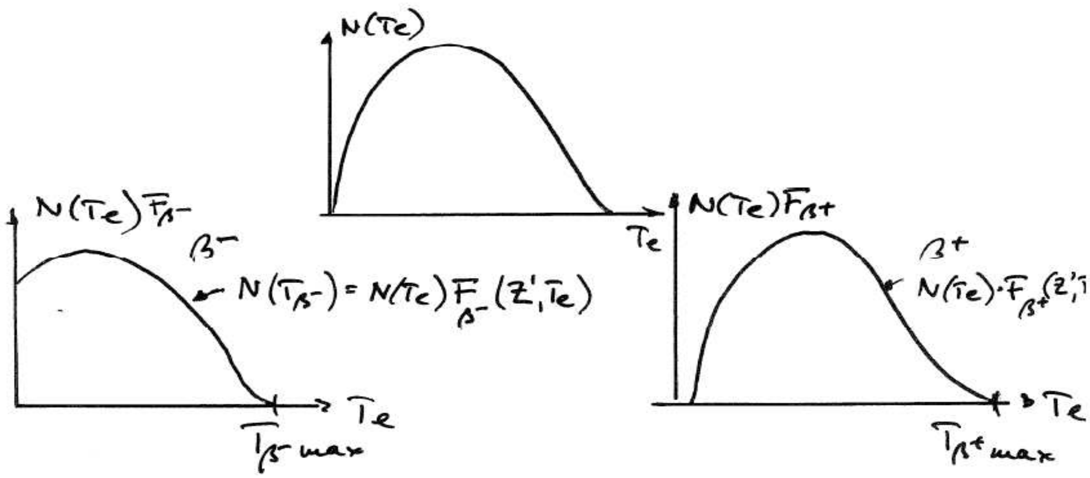
$$d\lambda(p) \propto N(p)dp = Cp^2q^2dp$$

Electron distribution: $N(p) = \frac{C}{c^2}p^2(Q-T_e)^2 = \frac{C}{c^2}p^2[Q - \sqrt{(pc)^2 + (mc^2)^2} + mc^2]^2$

$$N(p)dp = N(T_e)dT_e \Rightarrow \frac{dp}{dT_e} = \frac{1}{c^2p}(T_e + mc^2)$$

$\Rightarrow$ $N(T_e) = \frac{C}{c^5}\sqrt{T_e^2 + 2T_emc^2}(Q-T_e)^2(T_e+mc^2)$

The Fermi factor $F_{\beta^\pm}(Z', T_e)$ represents the Coulomb interactions with the nucleus:

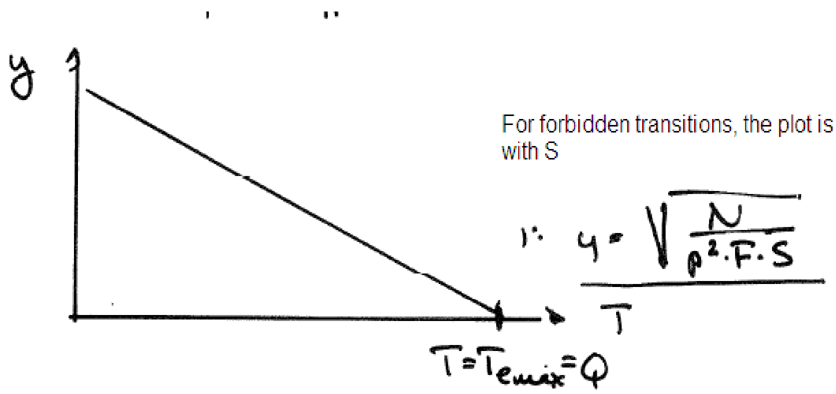


Electron distribution: $N(p) \propto p^2(Q-T_e)^2F(Z',p)\left|M_{fi}\right|^2S(p,q)$

Where the form factor $S(p,q) = \begin{cases} 1, & \text{for allowed transitions} \\ \neq 1, & \text{for forbidden transitions} \end{cases}$

Fermi-Curie-plot

$$y = \sqrt{\frac{N(p)}{p^2 F(Z', p)}} \propto (Q - T_e), \quad M_{fi} = constant$$



Total transition probability rate: $\lambda = \int_{p=0}^{p,max} d\lambda(p)$

The Fermi integral: $f = \frac{1}{(mc)^3} \frac{1}{(mc^2)^2} \int_0^{p,max} F(Z', p) p^2 (E_0 - E_e)^2 dp$

Conservation of energy: $E_0 - E_e = Q + mc^2 - (T_e + mc^2) = Q - T_e$

Comparable half-life: $ft_{\frac{1}{2}} = f \frac{\ln 2}{\lambda}$

$$ft_{\frac{1}{2}} = 0.693 \cdot \frac{2\pi^3 \hbar^7}{g^2 m_e^5 c^4 |M_{fi}|^2} \simeq 10^3 - 10^{20} s$$

For "super-allowed transitions:

$$log ft_{\frac{1}{2}} \in (3 - 4)$$

For $0^+ - 0^+$, $M_{fi} = \sqrt{2} \Rightarrow ft_{\frac{1}{2}}$-values for these transitions should be of equal magnitude. This corresponds with experiments performed. $log ft_{\frac{1}{2}}$ increases for increasing order of forbiddenness.

Selection rules

Conservation of angular momentum:	$\vec{I}_i = \vec{I}_f + \vec{L}_\beta + \vec{S}_\beta$
Parity:	$\pi_P = \pi_D(-1)^{L_\beta}$
Allowed transitions:	$\vec{L}_\beta = \vec{0}$
First forbidden:	$\vec{L}_\beta = \vec{1}$
Second forbidden:	$\vec{L}_\beta = \vec{2}$
Fermi transitions	$\vec{S} = \vec{0}$
Gamow-Teller transitions	$\vec{S} = \vec{1}$

Where $\vec{L}_\beta$ and $\vec{S}_\beta$ refer to the (β, ν) particle system.

1.) Allowed transitions:($\vec{L}_\beta = 0$, $\pi_P = \pi_D$)

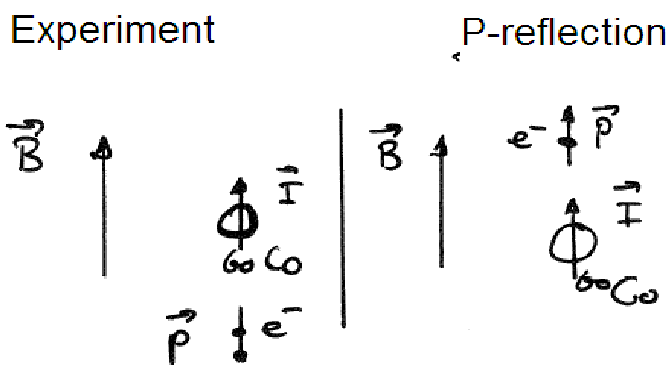
Fermi type: ($\vec{S} = \vec{0}$)	Gamow-Teller type: ($\vec{S} = \vec{1}$)
$\vec{I}_i = \vec{I}_f$	$\vec{I}_i = \vec{I}_f + \vec{1}$
$\Delta I = 0$	$\Delta I = 0, 1$; not $0^+ \to 0^+$
$0^+ \to 0^+$ Super-allowed.	$0^+ \to 1^+$ Pure Gamow-Teller.

2.) First forbidden transitions:($\vec{L}_\beta = \vec{1}, \pi_P = -\pi_D$)

Fermi type:($\vec{S} = \vec{0}$)	Gamow-Teller type:($\vec{S} = \vec{1}$)
$\vec{I}_i = \vec{I}_f + \vec{1}$	$\vec{I}_i = \vec{I}_f + \underbrace{\vec{1} + \vec{1}}_{\vec{0}, \vec{1}, \vec{2}}$
$\Delta I = 0, 1$	
	Three types:
	$\Delta I = 0$
	$\Delta I = 0, 1$
	$\Delta I = 0, 1, 2$

Violation of parity conservation during β-disintegration

When a physical law is invariant during a symmetry operation, there is a corresponding conserved quantity. Gravitation and electromagnetism are invariant during a spatial reflection (Parity operator P), charge (C) and time (T) $\Rightarrow$ Parity should be a conserved quantity.
$\Rightarrow < O_{PS} >= \int \Psi^* \hat{O}_{PS} \Psi d^3 r = 0$.
Where O_{PS} is an operator that is representing a pseudo-scalar quantity, for example $\vec{p} \cdot \vec{S}$, which is a product of a polar vector ($\vec{p}$) and an axial vector $\vec{S}$. $P(\vec{p}) = -\vec{p}$, $P(\vec{S}) = \vec{S}$. $< O_{PS} >= 0$ because the integrand is an odd function if parity is a conserved quantity.



The P-reflection experiment emits in the "forward" direction, while the original experiment emits backwards relative to $\vec{I}$. Wu et al. showed in 1957 that $< \vec{p} \cdot \vec{I} > < 0$ in this experiment, i.e. parity is not necessarily conserved in $\beta-$disintegration.

α-disintegration

α-disintegration takes place in nuclei with low $\frac{N}{P}$-ratio.

$$ {}^A_Z X \rightarrow {}^{A-4}_{Z-2} X' + \alpha $$

Energy released:

$$Q_\alpha = (m_P - m_D - m_{He})c^2 \text{ (atomic masses)}$$

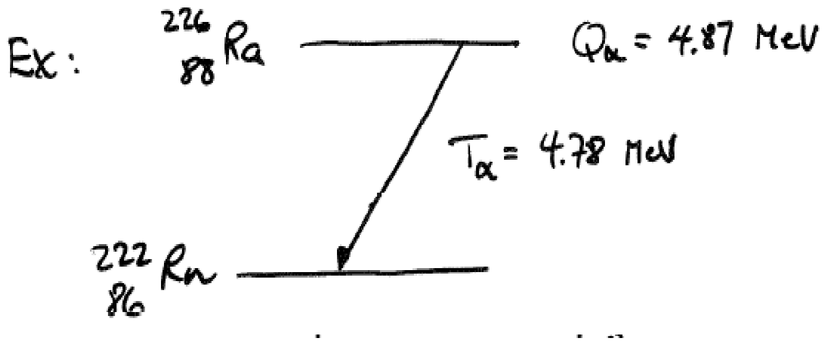
$$Q_\alpha = (\Delta_P - \Delta_D - \Delta_{He})c^2$$

$$Q_\alpha = T_{X'} + T_\alpha \text{(Assuming X is initially at rest)}$$

Conservation of momentum: $\vec{P}_{X'} + \vec{P}_\alpha = 0$

$\Rightarrow$ $T_\alpha = \frac{Q_\alpha}{1 + \frac{M_\alpha}{M_{X'}}}$

These α-energies are well defined, i.e monoenergetic, because this is a two-body problem.



Disintegration constant: $\lambda = f \cdot P \cdot A_\alpha^2$

Where f is the number of collisions with the potential barrier per second, P is the tunneling probability and A is the spectroscopical factor expressed below.

Spectroscopical factor: $A_\alpha^2 = \Big| < \Psi_f^*(A-4)\Psi_\alpha^*(4)|\Psi_i(A) > \Big|^2$

The physical interpretation of this spectroscopical factor is that it is the probability for creating an α-particle inside the nucleus.

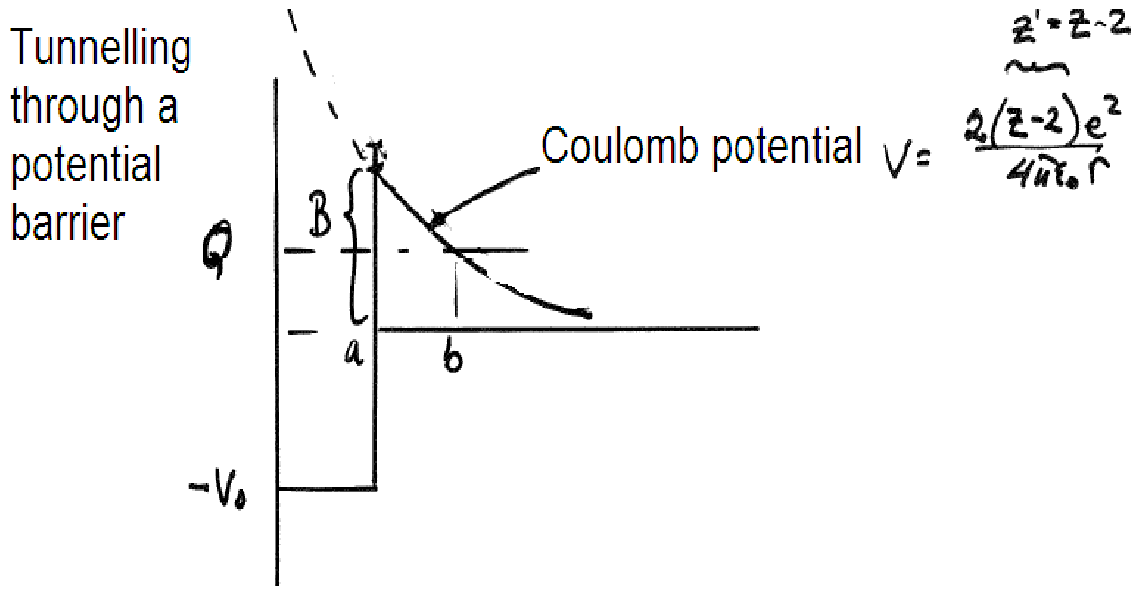
Gamow factor: $G = \int_a^b \sqrt{\frac{2m_\alpha}{\hbar^2}[V(r) - Q]}dr$

WKB-approximation solution: $G = \sqrt{\frac{2m_\alpha}{\hbar^2 Q}} \frac{zZ'e^2}{4\pi\varepsilon_0} \underbrace{\left[\arccos\sqrt{\frac{Q}{B}} - \sqrt{\frac{Q}{B}(1-\frac{Q}{B})}\right]}_{\simeq \frac{\pi}{2} - 2\sqrt{\frac{Q}{B}} \; for \; Q \ll B}$

Tunneling probability: $P = e^{-2G}$

Collision frequency: $f \simeq \frac{v}{a}$

Velocity : $v \simeq \sqrt{\frac{2(Q+V_0)}{m_\alpha c^2}} \cdot c$



Where v is the α-particle's velocity inside its nucleus-orbital, and a is the nuclear radius R. A_α^2 is assumed to be 1.

Geiger-Nuttals rule: $$t_{\frac{1}{2}} = 0.693\frac{a}{c}\sqrt{\frac{mc^2}{2(V_0+Q)}}exp\Big[2\sqrt{\frac{2mc^2}{(\hbar c)^2 Q}}\cdot\frac{zZ'e^2}{4\pi\varepsilon_0}(\frac{\pi}{2}-2\sqrt{\frac{Q}{B}})\Big]$$

This can again be simplified by introducing a few assumptions. $V_0 + Q \simeq V_0, \quad 2\sqrt{\frac{Q}{B}} \ll \frac{\pi}{2} \Rightarrow \lg t_{\frac{1}{2}} = C_1 + \frac{C_2}{\sqrt{Q}}$. See Lilley Fig.3.9.

Effects due to angular momenta

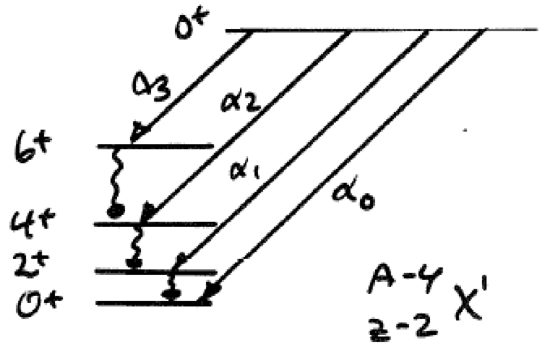
The centrifugal potential makes the potential barrier increase.

Selection rule: $\vec{I}_i = \vec{I}_f + \vec{l}_\alpha$

$|I_i - I_f| \leq l_\alpha \leq |I_f + I_i|$

Parity rule: $\pi_P = \pi_D(-1)^{l_\alpha}$

A typical example is a transition to rotational energy-states in deformed nuclei. $l_\alpha \in$even numbers because of symmetry and parity.



Deviation from Geiger-Nuttals rule:

1.) For deformed nuclei there is a higher probability for emitting through the poles,

because bigger $a(\equiv R) \Rightarrow$ lower potential barrier

2.) A_α^2 can be significantly ≤ 1, for example if the creation of an α-particle requires a break-up of nucleon bonds in filled shells.