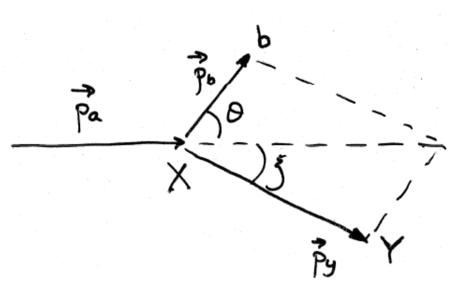
6.)
Nuclear reactions (Lilley Chap.4)

$$a + X \to Y + b$$
$$X(a, b)Y$$

Scattering process:	The particles do not change their identity.
Elastic scattering:	The kinetic energy is conserved $\Rightarrow$ no excitations.
Radiative capture:	$b \equiv \gamma$
Nuclear photo effect	$a \equiv \gamma$
Direct reactions:	Only a few nucleons participate in the process, while the rest of the nucleons remain passive.
"Compound nucleus" reactions:	An excited intermediate state is formed, and the memory of formation of this intermediate state is lost before de-excitation.
Conservation laws:	Total energy
	Total momentum
	Total angular momentum
	Proton numbers and neutron numbers (Not conserved in weak interactions)
	Parity

Process:



Conservation of energy:(relativistic) $m_X c^2 + T_X + m_a c^2 + T_a = m_Y c^2 + T_Y + m_b c^2 + T_b$

$$(m_a + m_X - m_Y - m_b)c^2 \equiv Q = T_Y + T_b - T_X - T_a$$

$$Q \equiv (m_{initial} - m_{final})c^2 = T_{final} - T_{initial}$$

If $Q < 0$, the reaction is called an endoterm reaction (requires an input of energy)
If $Q > 0$, the reaction is called an exoterm reaction (releases energy)

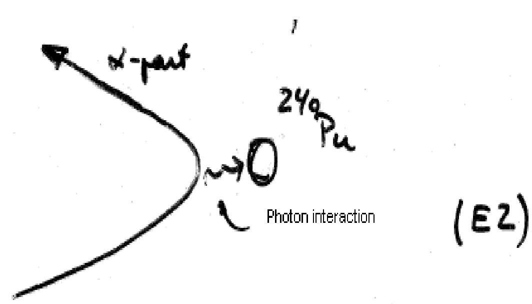
Conservation of momentum in the lab system: $p_a = p_b \cos\theta + p_Y \cos\xi$

$$0 = p_b \sin\theta - p_Y \sin\xi$$

Assuming $T_X = 0$. Furthermore, one defines the minimum energy required for the reaction to take place (Threshold energy), as the energy corresponding to a reaction where the final products are at rest in the CM system.

Threshold energy: $T_{th} = T_{a,min} = -Q\frac{m_Y + m_b}{(m_Y + m_b) - m_a}$

Inelastic Coulomb scattering (Coulomb excitation)



Inelastic Coulomb scattering:$Q_{ex} = (m_x + m_a - m_Y^* - m_b)c^2$ where $m_Y^* c^2 = m_Y c^2 + E_{ex}$ and $Q_{ex} = Q_0 - E_{ex}$.

Typical reaction:
Excitation of even-even nuclei from their ground state (0^+) to an excited state (2^+) via absorption/emission of virtual photons (E2).

$$Q_{ex} = Q_0 - E_{ex}$$

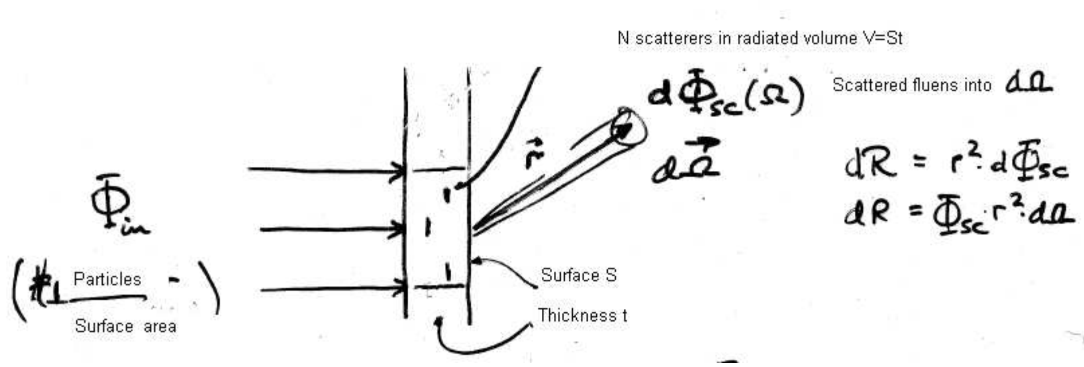
Nuclear force scattering(as opposed to Coulomb scattering)

$\Rightarrow$ Diffraction pattern in $\frac{d\sigma}{d\Omega}$ measured as a function of θ_{CM}

For neutron scattering: An evident diffraction pattern arise at all scattering angles (All energies)

For charged particles (protons): Diffraction pattern at high energies where the Coulomb potential is negligible, and for large scattering angles also at low energies.

Reaction cross section



Cross section contribution per "scatterer": $\sigma = \frac{1}{N}\frac{R_{sc}}{\Phi_{in}}$

Differential cross section: $\frac{d\sigma}{d\Omega} = \frac{\frac{dR_{sc}}{d\Omega}}{N\Phi_{in}}, [\frac{barn}{st.rad}]$

Total cross section: $\sigma = \int_\Omega \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int_0^\pi \frac{d\sigma}{d\Omega} \sin\theta d\theta$

Several reactions: $\sigma_{tot} = \sum_{b_i} \sigma_{b_i}$

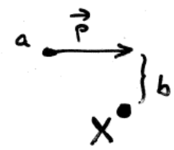
Energy dependence:

Double diff. cross section: $\frac{d^2\sigma}{d\Omega dE_b}$

$\frac{d\sigma}{dE_b}$

Where E_b represents the final energy of particle b.

Scattering and reaction cross sections



Semi-classical angular momentum: $l\hbar = pb$

$$b = \frac{l\hbar}{p} = \frac{l\hbar}{k\hbar} = l\frac{\lambda}{2\pi} = l\bar{\lambda}$$

For effective nuclear force scattering: $l_{max} = \frac{R}{\bar{\lambda}} = \frac{R_1+R_2}{\bar{\lambda}}$

Where $\bar{\lambda}$ represents the reduced de Broglie wavelength for particle a ($\lambda = h/p$).

Total semiclassical cross section:

$$\sigma = \sum_{l=0}^{\frac{R}{\bar{\lambda}}} (2l+1)\pi\bar{\lambda}^2 = \pi\left(R + \bar{\lambda}\right)^2$$

The particle's wave properties have a range $\bar{\lambda}$.

Quantum mechanically:

The wave function describing the incoming wave:

$$\Psi_{inc} = \frac{A}{2kr}\sum_{l=0}^{\infty} i^{l+1}\left(2l+1\right)\left[e^{-i\left(kr-\frac{l\pi}{2}\right)} - e^{i\left(kr-\frac{l\pi}{2}\right)}\right]P_l(\cos\theta)$$

Where the two exponential factors describe respectively an ingoing and an outgoing spherical wave. A superposition of the two waves results in an incoming plane wave.

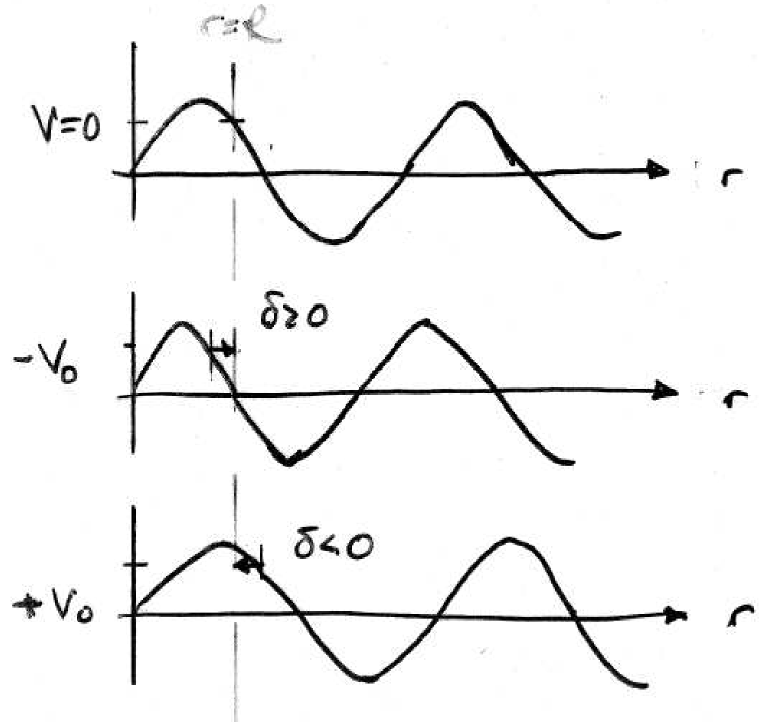
A scattered outgoing wave can have its phase and amplitude changed by the scattering process.

$$\Psi_{tot} = \Psi_{inc} + \Psi_{sc}$$

$$\Psi_{tot} = \frac{A}{2kr}\sum i^{l+1}\left(2l+1\right)\left[e^{-i\left[kr-\frac{l\pi}{2}\right]} - \eta e^{i\left[kr-\frac{l\pi}{2}\right]}\right]P_l(\cos\theta)$$

$$\Psi_{sc} = \frac{A}{2kr}\sum i^{i^{l+1}}\left(2l+1\right)\left(1-\eta_l\right)e^{i\left(kr-\frac{l\pi}{2}\right)}P_l(\cos\theta)$$

$$\Psi_{sc} = \frac{A}{2k}\frac{e^{ikr}}{r}\sum_{l=0}^{\infty}(2l+1)i(1-\eta_l)P_l(\cos\theta)$$



Scattered current density: $j_{sc} = \left(\Psi^*_{sc} \frac{\hbar}{im} \nabla \Psi_{sc}\right)$

$$= \frac{\hbar}{2im}\left(\Psi^*_{sc}\frac{\partial \Psi_{sc}}{\partial r} - \frac{\partial \Psi^*_{sc}}{\partial r}\Psi_{sc}\right)$$

$$j_{sc} = |A|^2 \frac{\hbar}{2mkr^2} \left|\sum_{l=0} (2l+1)\, i\, (1-\eta_l)\, P_l(\cos\theta)\right|^2$$

Incoming current density: $j_{inc} = \frac{\hbar k |A|^2}{m}$

Differential cross section: $\frac{j_{sc} r^2 d\Omega}{j_{inc}}$

$$\Rightarrow \frac{d\sigma_{sc}}{d\Omega} = \frac{1}{4k^2}\left|\sum_{l=0}^{\infty} (2l+1)\, i\, (1-\eta_l)\, P_l(\cos\theta)\right|^2$$

The total cross section is obtained by integrating over all possible angles.

Orthogonality requires: $\int P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta d\phi = \frac{4\pi}{2l+1}$ for $l = l'$

$$\int P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta d\phi = 0 \text{ for } l \neq l'$$

$$\Rightarrow \sigma_{sc} = \sum_{l=0}^{\infty} \pi \lambdabar^2 (2l+1)\, |1-\eta_l|^2, \quad \lambdabar = \frac{1}{k}$$

There is no scattering for $\eta_l = 1$. Only elastic scattering, i.e only a phase change and no reduction in amplitude is possible for $|\eta_l| = 1 \rightarrow \eta_l = e^{2i\delta_l}$

Total cross section: $\sigma_{sc} = \sum_{l=0}^{\infty} 4\pi \lambdabar^2 (2l+1) \sin^2 \delta_l, \quad |1 - e^{2i\delta_l}| = 2 \sin\delta_l$

Reaction cross section ($\equiv$ cross section concerning everything else than elastic interactions)

This is also denoted as the rate of loss of particles from energy channel k.

Rate of loss: $|j_{loss}| = |j_{in}| - |j_{out}|$

$$|j_{loss}| = \frac{|A|^2\hbar}{4mkr^2}\left[\left|\sum (2l+1)\, i^{l+1} e^{i\frac{l\pi}{2}} P_l\right|^2 - \left|\sum (2l+1)\, i^{l+1} e^{-i\frac{l\pi}{2}} \eta_l P_l\right|^2\right]$$

$$\Rightarrow \sigma_r = \sum_{l=0}^{\infty} \pi \lambda\!\!\!^{-}{}^2 \,(2l+1)\left(1 - |\eta_l|^2\right)$$

Total cross section: $\sigma_t = \sigma_{sc} + \sigma_r = \sum_{l=0}^{\infty} 2\pi \lambda\!\!\!^{-}{}^2 \,(2l+1)\,(1 - \Re\eta_l)$

Note that only inelastic scattering ($\sigma_r > 0, \sigma_{sc} = 0$) is impossible to achieve. To obtain inelastic scattering, $|\eta_l| < 1$. When this happens, $(1 - \eta_l) \neq 0$, *i.e.* $\sigma_{sc} > 0$.

"Black disc" absorber:

$\eta_l = 0$ for $l \leq \frac{R}{\lambda\!\!\!^{-}}$ i.e no outgoing wave for $l \leq \frac{R}{\lambda\!\!\!^{-}}$

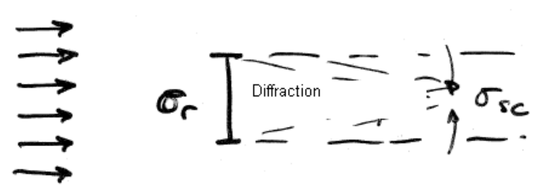
$\eta_l = 1$ for $l > \frac{R}{\lambda\!\!\!^{-}}$ i.e no scattering effect

Reaction cross section: $\sigma_r = \sum_{l=0}^{\frac{R}{\lambda\!\!\!^{-}}} \pi \lambda\!\!\!^{-}{}^2\,(2l+1) = \pi(R+\lambda\!\!\!^{-})^2$

Scattering: $\sigma_{sc} = \sum_{l=0}^{\frac{R}{\lambda\!\!\!^{-}}} \pi \lambda\!\!\!^{-}{}^2\,(2l+1) = \pi(R+\lambda\!\!\!^{-})^2$

Total: $\sigma_t = \sigma_r + \sigma_{sc} = 2\pi(R+\lambda\!\!\!^{-})^2 = 2 \cdot \sigma_{geometrical}$

$\sigma_{geometrical}$ is the semiclassical cross section.



Calculation method

1.) Choose a form of the nuclear potential $V(r)$.

2.) Solve the Schr. equation for the two regions, inside ($r \leq R$) and outside ($r \geq R$) the region of interaction.

3.) Ψ and $\frac{\partial \Psi}{\partial r}$ must be continuous over the boundary $r = R \Rightarrow \eta_l$

4.) Calculate σ_r and σ_{sc} and compare with experimental results. This result tells us whether $V(r)$ was reasonably chosen.

This is hard for everything else than elastic scattering, because all inelastic channels are coupled together. Both *in* and *out* scattering relative to channel k, i.e from all k' into k and from k to all k''.

Optical model of nuclear scattering:

Choose a particular potential as a model for elastic scattering + absorption.

Potential: $U(r) = \underbrace{V(r)}_{\textit{Elastic scattering}} + \underbrace{iW(r)}_{\textit{Absorption}}$

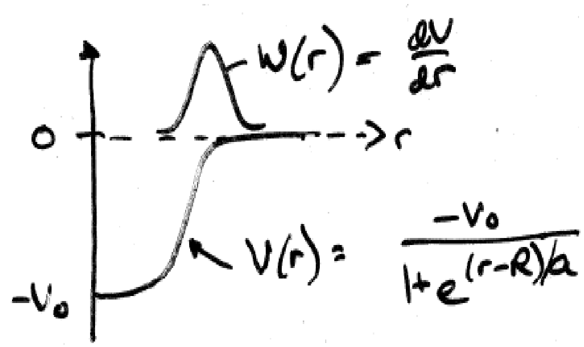
$k = \frac{1}{\hbar}\sqrt{2m(E-U)}$

Choose for example: $U(r) = -V_0 - iW_0$ for $r < R$

$= 0$ for $r > R$

Outgoing wave: $\Psi = \frac{e^{ikr}}{r} = e^{ik_r \cdot r} \cdot \frac{e^{-k_i \cdot r}}{r}$ for $r < R$

$$k = k_r + ik_i = \frac{1}{\hbar}\sqrt{2m(E+V_0)} + i\frac{W_0}{2\hbar}\sqrt{\frac{2m}{E+V_0}}, \quad W_0 \ll V_0$$



The only place where $W(r) \neq 0$ is close to the surface. This is because the internal nucleons cannot take part in absorption processes at moderate energies, because all the possible states are taken. This means that only the valence nucleons close to the surface can interact with incoming particles.

A realistic potential must also include spin-orbit coupling for valence nucleons, and Coulomb contribution if the incoming particle is charged. The optical model gives suprisingly good predictions (by calculating η_l) to experimental data, even though it only represents average nucleon properties. This model can only show that particles disappear from the elastic channel.

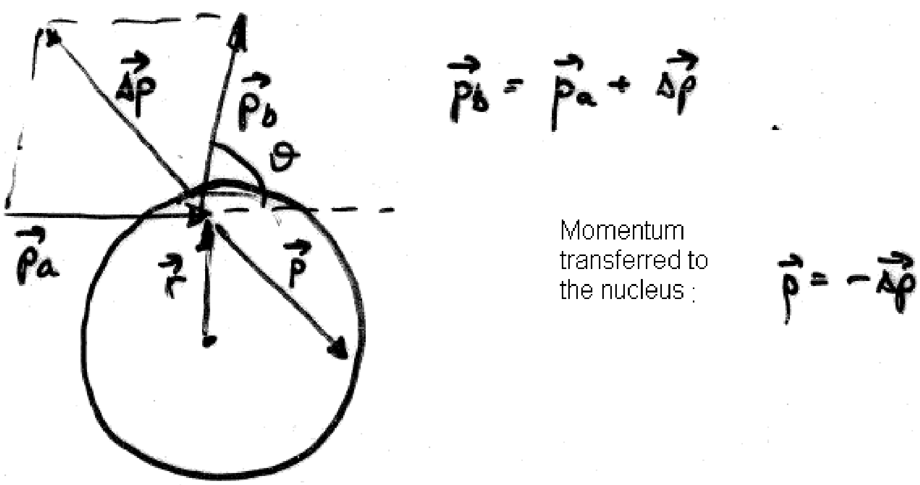
Direct reactions

An incoming particle interacts with single nucleons close to the surface of the nucleus. Typical incoming energies≥Coulomb barrier. Direct reactions show strong angular dependencies.

Selectivity:

Inelastic scattering reactions do not excite collective states. Transfer reactions result in excited states for single nucleons.

Ex.: Transfer of angular momentum by deuteron stripping reactions (d,n), (d,p)



$$p^2 = p_a^2 + p_b^2 - 2p_a p_b \cos\theta$$

$$l \cdot \hbar \simeq R \cdot p \Rightarrow l = \left[\frac{2c^2 p_a p_b (2\sin^2 \frac{\theta}{2})}{\frac{(\hbar c)^2}{R^2}} \right]^{\frac{1}{2}}$$

Large scattering angles for outgoing particle = large transfer of angular momentum, $l \propto \sin\frac{\theta}{2}$.

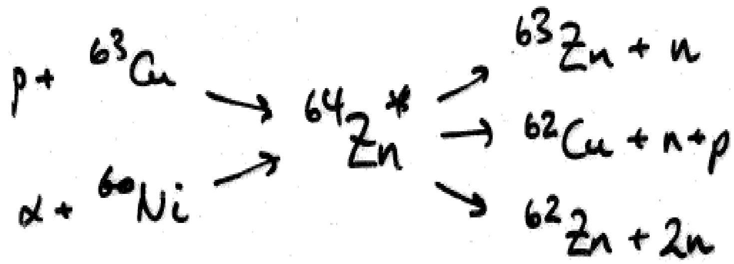
$l = 1, 3, 5....$(odd numbers)$\Rightarrow$ parity change for the nucleus
$l = 0, 2, 4...$(even numbers)$\Rightarrow$ leaves the parity unchanged

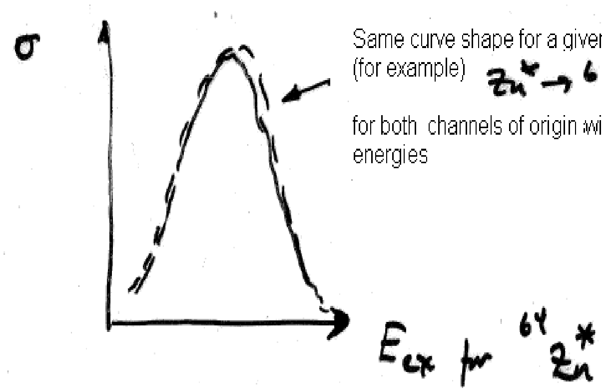
Nuclear spin: $I_f = I_i + l \pm \underbrace{\frac{1}{2}}_{n\ or\ p}$

Compound reactions

$$a + x \rightarrow C^* \rightarrow Y + b$$

Well defined intermediate state (Compound nucleus) with a lifetime long enough that the final reaction, $C* \rightarrow Y + b$, has forgotten (i.e. is not influenced by) how C^* was created.

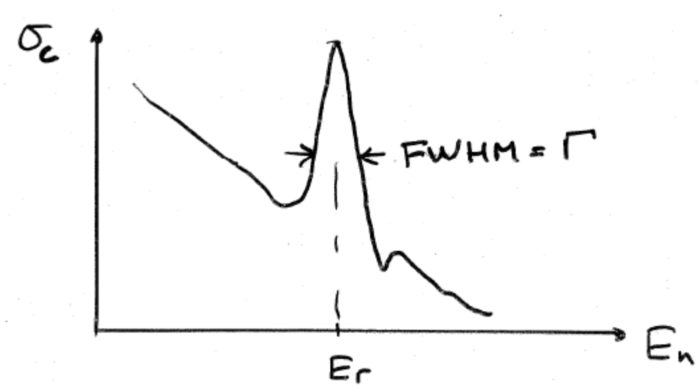




Same curve shape for a given outcome,
(for example) $Zn^* \rightarrow {}^{63}Zn + n$
for both channels of origin with equal excitation energies

Resonance reactions

They appear at well defined excitation levels for C^*



Example: Resonance peak in neutron capture cross section

Breit-Wigner formula:

$$\sigma_{\alpha,\beta} = g_\alpha(J)\frac{\pi}{k_\alpha^2}\frac{\Gamma_\alpha\Gamma_\beta}{(E-E_r)+(\frac{\Gamma}{2})^2}$$

$$\sigma_{\alpha\beta}(E = E_r \pm \frac{\Gamma}{2}) = \frac{1}{2}\sigma_{\alpha\beta}(E = E_r)$$

Spin factor: $g_\alpha(J) = \frac{2J+1}{(2i_a+1)(2i_A+1)}$

where i_a and i_A represent spin for the incoming particles.

$$g_\alpha = (2l+1) \text{ for } i_\alpha = i_A = 0$$

Generally: $\vec{I}_{C^*} = \vec{i_a} + \vec{l}_A + \vec{l}$

in this case, $\vec{l}$ represents the transferred angular momentum by (a, A).

Resonance level width: $\Gamma = \sum_{i\in\alpha,\beta}\Gamma_i = \hbar\sum\lambda_i = \hbar\lambda = \frac{\hbar}{\tau}$

τ is the mean lifetime of the intermediate state C^*.

Assuming $i_\alpha = i_A = 0$:

Maximum cross section for elastic scattering (At $E = E_r$): $\Gamma_\alpha \equiv \Gamma_\beta = \Gamma \Rightarrow \sigma_{\alpha\alpha}(max) = (2l+1)\frac{4\pi}{k_\alpha^2}$

Total absorption cross section: $\sigma_{abs} \propto \Gamma_\alpha\sum_{\beta\neq\alpha}\Gamma_\beta = \Gamma_\alpha(\Gamma - \Gamma_\alpha)$

$\Rightarrow \sigma_{abs}(max) = (2l+1)\frac{\pi}{k_\alpha^2}$

$\Gamma_\alpha(1-\Gamma_\alpha)_{max}$ for $\Gamma_\alpha = \frac{\Gamma}{2}$

Total capture cross section at $(E = E_\tau)$: $\sigma_C = (2l+1)\cdot\frac{4\pi}{k_\alpha^2}\frac{\Gamma_\alpha}{\Gamma}$

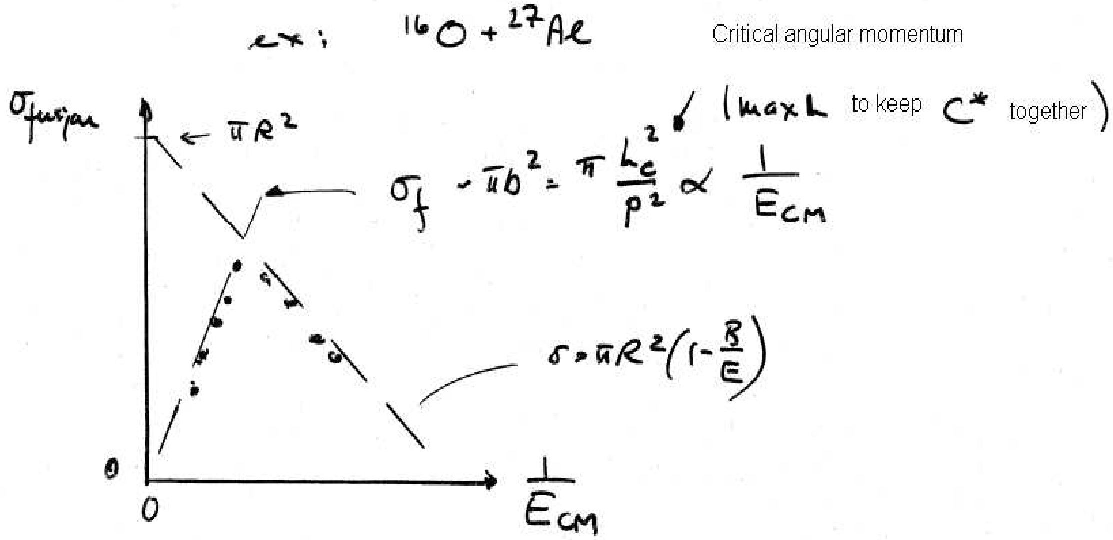
$$\sigma_{\alpha\beta} = \sigma_C\cdot\frac{\Gamma_\beta}{\Gamma} = \underbrace{(2l+1)\frac{4\pi}{k_\alpha^2}\frac{\Gamma_\alpha}{\Gamma}}_{\sigma_C}\cdot\underbrace{\frac{\Gamma_\beta}{\Gamma}}_{\textit{Exit channel }\beta}$$

Heavy ion reactions

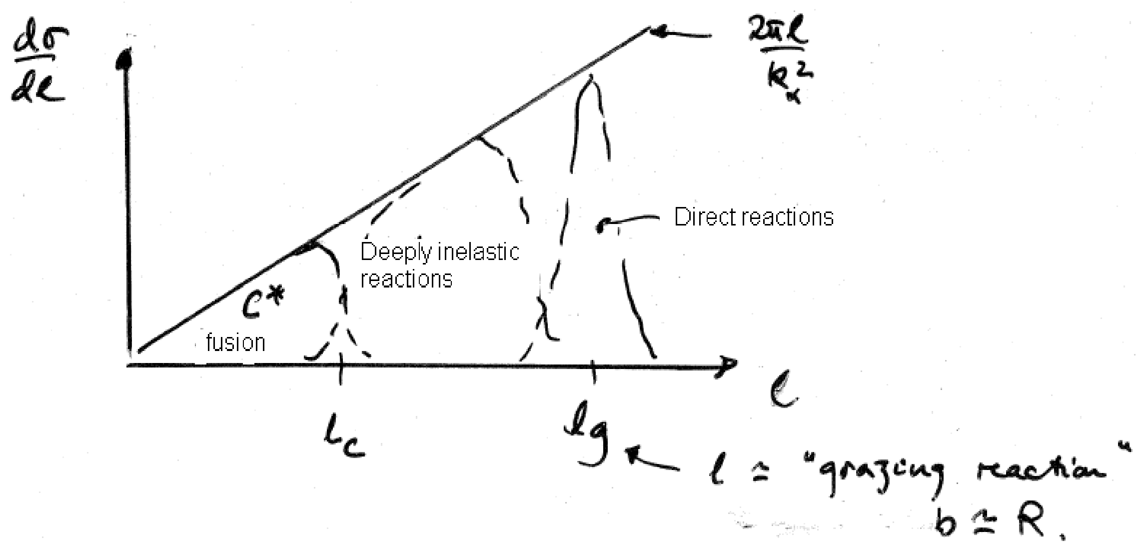
Ex:

$$^{16}O + ^{27}Al$$

Fusion



ex: $^{16}O + ^{27}Al$
Critical angular momentum
(max L to keep C* together)
σ_{fusion}
$\leftarrow \pi R^2$
$\sigma_f = \pi b^2 = \pi \frac{L_c^2}{p^2} \propto \frac{1}{E_{CM}}$
$\sigma = \pi R^2 (1 - \frac{B}{E})$
0
0
$\frac{1}{E_{CM}}$



$\frac{d\sigma}{dl}$
$\frac{2\pi l}{k_\alpha^2}$
Direct reactions
Deeply inelastic reactions
C*
fusion
l
l_c
l_g
$l \simeq$ "grazing reaction"
$b \simeq R$.